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COLBURN'S SERIES.—PART III.

THE

COMMON-SCHOOL ARITHMETIC:

A Practical Treatise

ON

THE SCIENCE OF NUMBERS.

BY

DANA P. COLBURN,

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PREFACE.

The title of this work indicates its character. It has been prepared for a text-book in Common Schools, and is designed to be a thoroughly practical treatise. It contains an exposition of the leading principles of Arithmetic, and illustrates their most important practical applications.

Its lessons are arranged in what seems to the author a natural and logical order; and their mutual dependencies and connections are, as far as possible, indicated. Its exercises and problems are numerous and varied, requiring, it is believed, such practice as will secure skill in performing numerical operations, and readiness in applying them.

The author has endeavored to illustrate clearly or demonstrate rigidly the truth of every principle and process; to make every definition and every statement clear and accurate; to avoid arbitrary rules and processes; and, in short, to prepare a work which shall be adapted to the wants and circumstances of pupils in Common Schools, and which shall at the same time tend to elevate the standard of elementary mathematical instruction.

Some subjects usually treated in School Arithmetics are omitted in this, and others of great practical importance are made very full and complete. Among the former are "Single and Double Position," "Circulating Decimals," "General Average," "Tonnage of Vessels," and "Permutations and Combinations"—subjects which are usually learned arbitrarily, if at all, and which, to the great mass of pupils, will never be of the slightest practical value. Among the latter are "Numeration," and the "Ground Rules," "Accounts," "Fractions," "Interest," and Problems pertaining to business life. The articles on "Bills," "Accounts," "Promissory Notes," "Orders," "Drafts," etc. will be found specially valuable.

The author claims for this, as for the other books of his series, that whatever be its merits or defects, it is the result of much careful thought and study, of considerable experience as a teacher, and of an honest effort to arrange such a course of lessons as shall tend to develop the youthful mind, and form correct habits of study.

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THE

COMMON-SCHOOL ARITHMETIC.

SECTION I.

1. Preliminary Definitions.

(a.) Anything which has value or size, is a quantity; or—

(b.) Quantity is whatever may be increased, diminished, or measured.

(c.) Every quantity is either a unit, or composed of units.

(d.) A unit is a single thing, or one.

Units may be either concrete or abstract. A concrete unit is any quantity which may be considered by itself, and made the measure of other similar quantities; as, *an apple, a foot, a dozen of eggs.* An abstract unit is *unity* or *one,* without reference to any particular kind of object or quantity.

(e.) Numbers are used to show how many units there are in any given quantity.

1. Numbers may be either concrete or abstract. A concrete number expresses concrete units; as, *five books, seven bushels.* An abstract number expresses abstract units; as, *four, eight, twelve.*

2. Numbers may be either simple or compound. A simple number expresses values in terms of a single denomination, as in *pounds,* in *shillings,* or in *pence.* All *abstract* numbers are simple. A compound number expresses values in terms of different denominations, as in pounds, shillings, *and* pence.

3. Numbers may be entire or fractional. An entire number involves only entire units. A fractional number either is a fraction or contains one.

4. Numbers may be either composite or prime. A composite number is one which has other factors besides itself and unity. A prime number is one which has no factors except itself and unity.

(f.) **ARITHMETIC IS THE SCIENCE OF NUMBERS AND THE ART OF NUMERICAL COMPUTATION.**

As a *science*, Arithmetic treats of the nature, the uses, the properties, and the relations of numbers. As an *art*, it includes all numerical operations, as *counting*, *adding*, and *multiplying*.

NOTE.—Arithmetic is a department of the science of MATHEMATICS. Everything which treats of quantity belongs to Mathematics. Indeed, Mathematics is the science of quantity.

2. Numerical Operations.

(a.) We may perform the following operations on numbers.

1st. We may COUNT, *i. e.* we may find how many units there are in any given quantity, by noting them one by one.

ILLUSTRATION.—One ball, two balls, three balls.

2d. We may ADD numbers, *i. e.* we may find how many units there are in two or more numbers considered together.

ILLUSTRATION.—In "five and four are nine," *five* is *added* to *four*.

3d. We may SUBTRACT one number from another, *i. e.* we may find how many units there are in the difference between two numbers.

ILLUSTRATION.—In "six from twelve leaves six," *six* is *subtracted* from *twelve*.

4th. We may MULTIPLY one number by another, *i. e.* we may find how many units there are in any number of times a number.

ILLUSTRATION.—In "eight times five are forty," *five* is *multiplied* by *eight*.

5th. We may DIVIDE one number by another, *i. e.* we may find how many times one number contains another.

ILLUSTRATION.—In "seven is contained three times in twenty-one," or "twenty-one equals three times seven," *twenty-one* is *divided* by *seven*.

6th. We may find some FRACTIONAL PART of a quantity or number, as "one-half of an apple," "one-fourth of eight." This requires the use of FRACTIONS.

7th. We may REDUCE numbers, *i. e.* we may change their form or denomination without changing their value.

3. Mathematical Signs.

(a.) Characters, or signs, are often used to indicate numerical operations and relations. Among them are the following:

1st. The sign of *Equality* (=) is two parallel lines. It is read EQUALS, or IS EQUAL TO, and signifies that the quantities between which it is placed are equal to each other.

ILLUSTRATION.—"4 pecks = 1 bushel," is read, "*Four pecks equal one bushel,*" or, "*Four pecks are equal to one bushel.*"

2d. The sign of *Addition* (+) is a horizontal line crossing a vertical one. It is read AND, or PLUS, and indicates that the quantities between which it is placed are to be added together.

ILLUSTRATION.—"$4 + 7 = 11$," is read, "*Four and seven are eleven,*" or, "*Four plus seven equals eleven.*"

3d. The sign of *Subtraction* (—) is a single horizontal line. It is read MINUS, or LESS, and indicates that the number *which follows it* is to be subtracted.

ILLUSTRATION.—"$7 - 4 = 3$," is read, "*Seven minus four equals three,*" or, "*Seven less four equals three.*"

4th. The sign of *Multiplication* (×) is a cross formed by two oblique lines. It is read TIMES, or MULTIPLIED BY, and indicates that the quantities between which it is placed are to be multiplied together.

ILLUSTRATION.—"$3 \times 4 = 12$," is read, "*Three times four are twelve,*" or, "*Three multiplied by four equals twelve.*"

5th. The sign of *Division* (÷) is a horizontal line with a dot above and a dot below it. It is read DIVIDED BY, and indicates that the number which precedes it is to be divided by that which follows it.

ILLUSTRATION.—"$8 \div 2 = 4$," is read, "*Eight divided by two equals four.*"

6th. Division may also be expressed by writing the number to be divided over the one by which it is to be divided, and drawing a line between them.

ILLUSTRATION.—$\frac{12}{3}$ is sometimes read, "*Twelve divided by three,*" but more commonly "*twelve-thirds,*" in which case it is regarded as a FRACTION. See 74.

SECTION II.

NOTATION AND NUMERATION.

4. Methods of Representing Numbers.

(a.) NOTATION and NUMERATION treat of the various methods of representing and expressing numbers.

(b.) Numbers may be expressed by visible objects or marks, by words, and by letters or other written characters.

(c.) Numbers are usually represented to the eye by characters called FIGURES, though sometimes by LETTERS OF THE ALPHABET.

(d.) The method by figures is called the ARABIC METHOD, because it was introduced into Europe by the Arabs.

(e.) The method by letters is called the ROMAN METHOD, because it was used by the ancient Romans.

5. Primitive Numbers and the Figures.

(a.) The first ten numbers are called the PRIMITIVE NUMBERS because in the DECIMAL SYSTEM OF NUMBERS, which is universally used, all other numbers are derived from them.

(b.) Their names are: *one, two, three, four, five, six, seven, eight, nine, ten.*

(c.) The following figures are used in writing numbers:

1, or *1*, called *one.*
2, or *2*, called *two.*
3, or *3*, called *three.*
4, or *4*, called *four.*
5 or *5*, called *five.*

6, or *6*, called *six.*
7, or *7*, called *seven.*
8, or *8*, called *eight.*
9, or *9*, called *nine.*
0, or *0*, called *zero*, or *cipher.*

The number *ten* is written 10, or *10*

6. The Derived or Higher Numbers.

(a) 1 + 10 = eleven = 11.
2 + 10 = twelve = 12.
3 + 10 = thirteen = 13.
4 + 10 = fourteen = 14.
5 + 10 = fifteen = 15.
6 + 10 = sixteen = 16.
7 + 10 = seventeen = 17.
8 + 10 = eighteen = 18.
9 + 10 = nineteen = 19.
10 + 10 = twenty = 20.

(b.) 1 ten = ten = 10.
2 tens = twenty = 20.
3 tens = thirty = 30.
4 tens = forty = 40.
5 tens = fifty = 50.
6 tens = sixty = 60.
7 tens = seventy = 70.
8 tens = eighty = 80.
9 tens = ninety = 90.
10 tens = one hundred = 100.

(c.) 20 + 1 = twenty-one = 21.
20 + 2 = twenty-two = 22.
20 + 3 = twenty-three = 23.
90 + 1 = ninety-one = 91.
90 + 8 = ninety-eight = 98.
90 + 9 = ninety-nine = 99.

(d.) Read the following numbers:

1. 27.
2. 58.
3. 26.
4. 95.
5. 37.
6. 51.
7. 70.
8. 36.

(e.) Analyze each of the above numbers.

MODEL.—Twenty-seven equals two tens and seven units; fifty-eight equals five tens and eight units, etc.

7. The Decimal Point and Decimal Places.

(a.) The dot or point (like a period) at the right of each of the above numbers, is called the DECIMAL POINT.

(b.) The first figure at the left of the decimal point is in the UNITS' PLACE, and represents units; the second figure at the left of the point is in the TENS' PLACE, and represents tens; the third figure at the left of the point is in the HUNDREDS' PLACE, and represents hundreds.

(c.) When the right-hand figure of a number is in the units' place, the decimal point is usually omitted.

(d.) The method of writing and reading numbers containing hundreds, tens, and units, may be seen below.

100 = 1 hundred.
200 = 2 hundred.
300 = 3 hundred.
287 = 2 hundred and 87.
509 = 5 hundred and 9.
480 = 4 hundred and 80.

(e.) Read each of the following numbers, and tell its denominations:

Model.—518 = five hundred and eighteen = five hundreds, one ten, and eight units.

1. 518.
2. 185.
3. 851.
4. 204.
5. 402.
6. 420.
7. 240.
8. 576.
9. 894.
10. 307.
11. 490.
12. 036.

(f.) Explain the use of the figures in the above numbers.

Model.—In 518, the 8 marks the units' place, and shows that there are 8 units; the 1 marks the tens' place, and shows that there is 1 ten; the 5 marks the hundreds' place, and shows that there are 5 hundreds.

(g.) Give the value of each figure in the above numbers.

Model.—In 518, the 8 = 8 units; the 1 = 1 ten = 10 units; the 5 = 5 hundreds = 50 tens = 500 units.

(h.) Give the value of each of the above numbers in tens and units.

Model.—518 = 51 tens and 8 units; 185 = 18 tens and 5 units.

Note.—Such exercises as the foregoing are of great value. The student who masters them will not be likely to find any serious difficulty in reading and understanding larger numbers, and decimal fractions.

(i.) Write each of the following numbers in figures:

1. Five hundred and eighty-nine.
2. Eight hundred and ninety-five.
3. Nine hundred and fifty-eight.
4. Six hundred and seven.
5. Seven hundred and six.
6. Eight hundred and eighty.
7. Eight hundred and eight.
8. Four hundred and sixty-one
9. Nine hundred and two.
10. Two hundred and ninety.

8. Higher Denominations and Places.

(a.) As 10 units = 1 ten, and 10 tens = 1 hundred, so 10 hundreds = 1 thousand, 10 thousands = 1 ten-thousand, etc.

(b.) The first figure at the left of the decimal point is in the units' place; the second figure is in the tens' place; and the third in the hundreds' place; and, in like manner, the fourth figure is in the thousands' place; the fifth in the ten-thousands' place, etc.; as illustrated in the following table:

Hundred-trillions.	0	15th place
Ten-trillions.	0	14th place
Trillions.	0,	13th place
Hundred-billions.	0	12th place
Ten-billions.	0	11th place
Billions.	0,	10th place
Hundred-millions.	0	9th place
Ten-millions.	0	8th place
Millions.	0,	7th place
Hundred-thousands.	0	6th place
Ten-thousands.	0	5th place
Thousands.	0,	4th place
Hundreds.	0	3d place
Tens.	0	2d place
Units.	0	1st place
Decimal Point.	•	

(c.) Name the above places in their order, from the point towards the left, and also from the left towards the point.

(d.) Large numbers can be most easily read by dividing the figures representing them into sets or periods of three figures each, beginning at the right.

(e.) The first period is called the UNITS' PERIOD, the second the THOUSANDS' PERIOD, the third the MILLIONS' PERIOD, etc. as is illustrated below:

8th, or Sextillions' Period.	7th, or Quintillions' Period.	6th, or Quadrillions' Period.	5th, or Trillions' Period.	4th, or Billions' Period.	3d, or Millions' Period.	2d, or Thousands' Period.	1st, or Units' Period.
0 0 0,	0 0 0,	0 0 0,	0 0 0,	0 0 0,	0 0 0,	0 0 0,	0 0 0.

(f.) Name the above periods in their order from right to left, and also from left to right.

(g.) Learn the following table:

1000 units = 1 thousand.
1000 thousands = 1 million.
1000 millions = 1 billion.
1000 billions = 1 trillion.
1000 trillions = 1 quadrillion.

NOTE.—The English usually divide numbers into periods of six figures each, thus making a billion equal a million millions, a trillion equal a million billions, etc. This method is now rarely used in this country.

9. Method of Reading Numbers.

(a.) To read a number expressed by figures, divide it into periods; and, beginning at the left, read the figures of each period as though they were followed by the name of the period.

ILLUSTRATION.—The number 34,068,250,006,789 is read as if written 34 trillion, 68 billion, 250 million, 6 thousand and 789.

(b.) Read each of the following numbers:

1. 4237.
2. 2406.
3. 3028.
4. 5870.
5. 9002.
6. 3209.
7. 5050.
8. 25050.
9. 625050.
10. 50625.
11. 500625.
12. 625,500.
13. 807,807.
14. 536,296.
15. 400,004.
16. 527,008.
17. 620,060.
18. 247,329.
19. 8,798,654.
20. 27,948,698.
21. 34,258,697.
22. 428,637,596.
23. 206,206,206.
24. 300,300,300.
25. 3,003,003.
26. 429,476,854.
27. 200,002,020.
28. 297,648,697,649.
29. 423,869,794,769,247.
30. 23,023,023,023,023.
31. 7,000,000,684,000.
32. 9,000,009,009.
33. 427,684,547,237,427,168.
34. 20,020,020,020.
35. 429,429,429,429,429,429.
36. 620,537,007,716,090,067,005.

10. Method of Writing Numbers.

(a.) To express numbers by figures, write in each successive period the figures which show how many there are of the denomination of that period, taking care to fill each place, which is otherwise unoccupied, by a zero. The left-hand period may contain one, two, or three figures, but each of the others must contain three.

(b.) Write each of the following numbers in figures:

1. Nine thousand six hundred and fifty-eight
2. Twenty-nine thousand, six hundred and fifty-eight.
3. Eight hundred and twenty-nine thousand, six hundred and fifty-eight.

4. Three hundred and two thousand, three hundred and two.

5. Six thousand and sixty.

6. Ninety thousand, three hundred and eighty-seven.

7. Seventy thousand and seventy.

8. Four hundred and eighty-six thousand, five hundred and eighty-six.

9. Twenty-seven million, one hundred and eleven thousand, seven hundred and twelve.

10. Eight million, one hundred and seventy-six thousand, three hundred.

11. Five hundred and sixty million, three hundred and fourteen thousand, two hundred and forty-three.

12. Two million, two thousand, and seven.

13. Twenty million, twenty thousand, and seventy.

14. Two hundred million, two hundred thousand, two hundred.

15. Seventy-five million, twenty-five thousand, and twenty-four.

16. Two hundred and fifty million, two hundred and forty thousand, two hundred and forty.

17. Four million and five.

18. Forty-six million and forty-six.

19. Four hundred and sixty-nine million, four hundred and sixty-nine.

20. Eighty-seven billion, nine hundred and forty-three million, two hundred and seventy-eight thousand, four hundred and thirty-two.

21. Six hundred and thirty billion, six hundred and thirty.

22. Ninety billion, ninety million, ninety thousand, and ninety.

23. Four hundred billion, four million, four hundred thousand and four.

24. Eight hundred and seventy-eight trillion, nine hundred and forty-three billion, seven hundred and thirty-six million, four hundred and nine thousand, eight hundred and thirteen.

25. Six hundred and thirteen quintillion, four hundred and sixty quadrillion, two hundred and three trillion, seven billion, sixty-eight thousand, and one.

11. Places at the Right of the Point.

(a.) The foregoing exercises and illustrations show that—

The figure occupying any place represents ten times the value it would represent if it stood one place farther to the right, and one-tenth of the value it would represent if it stood one place farther to the left; one hundred times the value it would represent if it stood two places farther to the right, and one hundredth of the value it would represent if it stood two places farther to the left, etc. etc.

(b.) These principles extended would make the first figure at the right of the point represent *tenths;* the second, *hundredths;* the third, *thousandths*, etc. as illustrated below:

Thousands.	Hundreds.	Tens.	Units.	Point.	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.	Ten-millionths.	Hundred-millionths.	Billionths.
0	0	0	0	.	0	0	0	0	0	0	0	0	0

1. What are the denominations of 37.684?

ANSWER.—3 tens, 7 units, 6 tenths, 8 hundredths, and 4 thousandths.

NOTE.—The pupil should be very careful, in reading and speaking, to distinguish between *tens* and *tenths, hundreds* and *hundredths*, etc.

Name the denominations of—

2. 4.26
3. 15.79
4. .368
5. .4276
6. .30492
7. .246873
8. 1.5867843
9. .006793645
10. .0006237

(c.) Numbers thus expressed by figures at the right of the point are called DECIMAL FRACTIONS.

They are so called because their denominator is always 10, 100, 1000, or some other power of ten—the word *"decimal"* being derived from the Latin word *"decem,"* which means ten. When the denominator is written beneath the numerator, they have the form of VULGAR FRACTIONS.

12. Method of Reading Decimal Fractions.

(a.) Numbers containing decimal fractions may be read by first reading the figures at the left of the point *as though they stood alone*, and then reading the figures at the right of the point *as though the name of the place occupied by the right-hand figure were written after them.*

ILLUSTRATIONS.—37.28 = 37 and 28 hundredths; .0067846 = 67846 ten-millionths; 6000.006 = 6000 and 6 thousandths.

(b.) Read each of the following:

1. 5.75
2. 28.61
3. .437
4. .03
5. 20.42
6. 37.06
7. .4986
8. 5.029
9. 4.36784
10. .259687
11. 23.86943
12. 14.379
13. 3.07954
14. .00386
15. 400.004
16. 305.000305
17. .67243867
18. 49.5958
19. .05958
20. .005958
21. .0005958
22. 24.03702
23. 60000.00006
24. 200.0000005

(c.) Numbers containing decimal fractions may also be read as if the point were omitted, and the name of the place occupied by the right-hand figure written after them.

ILLUSTRATIONS.—37.28 = 3728 hundredths; 429.678 = 429678 thousandths; 6000.006 = 6000006 thousandths.

(d.) Read the numbers under b, by the last method.

NOTE.—By this method they are read as IMPROPER FRACTIONS (see 76).

13. Method of Writing Decimal Fractions.

(a.) To represent decimal fractions by figures, write them as if they were whole numbers, and then mark the decimal point so as to bring the right-hand figure in the place bearing the name of the denominator of the given fraction.

(b.) There will always be as many places at the right of the point as there would be zeroes used in writing the denominator of the given fraction. When there are not figures enough for this in the numerator, zeroes must be prefixed to supply the deficiency.

ILLUSTRATIONS.—To write 479 hundredths, write 479 and place the point so that the 9 shall come in the hundredths' place; thus, 4.79. To write 6 hundred, and 47 millionths, write 600 and the point, and then write the 47 so that the 7 shall come in the millionths' place, which will require 4 zeroes between the point and the 47; thus, 600.000047.

(c.) Write each of the following numbers:

1. Six thousandths.
2. Forty-five hundredths.
3. Eight, and nine-tenths.
4. Eighty-nine tenths.
5. Eighty-nine hundredths.
6. Six, and five hundredths.
7. Twenty-nine, and four hundred and thirty-six thousandths.
8. Sixty-five ten-thousandths.
9. Eight hundred and thirteen tenths.
10. Eight hundred and thirteen hundredths.
11. Eight hundred and thirteen thousandths.
12. Sixty-one, and forty-two thousandths.
13. Five hundred and twenty-seven thousands, and five hundred and twenty-seven thousandths.
14. Nine hundred and fourteen million, two hundred and thirteen thousand, seven hundred and eight, and six hundred and eleven thousand, two hundred and sixty-eight millionths.
15. Seventy-one billionths.
16. Six thousand, and five tenths.
17. One hundred and forty-two thousand four hundred and forty-eight ten-thousandths.
18. Seventy million, and nine millionths.
19. Eighty-nine million, eighty-nine thousand and eighty-nine, and eighty-nine million, eighty-nine thousand and eighty-nine billionths.
20. Three million, and three-hundred-thousandths.

14. Multiplication and Division by 10, 100, 1000, etc.

(a.) A consideration of the foregoing principles shows that removing the figures of any number one place to the left, or the decimal point one place to the right, multiplies the number by 10; while removing the figures one place to the right, or the point one place to the left, divides the number by 10. A similar change of two places would multiply or divide by 100; a change of three places would multiply or divide by 1000, etc.

Illustrations.—Removing the point in 43.28 one place to the right, gives 432.8 = 10 times 43.28; while removing the point one place to the left, gives 4.328 = .1 of 43.28.

Removing the point in 2.36 three places to the right, leaves one place to be filled with a zero, or gives 2360 = 1000 times 2.36; while removing it three places to the left, leaves two places to be filled with zeroes, or gives .00236 = .001 of 2.36.

(b.) Hence, 1st, *To multiply a number by* 10, 100, 1000, *or any other power* of ten, remove the decimal point as many places to the right as there are zeroes used in writing the given multiplier.*

2d. *To divide a number by* 10, 100, 1000, *or any other power of ten, remove the decimal point as many places to the left as there are zeroes used in writing the given divisor.*

3d. *When by such change any places are left vacant between the number and the point, they should be filled by zeroes.*

Illustrations.—67 × 100 = 6700.
6.7 × 100 = 670.
.67 × 100 = 67.

67 ÷ 100 = .67
6.7 ÷ 100 = .067
.67 ÷ 100 = .0067

Note.—These operations are very important. If thoroughly mastered, they will remove in advance most of the difficulties connected with decimal fractions.

1. 36 × 10.
2. 423 × 100.
3. 62.7 × 10.
4. 539.6 × 100.
5. 247 ÷ 10.
6. 3697 ÷ 100.
7. 4.28 ÷ 1000.
8. 3 ÷ 100.
9. .04 × 100.
10. .7279 × 10.
11. 6597 × 1000.
12. .29 ÷ 100.
13. 67.3 ÷ 10.
14. 42.79 × 10.
15. .0006 ÷ 10.
16. 75000 × 100.
17. .0039 × 100.
18. 57.897 × 10.
19. 87 × 1000000.
20. 87 ÷ 1000000.

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* For definition of the word *power*, see 68.

15. Roman Method of Notation.

a.) The Roman method of notation represents numbers by letters of the alphabet.

(b.) The letters used are I, V, X, L, C, D, and M.

(c.) The letter I stands for one; V for five; X for ten; L for fifty; C for one hundred; D for five hundred; and M for one thousand.

(d.) If a letter is repeated, it indicates that the number for which it stands is to be repeated.

Illustrations.—I = 1, II = 2, III = 3, X = 10, XX = 20, XXX = 30. Letters are rarely repeated in this way more than three times.

(e.) If a letter representing a number stands before a letter representing a larger number, the former is to be subtracted from the latter; but if the letter representing the larger number be placed first, the values of the two are to be added together.

Illustrations.—IV = 1 from 5 = 4, XL = 10 from 50 = 40, etc.; while VI = 5 + 1 = 6, LX = 50 + 10 = 60, etc.

I. = 1.
II. = 2.
III. = 3
IV. = 4.
V. = 5.
VI. = 6.
VII. = 7.
VIII. = 8.
IX. = 9.
X. = 10.
XI. = 11.
XII. = 12.
XIII. = 13.
XIV. = 14.
XV. = 15.
XVI. = 16.
XVII. = 17.
XVIII. = 18.
XIX. = 19.
XX. = 20.
XXI. = 21.
XXII. = 22.
XXIX. = 29.
XXXVI. = 36.
XLIX. = 49.
XCVIII. = 98.
CCXLIX. = 249.
MDCCLXXVI. = 1776.
MDCCCXXXIII. = 1833.
MDCCCLVII. = 1857.

(f.) A dash placed over a number makes it represent thousands instead of ones.

Illustrations.—$\overline{\text{V}}$ = 5000 $\overline{\text{XI}}$ = 11 thousands; $\overline{\text{MDCCXLII}}$DCXXI = 1742621.

(g.) Read the following:

1. CCXLIX.
2. XLIV.
3. CCCLXXXVIII.
4. MDCXX.
5. $\overline{\mathrm{MCV}}\mathrm{MCV}$.
6. $\overline{\mathrm{XCIII}}\mathrm{XCIII}$.

SECTION III.

TABLES OF MONEYS, WEIGHTS, AND MEASURES.

16. United States Money.

(a.) The money used in the United States is called UNITED STATES MONEY, or FEDERAL MONEY.

TABLE.

10 mills = 1 cent.
10 cents = 1 dime.
10 dimes = 1 dollar.
10 dollars = 1 eagle.

(b.) The coins of the United States are — the CENT; the THREE-CENT PIECE; the HALF-DIME, worth 5 cents; the DIME, worth 10 cents; the QUARTER-DOLLAR, worth 25 cents; the HALF-DOLLAR, worth 50 cents; the DOLLAR, worth 100 cents; the THREE-DOLLAR PIECE; the EAGLE, worth 10 dollars; the DOUBLE-EAGLE, worth 20 dollars; the HALF-EAGLE, worth 5 dollars; the QUARTER-EAGLE, worth 2½ dollars; and the FIFTY-DOLLAR PIECE.

(c.) These coins are made of gold, silver, copper, and nickel. Those worth more than a dollar are made of gold, and those worth less than a dollar are made of silver, with the exception of the cent, which is either made of copper, or of copper and nickel. The dollar is coined both of gold and of silver, though most frequently of gold.

(d.) The character, $, placed at the left of figures, shows that they represent dollars, or values in United States money.

(e.) Values in Federal money are usually expressed in dollars and cents; or, in dollars, cents, and mills. The dollar is the unit; and, as dimes, cents, and mills are, respectively, tenths, hundredths, and thousandths of a dollar, the figures representing them are written as decimal fractions.

ILLUSTRATIONS.—$4.27 = 4 dollars and 27 cents; or 427 cents = 4 dollars, 2 dimes, and 7 cents.

$93.068 = 93 dollars, 6 cents, and 8 mills; or, 93068 mills.

$50.203 = 50 dollars, 20 cents, 3 mills = 50203 mills, etc.

(f.) Read each of the following:

1. $9.23
2. $ .09
3. $17.286
4. $40.043
5. $50.50
6. $ .007

17. English or Sterling Money.

(a.) This money is used in England.

FULL TABLE.	ABBREVIATED.
4 farthings = 1 penny.	4 far. or 4 qr. = 1*d.*
12 pence = 1 shilling.	12*d.* = 1*s.*
20 shillings = 1 pound.	20*s.* = 1£.

(b.) The coin which represents a pound is called a SOVEREIGN. Among the English coins are also the GUINEA, worth 21 shillings, and the CROWN, worth 5 shillings. The SOVEREIGN usually passes, in the United States, for $4.84, though it varies from $4.83 to $4.86.

(c.) We can form an approximate estimate of values in English money, by regarding the pound as *about* 5 dollars, the shilling as *about* 25 cents, and the penny as *about* 2 cents. An income of £200 per year is very nearly equivalent to an income of $1000 per year; a man who owes £500 owes not far from $2500, etc. etc.

(d.) The term SHILLING, as used in New England, New York, and some other parts of the Union, does not refer to the English shilling. A New York shilling is 12½ cents, while a New England shilling is 16⅔ cents. The NINEPENCE of New England equals the shilling of New York. These terms, thus used, are very inconvenient; and will, it is hoped, soon be entirely superseded by the decimal currency of the United States

18. Avoirdupois Weight.

(a.) Iron, flour, sugar, wool, coal, and almost all articles, except gold, silver, and jewels, are weighed by AVOIRDUPOIS WEIGHT.

FULL TABLE.	ABBREVIATED.
16 drams = 1 ounce.	16 dr. = 1 oz.
16 ounces = 1 pound.	16 oz. = 1 lb.
25 pounds = 1 quarter.	25 lb. = 1 qr.
4 quarters = 1 hundred-weight.	4 qr. = 1 cwt.
20 hundred-weight = 1 ton.	20 cwt. = 1 T.

(b.) Formerly, the quarter was reckoned at 28 pounds, the hundredweight at 112 pounds, and the ton at 2240 pounds; and they are thus reckoned, at the present time, in Great Britain, and at the United States' custom-house. Merchants, however, most frequently buy and sell by the SHORT TON, or the ton of 2000 pounds.

19. Troy Weight.

(a.) TROY WEIGHT is used in weighing gold, silver, and precious stones, and also in philosophical experiments. It is made the standard of weights.

FULL TABLE.	ABBREVIATED.
24 grains = 1 pennyweight.	24 gr. = 1 dwt.
20 pennyweights = 1 ounce.	20 dwt. = 1 oz.
12 ounces = 1 pound.	12 oz. = 1 lb.

20. Apothecaries' Weight.

(a.) This weight is used only in compounding and mixing medicines, Avoirdupois Weight being used in buying and selling them.

FULL TABLE.	ABBREVIATED.
20 grains = 1 scruple.	20 gr. = 1℈.
3 scruples = 1 dram.	3℈ = 1ʒ.
8 drams = 1 ounce.	8ʒ = 1℥
12 ounces = 1 pound.	12℥ = 1℔.

21. Comparison of Weights.

(a.) A pound Troy is equal to a pound Apothecaries', but is smaller than a pound Avoirdupois; while an ounce Troy is equal to an ounce Apothecaries', and is greater than an ounce Avoirdupois.

(b.) The following table shows the value, in *Troy grains*, of a unit of each denomination named in the last three tables, and thus furnishes ready means of comparing the different weights with each other.

TABLE OF COMPARISON.

1 lb. Avoirdupois = 7000 grains Troy.
1 lb. Troy = 1 ℔. = 5760 grains Troy.
1 oz. Avoirdupois = $437\frac{1}{2}$ grains Troy.
1 oz. Troy = 1℥ = 480 grains Troy.
1 dr. Avoirdupois = $27\frac{11}{32}$ grains Troy.
1ʒ = 60 grains Troy.
1℈ = 20 grains Troy.
1 dwt. = 24 grains Troy.
1 gr. Apothecaries' = 1 grain Troy.

(c.) It follows from the above, that—

144 lb. Avoirdupois = 175 lb. Troy.
192 oz. " = 175 oz. Troy.
1 lb. " = $\frac{175}{144}$ of 1 lb. Troy.
1 oz. " = $\frac{175}{192}$ of 1 oz. Troy.

22. Long Measure.

(a.) LONG MEASURE is used for measuring lengths and distances

FULL TABLE.	ABBREVIATED.
12 lines = 1 inch.	12 l. = 1 in.
12 inches = 1 foot.	12 in. = 1 ft.
3 feet = 1 yard.	3 ft. = 1 yd.
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet } = 1 rod or pole.	$5\frac{1}{2}$ yd. or $16\frac{1}{2}$ ft. } = 1 rd.
40 rods = 1 furlong.	40 rd. = 1 fur.
8 furlongs = 1 mile.	8 fur. = 1 m.
3 miles = 1 league	3 m = 1 le.

(b.) Surveyors usually measure distances by means of a chain 4 rods in length, called GUNTER'S CHAIN, or the SURVEYOR'S CHAIN. This chain contains 100 equal links; 25 links will, therefore, equal 1 rod, and 1 link will equal 7.92 inches.

23. Cloth Measure.

(a.) CLOTH MEASURE is used for measuring cloths, silks, etc.

FULL TABLE.	ABBREVIATED.
2¼ inches = 1 nail.	2¼ in. = 1 na.
4 nails = 1 quarter.	4 na. = 1 qr.
4 quarters = 1 yard.	4 qr. = 1 yd.

(b.) The yard and inch are the same in length as the yard and inch in Long Measure.

24. Square Measure.

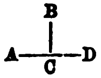
(a.) SQUARE MEASURE is used in measuring land, and all kinds of surfaces.

(b.) *Preliminary Definitions.* — An ANGLE is the difference in direction of two lines. The point where the lines meet is called the VERTEX OF THE ANGLE.

(c.) When the two angles formed by one straight line meeting another *are equal to each other*, they are RIGHT ANGLES.

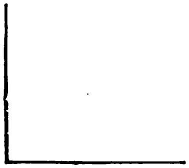
(d.) One line is PERPENDICULAR to another when it makes right angles with it.

The angle A C B is equal to the angle B C D, and hence they are right angles. Therefore, B C is perpendicular to A D.

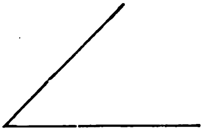


(e.) An angle greater than a right angle is called an OBTUSE ANGLE, and an angle less than a right angle is called an ACUTE ANGLE.

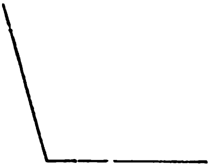
A RIGHT ANGLE.



AN ACUTE ANGLE.



AN OBTUSE ANGLE.

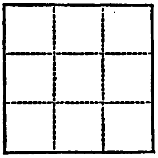


(f.) A four-sided figure, having all its angles right angles, is called a RECTANGLE.

(g.) A rectangle, having all its sides equal, is called a SQUARE. A SQUARE, then, has four equal sides, and four equal angles.

(h.) A SQUARE FOOT is a square, 1 ft. or 12 in. long, and 1 ft. or 12 in. wide, and hence contains 12 times 12, or 144 sq. in. In like manner, a SQUARE YARD is a square measuring 1 yd., or 3 ft., on each side, and hence contains 3 times 3, or 9 sq. ft.

This figure represents a square yard.



(i.) To measure a surface, we find how many squares of a given size it equals. Thus, a surface contains 5 sq. ft. when it is equivalent to 5 squares, each measuring 1 foot on a side.

(j.) Any rectangular surface contains as many square units as there are in the product obtained by multiplying its length by its breadth.

Table of Square Measure.

FULL TABLE.	ABBREVIATED.
144 square inches = 1 square foot.	144 sq. in. = 1 sq. ft.
9 square feet = 1 square yard.	9 sq. ft. = 1 sq. yd.
30¼ square yards, or 272¼ square feet = 1 square rod.	30¼ sq. yd. or 272¼ sq. ft. = 1 sq. rd.
40 square rods = 1 rood.	40 sq. rd. = 1 R.
4 roods = 1 acre.	4 R. = 1 A.
640 acres = 1 square mile.	640 A. = 1 sq. m

25. Cubic Measure.

(a.) Cubic Measure is used in measuring solids.

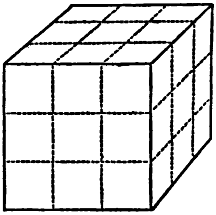
(b.) A solid is whatever has length, breadth, and thickness.

Note.—The term "*solid,*" as used in mathematics, refers to space, rather than to material substances.

(c.) A cube is a rectangular solid, whose length, breadth, and height are equal. It may also be defined as a solid bounded by six equal squares.

(d.) A cube 1 foot long, 1 foot wide, and 1 foot high, would be a cubic foot; a cube 1 yard long, 1 yard wide, and 1 yard high, would be a cubic yard, etc.

This figure represents a cubic yard.



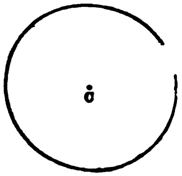
(e.) Any rectangular solid contains as many cubic units as there are in the product of its length multiplied by its breadth, multiplied by its height.

FULL TABLE.	ABBREVIATED.
1728 cubic inches = 1 cubic foot.	1728 cu. in. = 1 cu. ft.
27 cubic feet = 1 cubic yard.	27 cu. ft. = 1 cu. yd.
16 cubic feet = 1 cord foot.	16 cu. ft. = 1 cd. ft.
8 cord feet, or 128 cubic feet = 1 cord of wood.	8 cd. ft. or 128 cu. ft. = 1 cd.

26. Circular or Angular Measure.

(a.) Circular or Angular Measure is used to measure angles and the circumferences of circles.

(b.) A circle is a surface bounded by a curved line, which is everywhere equally distant from a point within, called the centre. The boundary line is called the circumference of the circle.



The figure represents a circle, of which C is the centre.

(c.) The distance from the centre of a circle to the circumference is called the RADIUS.

(d.) The distance from a point on one side of a circle through the centre to a point on the opposite side, is called the DIAMETER. Any portion of the circumference is called an ARC.

(e.) Every circumference of a circle, whether large or small, is supposed to contain 360 equal parts, called DEGREES; each degree is divided into 60 equal parts, called MINUTES; and each minute into 60 equal parts, called SECONDS.

NOTE.—The length of a degree, as well as that of its subdivisions, varies with the size of the circle.

FULL TABLE.	ABBREVIATED.
60 seconds = 1 minute.	$60'' = 1'$.
60 minutes = 1 degree.	$60' = 1°$.
360 degrees = 1 circumference.	$360° = 1$ circ.

27. Dry Measure.

(a.) DRY MEASURE is used for measuring grain, nuts, salt, etc.

FULL TABLE.	ABBREVIATED.
2 pints = 1 quart.	2 pt. = 1 qt.
8 quarts = 1 peck.	8 qt. = 1 pk.
4 pecks = 1 bushel.	4 pk. = 1 bu.

(b.) The chaldron of 36 bushels is sometimes used for measuring coal. Ch. is the sign for chaldron.

(c) The bushel contains $2150\frac{2}{5}$ cubic inches, and the quart contains $67\frac{1}{5}$ cubic inches.

28. Liquid Measure.

(a.) All kinds of liquids are measured by LIQUID MEASURE.

FULL TABLE.	ABBREVIATED.
4 gills = 1 pint.	4 gi. = 1 pt.
2 pints = 1 quart.	2 pt. = 1 qt.
4 quarts = 1 gallon.	4 qt. = 1 gal.

(b.) The HOGSHEAD of 63 gallons is used in estimating the contents of reservoirs, or other large bodies of water; but, in most other cases, the term hogshead is not a definite measure. Casks containing from 50 or 60 to 100 or 200 gallons, are called hogsheads.

(c.) A *barrel of cider* is usually reckoned at 31½ gallons.

(d.) The *gallon* contains 231 cubic inches; but the BEER GALLON, which is sometimes used in measuring beer, milk, and ale, contains 282 cubic inches.

29. Comparison of Dry, Liquid, and Beer Measures.

1 qt. Dry Measure = 67⅕ cubic inches.
1 qt. Liquid Measure = 57¾ "
1 qt. Beer Measure = 70½ "

30. Table of Time.

FULL TABLE.	ABBREVIATED.
60 seconds = 1 minute.	60 sec. = 1 min.
60 minutes = 1 hour.	60 min. = 1 h.
24 hours = 1 day.	24 h. = 1 d.
7 days = 1 week.	7 d. = 1 wk.
365¼ days, or 52 weeks, 1¼ days = 1 year.	365¼ d. or 52 wk. 1¼ d. = 1 y.

(a.) To avoid the inconvenience of reckoning ¼ of a day with each year, every fourth year is, with the exception named below, reckoned at 366 days, and called LEAP YEAR. The other years are reckoned at 365 days each.

(b.) The year, in reality, contains but 365 days, 5 h. 48 m. 48 sec.; so that, by reckoning 365¼ days, we make a slight error each year, which, in 100 years, amounts to about ¾ of a day. The centennial years are not, therefore, reckoned as leap years, unless the number of the year be divisible by 400. Thus, the year 2000 will be a leap year, but the year 1900 will not.

(c.) The year is divided into 12 months, which differ somewhat in length, as is seen in the following

(d.) TABLE OF THE MONTHS.

January has 31 days.
February has 28 days.*
March has 31 days.
April has 30 days.
May has 31 days.
June has 30 days.
July has 31 days.
August has 31 days.
September has 30 days.
October has 31 days.
November has 30 days
December has 31 days.

31. Miscellaneous Tables.

(a.) 12 things = 1 dozen.
12 dozen = 1 gross.
12 gross = 1 great gross.
20 things = 1 score.
24 sheets of paper = 1 quire.
20 quires of paper = 1 ream.

(b.) A barrel of beef or pork weighs 200 lb. A barrel of flour weighs 196 lb.

(c.) A book formed of sheets folded in
2 leaves is called a folio.
4 " " quarto, or 4to.
8 " " octavo, or 8vo.
12 " " duodecimo, or 12mo.
18 " " 18mo.
24 " " 24mo.

32. French Measures and Weights.

(a.) LONG MEASURE.

10 millimetres = 1 centimetre.
10 centimetres = 1 decimetre.
10 decimetres = 1 metre.
10 metres = 1 decametre.
10 decametres = 1 hectometre.
10 hectometres = 1 kilometre.
10 kilometres = 1 myriametre.

(b.) The metre is regarded as the unit of measure, and equals 39.371 of our inches. It is the twenty-millionth part of the distance measured on a meridian, from one pole to another.

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* Except in leap year, when it has 29 days.

(c.) WEIGHTS.

10 milligrammes = 1 centigramme.
10 centigrammes = 1 decigramme.
10 decigrammes = 1 gramme.
10 grammes = 1 decagramme.
10 decagrammes = 1 hectogramme.
10 hectogrammes = 1 kilogramme.
10 kilogrammes = 1 myriagramme.

(d.) The gramme is regarded as the unit of this weight, and equals 15.434 grains Troy.

(e.) The kilogramme is the weight most frequently used in business transactions, and equals very nearly 2½ pounds Avoirdupois.

(f.) MONEY.

10 centimes = 1 decime.
10 decimes = 1 franc.

(g.) The franc equals $.186, and the five-franc piece, often seen in the United States, is equal to 93 cents.

SECTION IV.

ADDITION.

33. Definitions and Explanations.

(a.) ADDITION is a process by which we ascertain how many units there are in two or more numbers taken together.

(b.) The result, or number found, is called the SUM or AMOUNT.

ILLUSTRATIONS. — In "3 and 4 are 7," or "3 + 4 = 7," 3 is *added* to 4, giving 7 for the *sum* or *amount*.

(c.) In order that numbers may be added, they must be of the same denomination, *i. e.* they must represent units of the same name or kind.

Illustrations.—6 pens and 3 pens are 9 pens; 6 pencils and 3 pencils are 9 pencils; but 6 pens and 3 pencils are neither 9 pens nor 9 pencils. They cannot be added, because they are not of the same denomination; but, if we should change their name or denomination, by calling them both *things*, we could add them; thus, 6 things and 3 things are 9 things.

(d.) Numbers consisting of several denominations, as units, tens, and hundreds, can only be added by adding the units with the units, the tens with the tens, etc.; hence, for convenience of addition, they should be so written that figures of the same denomination shall stand under each other, *i. e.* so that units shall stand under units, tens under tens, etc.

(e.) We add the columns separately, beginning at the right-hand or lowest denomination, and reduce as much as possible of the sum of each column to units of the next higher denomination, writing the remainder in its appropriate place in the amount, and adding the higher units obtained by the reduction to the next column.

(f.) The following example will illustrate this:

What is the value of 827 + 948 + 759?

$$\begin{array}{r} 827 \\ 948 \\ 759 \\ \hline 2534 \end{array}$$

Solution.—Writing the numbers as opposite, we add thus: 9 units and 8 units are 17 units, and 7 units are 24 units. But, as 24 units equal 2 tens and 4 units, we write the 4 units, and add the 2 tens with the tens. 2 tens and 5 tens are 7 tens, and 4 tens are 11 tens, and 2 tens are 13 tens. But, as 13 tens equal 1 hundred and 3 tens, we write the 3 tens and add the 1 hundred with the hundreds. 1 hundred and 7 hundreds are 8 hundreds, and 9 hundreds are 17 hundreds, and 8 hundreds are 25 hundreds, which, being the last sum, we write. The answer, then, is 2534.

(g.) The labor of adding may be lessened by omitting to name the separate numbers added.

Illustration.—In the above example, instead of saying "9 units and 8 units are 17 units, and 7 units are 24 units," it will be shorter to name only the *results*; thus, "nine, seventeen, twenty-four, or twenty-four units, equal to 2 tens and 4 units." Writing the 4 units, we add the 2 tens with the tens; thus, "two, seven, eleven, thirteen, or thirteen tens, equal," etc. as before.

(h.) *Methods of Proof.*—To test the correctness of the work—

1st. Add the numbers carefully a second time, as before.

2d. Add the numbers in a reverse direction from that taken in the first addition.

3d. Find the sum of two or more of the numbers, and add to it the remaining numbers.

The results should in each case equal the first result; otherwise, there is an error in one operation or the other, and possibly in both.

34. Examples for Practice.

Add the numbers in each of the following examples, and prove the work.

1.	2.	3.	4.	5.	6.
423	347	189	463	417	786
574	286	476	524	326	348
215	453	387	356	489	597

7.	8.	9.	10.	11.	12.
235	654	735	468	469	$6.27
124	123	144	246	875	4.85
521	356	327	357	638	3.96
342	246	656	753	904	6.84

13.	14.	15.	16.	17.
4571	5238	$1.548	$53.29	$41.87
3286	4597	2.697	61.84	93.25
2479	6285	6.084	24.97	46.78
4867	4329	3.297	63.21	69.32

18.	19.	20.	21.
571043	179486	432156	589786
243251	364527	729817	394695
324345	432615	854324	258676
132412	276143	979869	829584

22.	23.	24.	25.
$824.75	$15.863	$57.17	235.76 bu.
337.84	39.580	8.375	824.89
926.58	8.179	.25	427.06
367.25	43.269	38.258	328.59

26.	27.	28.	29.
543276	4.39	259.764	.794327
925138	63.28	43.849	.089465
423697	5.697	287.36	.321456
528437	38.423	12.785	.003287
629814	54.76	3.989	.695846

30.	31.	32.
247.968	1347.86	4279.43278
328.473	257.948	247.43986
549.613	3246.089	1484.3297
258.697	34.768	68.43954

33.	34.	35.
3496.13	3471.22	4782315
742.217	9436.819	14371096
36.042	732.45	827298
195.67	172.042	71564230
947.216	85417.786	3671786

36. 437 + 879 + 104 + 360 + 512 + 653.

37. 1708 + 4980 + 8470 + 3645 + 817 + 4765.

38. 90726 + 35076 + 18325 + 47009 + 63871.

39. $6.29 + $5.43 + $26.03 + $.625 + $4.376 + $7.36.

40. $37.50 + $84.42 + $13.27 + $59.07 + $90.09 + $3.50

41. 4367986 + 5942874 + 3275659 + 3128493 + 9874327.

42. 428 + 397 + 584 + 761 + 695 + 803 + 582 + 195 + 817

43. 5.79 + 57.9 + 5.79 + .579 + .0579 + .00579.

44. 694.60 + 84.325 + 572.93 + 42.847 + 3697.4 + 236.85.

45. 4.89 + 3.27 + 5.48 + 6.27 + 3.86 + 9.54 + 4.8 + 3.65.

46. 52.64 + 38.97 + 57.43 + 86.95 + 47.31 + 56.68 + 87.95.

47. 31.27 + 6.42 + .29 + 311.96 + 47.13 + 805.32 + 79.

48. 18365 + 739 + 4316 + 823 + 53871 + 7328 + 43797.

49. 415 + 39762 + 67 + 9346 + 185 + 25327 + 2811 + 27639 + 454 + 9088 + 4879 + 89076 + 532 + 8846.

50. 134.7 + 16.935 + 7.19 + 18.723 + 9.21 + 108.6 + 63.309 + 207.4 + 9.017 + 16.43 + .859 + 68.749 + 32.587.

35. Addition of Double Columns.

(a.) Accountants often add two or more columns, at one operation.

Model. — 63 plus 30 are 93, plus 8 are 101, plus 80 are 181, plus 4 are 185, plus 90 are 275, plus 7 are 282.

(b.) By adding the tens first, and then the units, as before, and omitting to name the separate numbers added, we can abbreviate the above.

97
84
38
63
282

Model. — 63, 93, 101, 181, 185, 275, 282.

(c.) A little practice will enable a person to abbreviate still farther; 63 plus 38 are 101, plus 84 are 185, plus 97 are 282; or, by omitting to name the numbers added, thus, 63, 101, 185, 282.

(d.) Let the pupil now perform the preceding examples in this way, or, at least, enough of them to make the principle and process familiar.

36. Practical Problems.

1. I paid $178 for a horse, $239 for a chaise, and $43 for a harness. What did I give for all?

2. A country trader bought cloth for $46.28, shoes for $37.93, West India goods for $98.46, and books for $86.37. What was the amount of his purchase?

3. A merchant bought 4 bales of cloth, the first containing 527 yd., the second 497 yd., the third 534 yd., and the fourth 549 yd. How many yards did he buy, in all?

4. My garden contains 3.265 acres, my orchard contains 5.238 acres, my meadow 9.648 acres, my pasture 18.973 acres, and my wood-lot as much as my garden, orchard, meadow, and pasture together. How many acres does my wood-lot contain? How many acres are there in all?

5. The receipts of cotton at the city of Savannah, Ga., have been as follows, viz.: 245496 bales in 1847, 406906 bales in 1848, 340025 bales in 1849, 312294 bales in 1850, 351566 bales in 1851, and 353068 bales in 1852. How many bales were exported in all this time?

6. Four men, A, B, C, and D, form a partnership. A puts in $4375; B puts in $1596; C, $6725; and D, $3950. How many dollars do all put in?

7. A merchant bought some coffee for $457.62, and some tea for $529.37: he sold the coffee at an advance of $119.87, and the tea at an advance of $186.98. How many dollars did he receive for both?

8. In 1850, the State of Massachusetts produced 31211 bushels of wheat, 481021 bushels of rye, 2345490 bushels of Indian corn, and 1165146 bushels of oats. How many bushels were produced of all these grains?

9. In 1850, Boston had a population of 136881, Charlestown, 17216, Cambridge, 15215, and Roxbury, 18364. What was the population of the four cities?

10. In 1850, Mississippi produced the following grains: 137990 bushels of wheat, 9606 bushels of rye, 22446552 bushels of Indian corn, and 1503288 bushels of oats. How many bushels were produced, of all these kinds?

11. Jan. 1st, 1856, I purchased a horse for $187.50, a chaise for $225, and a harness for $48.97. During the year, it cost me

$147.78 for keeping and taking care of the horse, and $28.37 for varnishing and repairing the chaise. What is the sum of the expenses thus incurred?

12. I bought two farms, paying $3988.67 for the first, and $1867.76 more for the second than for the first. How many dollars did I pay for the second, and how many for both?

13. A grain dealer sold 16784 bushels of corn to one man, 3251 to another, and, when he had sold 11868 to another, he had 24785 bushels left. How many bushels had he at first?

14. During the year ending Aug. 31st, 1854, 1603750 bales of cotton were exported to Great Britain, 374058 to France, 165172 to the north of Europe, and 176168 to other foreign ports. How many bales were exported?

15. During the year ending Dec. 1st, 1854, the United States' mails were transported an aggregate of 15433389 miles by railroad, 20890530 miles by coach, 5795483 by steamboats, and 21267603 by other conveyances. How many miles were they transported, in all?

16. A man invested $478.36 in real estate, $237.86 more in bank stock than in real estate, and $136.59 more in trade than in bank stock? How much did he invest in each, and how much in all?

17. A, B, and C divided a sum of money in such a way, that A had $427.83, B had $348.59 more than A, and C had $148.26 more than A and B together. How many dollars had B? what number had C? How many dollars were divided?

18. I own a rectangular field, 1389 feet long and 1246 feet wide. How many feet of fence will it take to enclose it?

19. My farm is worth $8794.63, my railroad stock is worth $3275.87, my bank stock is worth $7948.35, my insurance stock is worth $3298.46, and my stock of goods is worth $1985.73. What are all worth?

20. I have just paid $1273.82 on note, $4296.84 on bond and mortgage, and $1437.82 on account, and I still owe $948.62 on notes, $2578.63 on bond and mortgage, and $3258.25 on accounts. How much did I owe before paying any of those debts?

SECTION V.

COMPOUND ADDITION.

37. Definitions and Explanations.

Note. — This and the other sections on Compound Numbers may be omitted till after the "Simple Rules" are mastered, if, in the opinion of the teacher, the circumstances of the class render it desirable.

(a.) Compound Addition is the addition of Compound Numbers.

(b.) Compound Addition involves the same principles as Simple Addition.

(c.) In both, numbers of the same denomination are added together; in both, the numbers should be so written as to bring figures of the same denomination under each other; in both, the addition is commenced with the right-hand or lowest denomination; and, in both, as much as possible of the sum of each column is reduced to units of the next higher denomination. Moreover, the methods of proof are the same in both. The only difference of any kind between them is, that, in simple numbers, 10 units of any denomination equal 1 of the next higher; while, in compound numbers, there is no uniformity in this respect.

(d.) In Compound Addition, the abbreviations of the denominations should be marked over the several columns.

(e.) The application of these principles is illustrated in the following problem and solution:

What is the value of £27 18s. 8d. + £47 19s. 6d. + £86 17s. 11d. + £57 13s. 10d.?

1st Solution.—Writing the numbers so that figures of the same denomination shall fall under each other, we begin at the right, and, observing that there will be 1 shilling for every 12 pence, we add thus:

£	s.	d.
27	18	8
47	19	6
86	17	11
57	13	10
220	9	11

10d. and 11d. are (by uniting 1 of the 10d. with the 11d.) 12d. + 9d. = 1s. 9d., and 6d. are (by uniting 3 of the 6d. with the 9d.) 1s. 12d. + 3d. = 2s. 3d., and 8d. are 2s. 11d.

Writing the 11d., and, observing that there will be £1 for every 20s., we add the 2s. with the shillings, thus:

2s. and 13s. are 15s., and 17s. are (by uniting 3 of the 15s. with the 17s., or else 5 of the 17s. with the 15s.) 20s. + 12s. = £1 12s., and 19s. are (by uniting 1 of the 12s. with the 19s.) £2 11s., and 18s. are (by uniting 2 of the 11s. with the 18s.) £3 9s.

Writing the 9s., we add the £3 to the pounds, as in simple numbers, which gives for the answer, £220 9s. 11d.

2D SOLUTION.—Writing the numbers as before, we add thus:

10d. + 11d. + 6d. + 8d. = 35d., which, since 12d. = 1s., must equal as many shillings as there are 12 in 35. This, found by dividing 35 by 12 gives 2s. 11d.

Writing the 11d., we add the 2s. to the column of shillings, thus:

2s. + 13s. + 17s. + 19s. + 18s. = 69s., which, since 20s. = £1, must equal as many pounds as there are times 20 in 69. This, found by dividing 69 by 20, gives £3 9s.

Writing the 9s., we add the £3 to the column of pounds, thus:

£3 + £57 + £86 + £47 + £27 = £220. Hence, the required sum is £220 9s. 11d.

NOTE.—We think that, as a general thing, the first solution will be found the most convenient. The teacher alone can judge of the expediency of having the class attempt to master both.

38. Examples for Practice.

1.

£	s.	d.	far.
28	16	11	3
37	18	10	2
23	19	8	3
42	13	9	3

2.

£	s.	d.	far.
26	16	11	2
35	14	3	0
92	19	10	0
48	8	9	3

3.

yd.	qr.	na.
24	3	2
53	1	3
48	2	3
27	0	2

4.

yd.	*ft.*	*in.*
84	2	7
95	1	9
27	2	10
36	2	11

5.

bu.	*pk.*	*qt.*	*pt.*
33	2	5	1
47	3	7	1
57	1	6	0
38	2	5	1

6.

gal.	*qt.*	*pt.*	*gi.*
5	3	1	2
4	3	0	3
7	2	1	3
4	3	0	2

7.

A.	*R.*	*sq. rd.*
24	1	26
62	3	39
37	2	16
58	0	23

8.

sq.yd.	*sq.ft.*	*sq.in.*
8	6	127
3	7	100
5	5	96
2	8	138

9

m.	*fur.*	*rd.*
226	6	27
113	7	36
427	5	28
336	6	35

10.

lb.	*oz.*	*dwt.*	*gr.*
43	7	13	20
27	11	16	22
45	10	18	21
36	8	7	22
46	9	18	23

11.

lb.	*oz.*	*dwt.*	*gr.*
16	10	11	4
21	3	18	22
15	11	17	19
14	8	17	21
42	9	14	17

12.

°	′	″
14	13	46
11	45	37
10	58	49
8	17	57
3	32	25

13.

℔	℥	ʒ	℈	*gr.*
13	11	5	2	18
21	8	6	1	9
32	10	7	0	13
25	7	4	2	15
32	6	7	0	18

14.

℔	℥	ʒ	℈	*gr.*
21	11	7	2	19
35	10	6	1	18
27	9	5	0	17
18	10	6	1	18
94	11	7	2	19

15.

T.	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
385	17	2	24	14	13
26	16	0	16	15	10
437	14	1	13	10	11
227	19	3	21	8	15
147	8	2	23	14	14

16.

T.	*cwt.*	*qr.*	*lb.*	*oz.*	*dr*
8	18	0	20	15	7
5	13	1	21	9	14
9	9	3	24	7	12
7	10	3	6	10	13
6	16	3	23	11	15

17.

wk.	*da.*	*h.*	*min.*	*sec.*
3	4	21	30	46
5	3	15	25	15
8	6	16	46	45
7	5	22	52	37
9	2	8	13	56

18.

wk.	*da.*	*h.*	*min.*	*sec.*
37	5	23	35	59
43	6	20	20	37
82	3	15	56	48
91	6	17	49	26
43	1	19	53	30

19.*

rd.	*yd.*	*ft.*	*in.*
7	4	2	11
8	3	1	10
6	4	2	8
9	5	0	6
5	2	0	9

20.

m.	*fur.*	*rd.*	*yd.*	*ft.*	*in.*
37	7	22	5	0	8
69	5	37	4	1	10
38	2	14	3	2	7
47	5	25	5	1	3
36	7	36	4	0	10

21.

rd.	*yd.*	*ft.*	*in.*
8	4	2	10
9	5	1	5
7	2	2	10
4	5	0	9
3	4	2	8
6	4	2	10

22.

m.	*fur.*	*rd.*	*yd.*	*ft.*	*in.*
326	7	39	5	0	11
483	4	27	4	2	8
527	3	38	3	1	3
695	7	20	5	2	10
839	6	36	1	2	9
628	7	39	2	0	11

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* In adding yards, the reductions can generally best be made by regarding every 11 yards as 2 rods. It should be also borne in mind that ½ yard equals 1 ft. 6 in., and that if, in any case, there are 5 yd. and 1 ft. 6 in. besides, the value can best be expressed as 1 rod.

23.*

s.rd.	*s.yd.*	*s.ft.*	*s.in.*
5	29	8	140
9	28	3	26
7	30	6	100
8	27	7	136
6	30	8	142

24.

A.	*R.*	*s.rd.*	*s.yd.*	*s.ft.*	*s.in.*
46	3	25	15	8	128
39	2	17	17	6	135
28	0	36	26	0	36
59	2	28	30	7	72
47	3	35	28	8	130

25.

C.	*cd.ft.*	*cu.ft.*	*cu.in.*
58	3	14	1720
96	6	13	896
37	7	15	1725
54	5	10	597

26.

C.	*cd.ft.*	*cu.ft.*	*cu.in.*
47	7	14	1650
83	0	8	1722
58	6	9	1528
49	7	13	1628

27. £15 17s. 10d. 3 far. + £18 16s. 11d. 2 far. + £23 11s. 8d. 3 far. + £34 19s. 6d. 1 far.?

28. 85 lb. 10 oz. 19 dwt. 22 gr. + 43 lb. 6 oz. 17 dwt. 21 gr. + 49 lb. 8 oz. 14 dwt. 13 gr. + 87 lb. 7 oz. 18 dwt. 23 gr.?

29. 14℔ 8℥ 6ʒ 2℈ 14 gr. + 7℔ 10℥ 7ʒ 1℈ 18 gr. + 19℔ 9℥ 5ʒ 1℈ 16 gr. + 34℔ 11℥ + 7ʒ + 1℈ + 17 gr.?

30. 8 bu. 2 pk. 3 qt. 1 pt. + 5 bu. 3 pk. 7 qt. 0 pt. + 5 bu. 3 pk. 6 qt. 1 pt. + 9 bu. 1 pk. 3 qt. 0 pt. + 3 bu. 2 pk. 7 qt. 1 pt.?

31. 6 T. 13 cwt. 2 qr. 24 lb. 5 oz. 13 dr. + 8 T. 18 cwt. 3 qr. 20 lb. 15 oz. 15 dr. + 9 T. 13 cwt. 1 qr. 22 lb. 8 oz. 9 dr. + 2 T. 11 cwt. 1 qr. 15 lb. 7 oz. 14 dr. + 4 T. 19 cwt. 2 qr. 21 lb. 14 oz. 11 dr.?

32. 23 C. 6 cd. ft. 14 cu. ft. 1379 cu. in. + 87 C. 4 cd. ft. 11 cu. ft. 1600 cu. in. + 68 C. 7 cd. ft. 9 cu. ft. 1425 cu. in.?

33. 9 wk. 6 da. 13 h. 50 min. 35 sec. + 12 wk. 4 da. 21 h. 48 min. 37 sec. + 5 wk. 4 da. 22 h. 53 min. 25 sec. + 2 wk. 5 da. 17 h. 38 min. 49 sec. + 9 wk. 4 da. 13 h. 22 min. 13 sec.?

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* In adding square yards, observe that 60½ sq. yd. = 2 sq. rd.; that 90¾ sq. yd. = 3 sq. rd.; and that 121 sq. yd. = 4 sq. rd. Observe, farther, that ¼ of a sq. yd. = 2 sq. ft. 36 sq. in.; ½ of a sq. yd. = 4 sq. ft. 72 sq. in.; and that ¾ of a sq. yd. = 6 sq. ft. 108 sq. in. This will avoid serious difficulty in the use of fractions.

34. 18 m. 5 fur. 26 rd. 4 yd. 2 ft. 7 in. + 23 m. 7 fur. 37 rd. 5 yd. 1 ft. 2 in. + 31 m. 5 fur. 32 rd. 4 yd. 2 ft. 10 in. + 17 m. 4 fur. 31 rd. 3 yd. 2 ft. 11 in. + 3 m. 4 fur. 39 rd. 0 ft. 7 in.?

35. 5 bu. 3 pk. 2 qt. 1 pt. + 8 bu. 1 pk. 7 qt. 0 pt. + 4 bu. 3 pk. 6 qt. 1 pt. 3 gi. + 5 bu. 2 pk. 5 qt. 0 pt. 2 gi. + 7 bu. 2 pk. 5 qt. 1 pt. 1 gi. + 4 bu. 2 pk. 4 qt. 1 pt. 3 gi.?

36. £278 13s. 6d. 1 far. + £523 18s. 11d. 3 far. + £436 14s. 8d. 2 far. + £142 17s. 10d. 3 far. + £203 10s. 8d. 1 far. + £850 15s. 7d. 3 far. + £312 18s. 11d. 2 far.?

37. I have 3 bars of silver: the first weighs 6 lb. 3 oz. 5 dwt. 17 gr.; the second weighs 5 lb. 10 oz. 13 dwt. 17 gr.; and the third weighs 5 lb. 11 oz. 18 dwt. 22 gr. What is the weight of the whole?

38. After paying £17 13s. 6d. to one man, £12 16s. 3d. 1 far. to another, £21 19s. 11d. 3 far. to another, and £10 14s. 9d. 3 far. to another, I had £58 10s. 7d. 1 far. left. How much had I at first?

SECTION VI.

SUBTRACTION.

39. Definitions and Explanations.

(a.) Subtraction is a process by which we find how many units there are in the difference of two numbers, or in the excess of one number over another.

(b.) The larger given number, or the one from which we subtract, is called the Minuend; the smaller number, or the number subtracted, is called the Subtrahend; and the result is called the Difference or Remainder.

Illustrations. — In "3 from 7 leaves 4," or "7 — 3 = 4," 7 is the *minuend*, 3 is the *subtrahend*, and 4 is the *difference* or *remainder*

(c.) In order that numbers may be subtracted, they must be of the same name or denomination, *i. e.* they must represent units of the same name or kind.

ILLUSTRATIONS.—3 pens from 7 pens leave 4 pens, and 3 pencils from 7 pencils leave 4 pencils; but 3 pens cannot be subtracted from 7 pencils, because they are not of the same denomination.

(d.) For convenience in subtracting, numbers are so written that figures of the same denomination shall come under each other, the minuend being usually written above the subtrahend, and the remainder beneath. The denominations are subtracted separately, beginning at the right.

1. What is the value of 8.795 — 3.281?

8.795
3.281
―――
5.514

SOLUTION.—Writing the numbers as opposite, we have 1 thousandth from 5 thousandths leaves 4 thousandths; 8 hundredths from 9 hundredths leave 1 hundredth; 2 tenths from 7 tenths leave 5 tenths; 3 units from 8 units leave 5 units. Therefore, the remainder is 5 units, 5 tenths, 1 hundredth, and 4 thousandths, or, 5.514.

(e.) *Methods of Proof.*—From the nature of subtraction, it is evident that, if the work be correct—

1st. The sum of the subtrahend and remainder will equal the minuend.

2d. The difference between the minuend and remainder will equal the subtrahend.

(g.) Perform the following subtractions, and prove the work.

2. 5786 — 2351?
3. 49.58 — 16.27?
4. .8943 — .6512
5. 7898 — 3426?
6. 6478 — 1352?
7. 8397 — 2064?
8. 3988 — 1437?
9. 5678 — 1234?
10. 9876 — 2345?
11. 5798 — 3217?

40. Reductions sometimes Necessary.

(a.) When, as is frequently the case, a figure of the subtrahend is larger than the corresponding figure of the minuend, we reduce a unit of the next higher denomination of the minuend to the denomination of the figures we are considering, and add its value to the value of the given minuend figure. From the sum thus obtained, we subtract the subtrahend figure.

(b.) Allowance should be made for the reduced unit, either by calling the next minuend figure one less, or the next subtrahend figure one greater than it is.

1. What is the value of 5283 — 1468?

5283
1468
3815

SOLUTION.—Writing the numbers as opposite, and beginning at the right hand, we proceed thus: since 8 units cannot be subtracted from 3 units, we reduce 1 of the 8 tens of the minuend to units; 1 ten = 10 units, which, added to the 3 units, gives 13 units. 8 units from 13 units leave 5 units, which we write. 6 tens from the 7 tens left in the minuend leave 1 ten, which we write. Since 4 hundreds cannot be subtracted from 2 hundreds, we reduce one of the 5 thousands to hundreds; 1 thousand = 10 hundreds, which, added to the 2 hundreds, gives 12 hundreds. 4 hundreds from 12 hundreds leave 8 hundreds, which we write. 1 thousand from the 4 thousands left in the minuend leaves 3 thousands, which we write.

The answer, then, is 3 thousands, 8 hundreds, 1 ten, and 5 units; or, 3815.

(c.) The following form of writing the work requires more figures, but shows the reduction more clearly than did the preceding.

4 12 7 13 = Minuend, changed in form.
5 2 8 3 = Minuend.
1 4 6 8 = Subtrahend.
3 8 1 5 = Remainder.

2. What is the value of 4003 — 2715?

4003
2715
1288

SOLUTION.—Writing the numbers as opposite, and beginning at the right hand, we subtract thus: since 5 units cannot be subtracted from 3 units, and there are no tens or hundreds in the minuend, we reduce 1 of the 4 thousands to hundreds, giving 10 hundreds; then reduce 1 of the 10 hundreds to tens, giving 10 tens; and then reduce 1 of the 10 tens to units, giving 10 units. 10 units plus 3 units equal 13 units, and 5 units from 13 units leave 8 units; 1 ten from the 9 tens left in the minuend leaves 8 tens; 7 hundreds from the 9 hundreds left in the minuend leave 2 hundreds; 2 thousands from the 3 thousands left in the minuend leave 1 thousand. The answer, then, is 1 thousand, 2 hundreds, 8 tens, and 8 units; or, 1288.

The following form of writing the changed minuend illustrates clearly the nature of the reductions made:

3 9 9 13 = Minuend, changed in form.
4 0 0 3 = Minuend.
2 7 1 5 = Subtrahend.
1 2 8 8 = Remainder.

(d.) In the preceding explanation, we subtracted 1 from the minuend figure from which a unit was taken for reduction, before subtracting the subtrahend figure; but we might as well have subtracted the 1 with the subtrahend figure.

ILLUSTRATION. — In the first example, instead of subtracting 1 ten, on account of the reduction, from the 8 tens, and then subtracting the 5 tens, we might have added the 1 ten to the 5 tens, and subtracted both together.

(e.) Many always subtract by the latter method. One method is as convenient as the other, but the one to which we are accustomed will seem the easiest.

41. Examples and Practical Problems.

1. 4753 — 2897.
2. 6984 — 3796.
3. 10000 — 3.
4. 21057 — 19861.
5. 70502 — 69699.
6. 12311 — 9872.
7. 4010 — 3907.
8. $1051.43 — $761.49.
9. $6710.81 — $1382.93.
10. 400500 — 376476.
11. 9057 — 3288.
12. $15.27 — $9.49.
13. $20.05 — $19.91.
14. $199.01 — $20.99.
15. 7059 — 6447.
16 9076 — 987.
17. 11231 — 9846.
18. 10000 — 97.
19. 876.543 — 345.678.
20. 5031.706 — 94.0798.
21. 30196 — 27463.
22. 7364 — 4637.
23. 4321 — 1234.
24. 81965 — 63079.
25. 432.1 — 43.21.
26. 87.65 — 8.765.
27. 10101 — 9093.
28. 7654 — 76.54.
29. 100.001 — 10.0207.
30. 6074.832 — 999.888.

31. A man bought a farm for $4728, and sold it for $6253. How many dollars did he gain?

32. Boston contained 93383 inhabitants in 1840, and 136881 in 1850. How many more were there in 1850 than in 1840?

33. Mount Washington, in New Hampshire, is 6226 feet high; Mount Monadnock, in the same State, is 3718 feet high. How many feet higher is the former than the latter?

34. How many more Troy grains are there in a pound Avoirdupois than in a pound Troy? See 21.

35. A man who owned 2578 acres of Western land, sold 1389 acres. How many had he left?

36. When a yard of broadcloth costs $5.375, and a yard of cassimere costs $1.25, how much more does a yard of broadcloth cost than a yard of cassimere?

37. If I have in my possession $3206.36, and owe $140.489, how many dollars shall I have left after paying my debts?

38. By the Tribune Almanac for 1857 it appears that, in the presidential election of 1856, Buchanan received 1,834,337 votes Fremont 1,341,812, and Fillmore 873,055. How many more votes did Buchanan receive than Fremont?

39. How many more did Buchanan receive than Fillmore?

40. How many more did Fremont receive than Fillmore?

41. How many more votes did Buchanan and Fremont together receive than Fillmore?

42. How many more votes did Buchanan and Fillmore together receive than Fremont?

43. How many more votes did Fremont and Fillmore together receive than Buchanan?

44. By selling a lot of land for $975.36, I gained $197.84. How much did the land cost me?

45. If one man travels 271 miles, and another travels 320 miles how much farther does the second travel than the first?

46. I bought goods to the amount of $71.58, giving in paymen a hundred-dollar bill. How much change ought I to receive back?

47. Edward has $675, George has $813, and William has as many dollars less than Edward, as Edward has less than George How many dollars has William?

48. Joseph says that he has 837 cents, and that if he should spend 198 cents, he would still have 237 cents more than Otis. How many cents has Otis?

42. Subtraction of Several Numbers.

(a.) The best method of subtracting several numbers from a single number, is to write them under the minuend, and then to subtract the sum of each column of the subtrahends from the appropriate part of the minuend, reducing from the higher denominations, as before explained.

1. What is the value of 651 — 87 — 49 — 58 — 79?

WRITTEN WORK.

651 Minuend.
87
49 Subtrahends.
58
79
———
378

EXPLANATION.—The sum of the units of the subtrahends is 33, which cannot be taken from 1 unit. To obtain units enough to perform the subtraction, we must reduce 4 tens to units. 4 tens = 40 units, and adding the 1 unit gives 41 units, from which subtracting 33 units leaves 8 units.

Now uniting 4 tens (on account of the reduction) with the tens of the subtrahends, gives 28 tens, which cannot be subtracted from the 5 tens of the minuend. To obtain tens enough to perform the subtraction, we must reduce 3 hundreds to tens. 3 hundreds = 30 tens, and adding the 5 tens, gives 35 tens, from which subtracting the 28 tens leaves 7 tens.

Subtracting 3 hundreds (on account of the last reduction) from the 6 hundreds of the minuend leaves 3 hundreds.

The answer, then, is 3 hundreds, 7 tens, and 8 units, or 378.

(b.) *Proof.*—To test the correctness of the work, see if the remainder added to the subtrahends equals the minuend.

(c.) Practically, the subtraction may be performed thus:

9, 17, 26, 33, from 41, leaves 8.
4, 11, 16, 20, 28, from 35, leaves 7.
3 from 6 leaves 3.
Giving for an answer, 378, as before.

(d.) What is the value of—

2. 9748 — 879 — 643 — 297 — 856?

3. 5246 — 472 — 631 — 842 — 319?

4. 3976 — 937 — 659 — 813?

5. 2927 — 132 — 471 — 329 — 781 — 943?

6. 6878 — 1462 — 321 — 925 — 1176?

7. $75.16 — $15.27 — $13.26 — $14.17 — $9.24?

8. $91.18 — $37.22 — $23.19 — $17.16 — $9.43?

9. 37906 — 5321 — 986 — 1732 — 8521 — 1769?

10. 56125 — 8754 — 9326 — 4721 — 15372?

11. 2501.38 — 53.12 — 6.125 — 143.7 — 19.83?

12. 189.26 — 43.21 — 74.32 — 9.365 — 18.4?

13. $785.34 — $150 — $231.92 — $50 — $9.758?

14. $1927 — $563.71 — $87.19 — $1122.42 — $153.59?

15. $75 — $26.153 — $11.49 — $.125 — $19.336 — $7?

16. 19273 — 4234 — 10721 — 1362 — 796?

17. 73647 — 63132 — 7318 — 329 — 45 — 7?

18. 35.46 — 11.23 — 9.18 — .4326 — 12.091?

19. 1632 — 163.2 — 16.32 — 1.632 — .1632 — .01632?

20. 7218.1 — 4321.6 — 708.003 — 1094 — 11.81?

21. $34.18 — $13.77 — $.095 — $9.03 — $9.199?

SECTION VII.

COMPOUND SUBTRACTION.

43. Definitions and Explanations.

(a.) Compound Subtraction is the subtraction of compound numbers.

(b.) It involves the same principles that simple subtraction does.

(c.) Subtract 8 lb. 10 oz. 18 dwt. 21 gr., from 12 lb. 3 oz. 16 dwt. 14 gr.

lb.	*oz.*	*dwt.*	*gr.*
12	3	16	14
8	10	18	21
3	4	17	17

Solution.—Writing the numbers as opposite, we subtract thus: since 21 gr. cannot be subtracted from 14 gr., we reduce 1 of the 16 dwt. to grains. 1 dwt. = 24 gr., which, added to the 14 gr., gives 38 gr., from which subtracting 21 gr. leaves 17 gr.

Since 18 dwt. cannot be subtracted from the 15 dwt. (left after the reduction), we reduce 1 of the 3 oz. to pennyweights. 1 oz. = 20 dwt., which, added to the 15 dwt., gives 35 dwt., from which subtracting 18 dwt. leaves 17 dwt.

Since 10 oz. cannot be subtracted from the 2 oz. (left after the reduction), we reduce 1 lb. to ounces. 1 lb. = 12 oz., which, added to the 2 oz., gives 14 oz., from which subtracting the 10 oz. leaves 4 oz.

8 lb. from 11 lb. leave 3 lb.

Hence, the answer is, 3 lb. 4 oz. 17 dwt. 17 gr.

(d.) When reductions are made, the remainder can often be found most easily by subtracting from the reduced unit, and adding the remainder thus obtained to the minuend figure.

Illustration.—In the above example, by the first reduction, we have 1 dwt. = 24 gr.; and 21 gr. from 24 gr. leave 3 gr. to be added to the 14 gr., giving 17 gr.

By the second reduction, we have 1 oz. = 20 dwt.; and 18 dwt. from 20 dwt. leave 2 dwt., to which adding the 15 dwt. (left after the former reduction), gives 17 dwt., etc. etc.

(e.) Another method, which is often found convenient, is to subtract as much of each denomination of the subtrahend as possible, from the corresponding denomination of the minuend, and then subtract the rest from the reduced unit.

Illustration.—In the above example, we can subtract 14 of the 21 gr. from the 14 gr. of the minuend, and then the remaining 7 gr. from the 1 dwt. or 24 gr.

44. Examples for Practice.

1. £27 13s. 6d. 1 far. — £13 16s. 3d. 1 far.
2. £54 8s. 3d. 2 far. — £16 13s. 8d. 3 far.
3. 16 lb. 4 oz. 14 dr. — 7 lb. 9 oz. 13 dr.

4. 5 cwt. 2 qr. 13 lb. 11 oz. 6 dr. — 2 cwt. 3 qr. 20 lb. 13 oz. 12 dr.

5. 36 T. 10 cwt. 2 qr. 20 lb. 5 oz. 8 dr. — 14 T. 16 cwt. 3 qr. 24 lb. 7 oz. 2 dr.

6. 38 bu. 2 pk. 3 qt. 0 pt. — 15 bu. 3 pk. 6 qt. 1 pt.

7. 33 bu. — 6 bu. 2 pk. 4 qt. 1 pt.

8. 93 w. 4 da. 15 h. 32 m. 14 sec. — 75 w. 4 da. 18 h. 42 m. 37 sec.

9. 52 w. 1 da. 6 h. — 26 w. 4 da. 3 h. 36 m. 15 sec.

10. 19℔ 9℥ 3ʒ 2℈ 16 gr. — 3℔ 6℥ 1ʒ 1℈ 5 gr.

11. 28℔ 8℥ 6ʒ 1℈ 8 gr. — 27℔ 11℥ 7ʒ 2℈ 19 gr.

12. 19 sq. yd. 5 sq. ft. 17 sq. in. — 4 sq. yd. 2 sq. ft. 100 sq. in.

13. 96 cd. 3 cd. ft. 12 cu. ft. 1473 cu. in. — 83 cd. 2 cd. ft. 15 cu. ft. 1697 cu. in.

14. 23 cd. 5 cd. ft. 13 cu. ft. 247 cu. in. — 8 cd. 6 cd. ft. 3 cu. ft. 1549 cu. in.

15. 48° 17′ 28″ — 37° 29′ 53″.

16. 57° 26′ 45″ — 32° 42′ 15″.

17. 279 lb. 6 oz. 13 dwt. 5 gr. — 82 lb. 9 oz. 7 dwt. 18 gr.

18. £24 — £13 6s. 2d. 3 far.

19 96 T. — 4 T. 13 cwt. 2 qr. 15 lb.

20. 367 bu. — 239 bu. 2 pk. 7 qt. 1 pt.

21. 89 yd. — 53 yd. 2 qr. 3 na.

45. Method of reducing Fractional Denominations.

(a.) If, in Long or Square Measure, it is necessary to reduce a *rod* to *yards*, the reduction should be made, and the ½ or ¼ of a yard reduced to feet and inches, *before commencing the subtraction.*

1. Subtract 14 rd. 4 yd. 2 ft. 8 in., from 28 rd. 3 yd. 0 ft. 4 in.

27	8	1	10	= Reduced minuend.
rd.	*yd.*	*ft.*	*in.*	
28	3	0	4	= Minuend.
14	4	2	8	= Subtrahend.
13	3	2	2	= Remainder.

SOLUTION.—Writing the work as opposite, and observing that the subtraction will require that 1 rod shall be reduced, we proceed thus: 1 rd. = 5½ yd.; adding the 5 yd. to the 3 yd. gives 8 yd.; ½ yd. = 1 ft. 6 in., which, added to the 0 ft. 4 in., gives 1 ft. 10 in. Now, subtracting 8 in. from 10 in. leave 2 in. Since 2 ft. cannot be subtracted from 1 ft., we reduce 1 of the 8 yd. to feet; 1 yd. = 3 ft., which, added to the 1 ft., gives 4 ft., from which subtracting 2 ft. leave 2 ft.

4 yd. from 7 yd. leave 3 yd.; 14 rd. from 27 rd. leave 13 rd.

The answer, then, is 13 rd. 3 yd. 2 ft. 2 in.

(b.) Perform the following examples:

2. 27 rd. 2 yd. 1 ft. 4 in. — 4 rd. 4 yd. 2 ft. 11 in.

3. 8 m. 1 fur. 36 rd. 2 yd. 1 ft. 4 in. — 8 m. 1 fur. 35 rd. 5 yd. 1 ft. 5 in.

4. 64 m. 0 fur. 13 rd. 0 yd. 0 ft. 2 in. — 28 m. 7 fur. 39 rd. 5 yd. 1 ft. 3 in.

5. 111 m. 1 fur. 1 rd. 1 yd. 1 ft. 1 in. — 22 m. 2 fur. 2 rd. 2 yd. 2 ft. 2 in.

6. 48 m. 6 fur. 29 rd. 0 yd. 2 ft. 8 in. — 27 m. 2 fur. 13 rd. 1 yd. 0 ft. 1 in.

7. 37 rd. 1 ft. 2 in. — 36 rd. 5 yd. 2 ft. 7 in.

8. 19 m. — 18 m. 7 fur. 39 rd. 5 yd. 1 ft. 5 in.

9. 15 rd. — 8 rd. 0 yd. 2 ft. 11 in.

10. 137 m. — 2 ft.

11. 39 sq. rd. 3 sq. yd. 2 sq. ft. 43 sq. in. — 30 sq. rd. 30 sq. yd. 3 sq. ft. 78 sq. in.

12. 96 A. 2 R. 29 sq. rd. 3 sq. yd. 6 sq. ft. 107 sq. in. — 14 A. 1 R. 12 sq. rd. 25 sq. yd. 1 sq. ft. 31 sq. in.

13. 538 A. 3 sq. yd. 1 sq. in. — 249 A. 1 sq. rd. 5 sq. yd. 7 sq. ft.

14. 32 sq. rd. 5 sq. yd. 1 sq. ft. 72 sq. in. — 13 sq. rd. 29 sq. yd. 8 sq. ft. 140 sq. in.

15. 27 A. 1 R. 26 sq. yd. 80 sq. in. — 26 A. 3 R. 39 sq. rd. 30 sq. yd. 8 sq. ft. 143 sq. in.

16. 19 A. — 5 A. 3 R. 39 sq. rd. 30 sq. yd. 2 sq. ft. 35 sq. in.

17. 48 A. — 6 sq. ft.

18. 5 A. — 3 A. 1 R. 39 sq. rd. 30 sq. yd. 8 sq. ft. 137 sq. in.

19. 35 sq. rd. 6 sq. ft. 93 sq. in. — 34 sq. rd. 30 sq. yd. 8 sq. ft. 129 sq. in.

20. What is the difference between 24 rd. 1 ft. 3 in. and 23 rd. 5 yd. 2 ft. 11 in.?

21. What is the difference between 28 sq. rd. 5 sq. ft. 63 sq. in. and 27 sq. rd. 30 sq. yd. 7 sq. ft. 107 sq. in.?

46. Miscellaneous Problems.

1. A man bought 17 cwt. 2 qr. 13 lb. of sugar, and sold 14 cwt. 2 qr. 21 lb. How much had he left?

2. An English merchant bought goods for £327 14s. 6d., and sold them for £596 10s. What was his gain?

3. My wood-lot is 32 rd. long, and 27 rd. 2 yd. 2 ft. wide. What is the difference between its length and its breadth?

4. I bought a bar of silver weighing 7 lb., and made 5 lb. 7 oz. 5 dwt. 14 gr. of it into tea-spoons, and the rest into salt-spoons. How much did I make into salt-spoons?

5. A farmer, carrying a load of hay to market, found that the hay and the cart together weighed 2 T. 7 cwt. 3 qr. 16 lb., and the cart alone weighed 18 cwt. 1 qr. 22 lb. What was the weight of the hay?

6. A trader purchased some sugar for $327.13, some molasses for $125.50, some flour for $520.62, some butter for $132.28, and some cheese for $86.72, giving in payment a lot of cloth worth $625.47, and the rest in money. How much money did he pay?

7. A man who has in his possession $6978, owes $1364 to one man, $965 to another, $1167 to another, and $847 to another. How many dollars will he have left after paying his debts?

8. I bought a carriage for $147.13, and paid $11.48 for having it repaired, and $13.27 for having it painted. I let it enough to come to $19.75, and then sold it for $175. What did I gain by the transaction?

9. A London merchant bought some merchandise for £247 8s. 11d. For how much must he sell it, to gain £89 15s. 7d.?

10. I bought a lot of flour for $912.46, and sold it for $1258.93. I afterwards bought a lot of wheat for $1678.27. For how much must I sell the wheat, to gain as much as I gained on the flour?

11. The distance from Albany to Buffalo, by railroad, is 325 miles. If A should start from Albany, and B should at the same time start from Buffalo, and travel towards each other, how far apart will they be when A has travelled 97 miles, and B has travelled 113 miles?

12. A man started from Boston on a journey, taking with him $100. He took the cars for Providence, paying $1.50 for his ticket. He stayed in Providence 2 days, at an expense of $5.37, and then purchased a ticket for New York, for $3. He remained in New York 5 days, at an expense of $15.62, and then went to Philadelphia, paying $3 for his ticket, and $1.25 for hack hire. His expenses at Philadelphia were $20.75. The total expense of his homeward trip was $15.27. How much money had he left?

13. How much more money must a person get who has $1378.56, in order to purchase a house worth $2538, and still have $893 left with which to purchase furniture?

14. I bought a wood-lot for $639.46, from which I obtained 123 cd. 5 cd. ft. of wood, worth $370.50; 75 cd. 6 cd. ft. worth $189.375; 84 cd. 4 cd. ft. worth $253.50; and 87 cd. 2 cd. ft. worth $174.50. How much wood did I obtain, in all? How many dollars was it worth?

15. Allowing that it cost me $178.25 to have the wood named in the preceding problem cut; that my other expenses on account of it amounted to $48.27; and that, after cutting the wood, I sold the land for $113, did I gain or lose by the whole transaction, and how much?

SECTION VIII.

MULTIPLICATION.

47. Definitions and Explanations.

(a.) Multiplication is a process by which we ascertain how many units there are in any number of times a given number.

(b.) The number to be taken is called the Multiplicand; the number showing how many times it is to be taken is called the Multiplier; and the result is called the Product.

(c.) The multiplier and multiplicand are called Factors of the product. The product is called a Multiple of its factors.

Illustration.—In "7 times 8 = 56," or, "8 multiplied by 7 = 56," 8 is the multiplicand, 7 is the multiplier, and 56 is the product. 7 and 8 are factors of 56, and 56 is a multiple of 7 and 8.

(d.) The following tables should be thoroughly mastered.

2 times 1, or once 2 = 2.
2 times 2 = 4.
2 times 3, or 3 times 2 = 6.
2 times 4, or 4 times 2 = 8.
2 times 5, or 5 times 2 = 10.
2 times 6, or 6 times 2 = 12.
2 times 7, or 7 times 2 = 14.
2 times 8, or 8 times 2 = 16.
2 times 9, or 9 times 2 = 18.
2 times 10, or 10 times 2 = 20.

3 times 3 = 9.
3 times 4, or 4 times 3 = 12.
3 times 5, or 5 times 3 = 15.
3 times 6, or 6 times 3 = 18.
3 times 7, or 7 times 3 = 21.
3 times 8, or 8 times 3 = 24.
3 times 9, or 9 times 3 = 27.
3 times 10, or 10 times 3 = 30.

4 times 4 = 16.
4 times 5, or 5 times 4 = 20.
4 times 6, or 6 times 4 = 24.
4 times 7, or 7 times 4 = 28.
4 times 8, or 8 times 4 = 32.
4 times 9, or 9 times 4 = 36.
4 times 10, or 10 times 4 = 40.

5 times 5 = 25.
5 times 6, or 6 times 5 = 30.
5 times 7, or 7 times 5 = 35.
5 times 8, or 8 times 5 = 40.
5 times 9, or 9 times 5 = 45.
5 times 10, or 10 times 5 = 50.

6 times 6 = 36.
6 times 7, or 7 times 6 = 42.
6 times 8, or 8 times 6 = 48.
6 times 9, or 9 times 6 = 54.
6 times 10, or 10 times 6 = 60.

7 times 7 = 49.
7 times 8, or 8 times 7 = 56.
7 times 9, or 9 times 7 = 63
7 times 10, or 10 times 7 = 70.

8 times 8 = 64.
8 times 9, or 9 times 8 = 72.
8 times 10, or 10 times 8 = 80.

9 times 9 = 81.
9 times 10, or 10 times 9 = 90.
10 times 10 = 100.

(e.) The above table shows that a change in the *order* of factors does not affect their product.

NOTE.—When the multiplicand is a concrete number, the principle is applied thus; 8 times 6 books = 6 times 8 books; 9 times 7 cents = 7 times 9 cents.

(f.) The multiplier must always be an abstract number.

ILLUSTRATIONS.—We can multiply a number by 5 or 7, *i. e.* we can take it 5 times or 7 times; but it would be absurd to speak of multiplying it by 5 cents or 7 bushels, *i. e.* of taking it 5 cents or 7 bushels times.

(g.) The product is always of the same denomination as the multiplicand.

ILLUSTRATIONS.—9 times 8 = 72; 9 times .08 = .72; 9 times 8 bushels = 72 bushels.

(h.) In multiplying, we consider each denomination separately, beginning at the right. We reduce as much of each product as possible to units of the next higher denomination, writing the remainder, and adding the reduced units to the product of the next denomination.

1. What is the product of 86.3 multiplied by 7?

86.3 Multiplicand.
7 Multiplier.
———
604.1 Product.

SOLUTION.—Writing the numbers as opposite, we multiply thus:

7 times 3 tenths = 21 tenths = 2 units and 1 tenth. Writing the 1 as the tenths figure of the product, we add the 2 units to the product of the units, thus:

7 times 6 units = 42 units, and 2 units are 44 units = 4 tens and 4 units. Writing 4 as the units figure of the product, we add the 4 tens to the product of the tens, thus:

7 times 8 tens = 56 tens, and 4 tens = 60 tens, which, being the last denomination, we write.

The answer, then, is 604.1.

(i.) *Methods of Proof.* — 1st. Perform the work again carefully as before.

2d. Change the order of the factors.

3d. Separate the multiplier into two or more parts, multiply by the parts separately, and find the sum of the products, thus; 7 times 86.3 = 4 times 86.3 + 3 times 86.3.

NOTE.—Each result should equal that first obtained; otherwise, there is an error, which should be discovered and corrected.

4th. Those who understand Division, may prove the work by dividing the product by one of the factors. The quotient should equal the other factor.

48. Examples and Practical Problems.

(a.) How many are—

1. 3 times 8543?
2. 9 times 2435?
3. 4 times 9378?
4. 6 times 58794?
5. 2 times 930.67?
6. 5 times 279.48?
7. 8 times 6583.2?
8. 7 times 400.674?
9. 4 times 909.99?
10. 5 times 376.804?
11. 9 times .07086?
12. 3 times 777.898?
13. 7 times 1234321?
14. 6 times 875908?
15. 4 times 96.369?
16. 9 times 987654?
17. 2 times 135796?
18. 5 times .0075937?
19. 3 times 867.149?
20. 8 times .008672?
21. 6 times 400.597?
22. 7 times 93.1425?
23. 4 times 796896?
24. 8 times 93472?
25. 8 times 8976?
26. 9 times 10438?
27. 5 times 18.198?
28. 9 times .07658?
29. 4 times 43.019?
30. 8 times 7958.99?

31. What will 7 acres of land cost at $428.36 per acre?

REASONING PROCESS.—If 1 acre of land costs $428.36, 7 acres will cost 7 times $428.36, which may be found by multiplying by 7.

(b.) How much will—

32. 9 barrels of beef cost at $23.125 per bbl.?

33. 7 quintals of fish cost at $3.75 per quintal?

34. 6 bales of cotton weigh if each weighs 296 lb.?

35. 4 acres of land cost at $245.70 per acre?

36. 7 barrels of flour weigh if each weighs 196 lb.?

37. 5 lots of land contain if each contains 3.795 acres?

38. 4 stacks of hay weigh if each weighs 6478 lb.?

39. A man earn in 6 months if he earns $37.75 per month?

40. 2 lots of land cost at $1397 each?

41. 247 barrels of flour cost at $8 per bbl.?

Reasoning Process.—If one bbl. of flour costs $8, 247 bbl. will cost 247 times $8, which is equivalent to 8 times $247. Hence, the answer may be found by multiplying $247 by 8.

Note.—The change in the order of the factors is made because the multiplier is larger than the multiplicand.

How much will—

42. 5794 soldiers earn in 1 month if each earns $8?

43. 1147 bottles hold if each holds 3 pints?

44. 6978 ears of corn contain if each ear contains 4 gi.?

45. 1746 paces measure if each pace measures 3 feet?

46. 957 pounds of sugar cost at $.09 per lb.?

Reasoning Process.—If 1 pound of sugar cost 9 cents, 957 lb. will cost 957 times 9 cents, which is equivalent to 9 times 957 cents, or to 9 times $9.57. Hence, the answer may be found by multiplying $9.57 by 9.

How much will—

47. 1849 pounds of iron cost at $.05 per lb.?

48. 1296 paper boxes cost at $.08 each?

49. 1397 pounds of raisins cost at $.07 per lb.?

50. 249 oranges cost at $.03 each?

51. 2050 bars of soap weigh if each bar weighs 2 lb.

52. 578 papers of pins cost at $.08 per paper?

53 1756 miles = how many furlongs?

Reasoning Process.—Since 1 mile equals 8 furlongs, 1756 miles must equal 1756 times 8 furlongs, equivalent to 8 times 1756 furlongs?

54. 2785 gallons = how many quarts?

55. 5947 square yards = how many square feet?

56. 814 pence = how many farthings?

57. 628 weeks = how many days?

58. 476 acres = how many roods?

59. 1437 pecks = how many quarts?

Note.—The reduction of numbers from a higher denomination to a lower is called **Reduction Descending**. The last seven examples are illustrations of it, as are several of the following.

60. 46 bu. 2 pk. 5 qt. = how many quarts?

Solution.—Since 1 bushel = 4 pecks, 46 bushels must equal 46 times 4 pecks, which is equivalent to 4 times 46 pecks. Hence, multiplying 46 pecks by 4, and adding the 2 pecks gives 186 pecks as the value of 46 bu. 2 pk.

But since 1 peck = 8 quarts, 186 pecks must equal 186 times 8 quarts, which is equivalent to 8 times 186 quarts. Hence, multiplying 186 quarts by 8, and adding in the 5 quarts, gives 1493 quarts as the value of 186 pk. 5 qt., or of 46 bu. 2 pk. 5 qt.

WRITTEN WORK.

46 bu. 2 pk. 5 qt.
186 pk. = 46 bu. 2 pk.
1493 qt. = 46 bu. 2 pk. 5 qt.

61. 87 yd. 3 qr. 2 na. = how many nails?

62. 96 sq. yd. 5 sq. ft. = how many square feet?

63. 46 bu. 3 pk. 7 qt. 1 pt. = how many pints?

64. 42℥ 6ʒ 2℈ = how many scruples?

65. 287 le. 1 m. 3 fur. = how many furlongs?

66. How much will 14 bu. 2 pk. 5 qt. 1 pt. of nuts cost at 3 cents per pint?

67. How much will 42 yd. 1 qr. 3 na. of cloth cost at $.08 per nail?

68. How much will it cost to build a wall 1 m. 1 fur. in length, at $62.50 per furlong?

69. How many square feet are there in a surface 49 ft. long and 7 ft. wide?

70. A garden 96 ft. long and 83 ft. wide is surrounded by a tight fence 7 ft. high. How many square feet does the fence contain?

71. How many square rods are there in a road 1537 rd long, and 4 rd. wide?

72. How many square feet are there in the four walls of a room 18 ft. long, 16 ft. wide, and 9 ft. high?

73. How many cubic feet are there in a pile of wood 43 ft. long, 4 ft. wide, and 4 ft. high?

74. How many cubic inches are there in a block of wood 17 in. long, 9 in. wide, and 8 in. high?

75. How many cubic feet are there in a wall 328 ft. long, 3 ft. thick, and 6 ft. high?

49. Multiplication by Factors.

(a.) When the multiplier is the product of factors, we may find the required product by multiplying by one of the factors, then this product by another, and so on until all the factors are used.

ILLUSTRATION. — Since 18 = 6 times 3, 18 times a number must equal 6 times 3 times the number, which may be found by multiplying by 3, and the product thus obtained by 6.

1. What is the product of 5943 multiplied by 36?

SOLUTION. — Since 36 = 4 × 9, we may find the product of 5943 multiplied by 36 by multiplying by 9, and that product by 4, as in the work written opposite.

$$\begin{array}{r} 5943 \\ 9 \\ \hline 53487 \\ 4 \\ \hline 213948 \end{array}$$

(b.) Find the following products.

2. 9376 × 32.
3. 6794 × 12.
4. 8723 × 24.
5. 5876 × 42.
6. 8967 × 64.
7. 4358 × 63.
8. 2761 × 49.
9. 9547 × 18.
10. 3254 × 21.
11. 6795 × 45.
12. 25.79 × 15.
13. 32.67 × 25.
14. 4.289 × 48.
15. 347.6 × 16.
16. 2974 × 36.
17. 3598 × 42.
18. 4861 × 54.
19. 31.87 × 56.
20. .0059 × 35.
21. 479.5 × 81.

(c.) The most important application of the foregoing principle is made when the multiplier contains 10, 100, 1000, or some other power of 10 as a factor.

Illustrations. — To multiply by 800, or 8 times 100, we may first multiply by 100 by removing the decimal point two places to the right (see **14**), and then multiply that product by 8; or we may simply multiply by 3 and remove the point two places further to the right.

To multiply by 7000, we merely multiply by 7 and remove the point three places to the right.

22. What is the product of 694.7 × 4000?

Solution. — Since 4000 = 4 times 1000, we may find the required product by multiplying 694.7 by 4, and removing the point three places to the right, as illustrated in the written work opposite.

$$\begin{array}{r}694.7\\4000\\\hline 2778800.\end{array}$$

(d.) Find the following products.

23. 5794 × 20.
24. 36.85 × 700.
25. 4.396 × 40.
26. 5764 × 3000.
27. 8342 × 500.
28. 8427 × 60.
29. .0049 × 3000.
30. 73648 × 60.
31. 4957.3 × 300.
32. 2796 × 8000.
33. 356.7 × 4000.
34. .079 × 50000.
35. 81.23 × 90000.
36. .0547 × 800000.
37. 52.78 × 500.
38. 327.1 × 9000.
39. 4358 × 600.
40. .329 × 5000000.
41. 6275 × 4000000.
42. 5279 × 300000.

50. Multiplication by Large Numbers.

(a.) The product of two numbers is equal to the sum of the products obtained by multiplying one of them by the parts into which the other may be divided. (See illustration under 3d method of proof.)

(b.) This principle and the one illustrated in **49**, c, are most frequently applied when the multiplier is a large number.

Illustrations. — To multiply a number by 28, we add 20 times the number to 8 times the number. To multiply a number by 576, we add together 500 times the number, 70 times the number, and 6 times the number.

$$\begin{array}{r} 58.67\\ 635\\ \hline 293.35\\ 1760.1\\ 35202.\\ \hline 37255.45 \end{array}$$

1. What is the product of 58.67 × 635?

Solution. — 635 times 58.67 = 600 times 58.67 plus 30 times 58.67 plus 5 times 58.67; to find which, we first multiply by 5, then by 30, then by 500, and add the several products together, as in the written work opposite.

Note. — The attentive student will observe that the right-hand figure of each product after the first stands under the second figure of the preceding product.

2. 6497 × 83.
3. 2578 × 61.
4. 4379 × 23.
5. 5847 × 45.
6. 6948 × 96.
7. 5387 × 48.
8. 6526 × 59.
9. 3984 × 235.
10. 4295 × 320.
11. 7328 × 170.
12. 2457 × 257.
13. 3869 × 209.
14. 3427 × 234.
15. 5868 × 918.
16. 6258 × 308.
17. 2794 × 437.
18. 3865 × 2536.
19. 6978 × 3215.
20. 2587 × 4208.
21. 6842 × 3007.

22. How much will 24 acres of land cost at $143.75 per acre?

23. How many quarts are there in 19 casks, each containing 237 quarts?

24. How many square rods are there in a field 48 rods long and 36 rods wide?

25. How many square feet are there in a house lot 137 feet long and 98 feet wide?

26. I bought 75 casks of oil, each containing 186 gallons. How many gallons did I buy?

27 I bought a pile of wood 37 ft. long, 4 ft. wide, and 5 ft. high. How many cubic feet did it contain?

28. How many cubic inches are there in a box 37 in. long, 29 in. wide, and 23 in. high?

29. My garden is 96 ft. long and 87 ft. wide, and I wish to build a tight board fence 13 ft. high around it. How many square feet of boards will it contain?

30. In a certain cornfield there are 149 rows, with 136 hills in each row. How many hills are there in the field?

31. A ship's crew of 38 men have provisions enough to allow each man 28 ounces per day for 75 days. How many ounces have they?

32. I bought 37 barrels of flour at $8.375 per bbl., and sold the lot for $369.63. What was my gain?

33. How much must be paid for 18 pounds of tea at 46 cents per pound, and 9 pounds of coffee at 18 cents per pound?

34. I bought 86 hogsheads of molasses at $73.285 per hogshead. How much did it cost me?

35. 23 sq. ft. = how many square inches?

36. 3 T. 15 cwt. 2 qr. 13 lb. = how many pounds?

37. £58 13s. 6d. 2 far. = how many farthings?

38. 15 w. 3 da. 5 h. = how many hours?

39. How much will 15 hogsheads of wine, each containing 63 gallons, cost at $1.42 per gallon?

40. I bought 17 pounds of sugar at $.13 per pound, 16 gallons of molasses at $.37 per gallon, and 23 pounds of tea at $.42 per pound. What was the amount of my purchase?

41. I bought a lot of land 89 ft. long and 53 ft. wide, at $1.48 per square foot. How much did it cost?

42. I sold 7 houses at $2478 each, receiving in payment 147 acres of land at $97.25 per acre, and the rest in money. How much money did I receive?

43. I bought 42 cords of oak wood at $6 per cord, 28 cords of maple at $5.50 per cord, and 24 cords of pine at $3.50 per cord. What was the amount of my purchase?

44. A silversmith bought 13 lb. 5 oz. 15 dwt. of silver at $.05 per dwt. How much did it cost him?

45. I bought 328 casks of nails, each containing 100 pounds, at $.05 per pound, and sold 129 casks at $.07 per pound, and the rest at $.08 per pound. How much did I gain?

46. How many cubic feet in a block 47 ft. long, 35 ft. wide, and 27 ft. high?

[illegible] [illegible] ft. long, [illegible] ft. wide, and 5 ft. [illegible]?

[illegible] I bought [illegible] boxes of sugar, each containing [illegible] pounds, at $[illegible] per lb. What was the amount of my purchase?

[illegible] I bought a house and [illegible] for $[illegible]. I paid $[illegible] for improvements and repairs, $[illegible] for having it painted, and then sold it for a sum equal to three times as much as it had cost me in all. For how much did I sell it?

11. A man bought a bar of gold weighing 1 lb. [illegible] dwt. 13 gr., at $[illegible] per grain. What did it cost him?

SECTION IX.

COMPOUND MULTIPLICATION.

51. Definitions, Explanations, and Problems.

52. Compound Multiplication is the multiplication of Compound Numbers. It does not differ in principle from Simple Multiplication.

1. What is the product of 18lb 5℥ 6ʒ multiplied by 6?

lb	℥	ʒ
18	5	6
		6
110	10	4

Solution.—6 times 6ʒ = 36ʒ, which, since 8ʒ = 1℥, must equal as many ounces as there are times 8 in 36, which are 4 times and 4 remainder. Hence 36ʒ = 4℥ and 4ʒ. Writing the 4ʒ, we add the 4℥ to the product of the ounces, thus:

6 times 5℥ = 30℥, and 4℥ are 34℥, which, since 12℥ = 1lb, must equal as many pounds as there are times 12 in 34, which are 2 times and 10 remainder. Hence 34℥ = 2lb and 10℥. Writing the 10℥, we add the 2lb to the product of the pounds, thus:

6 times 18lb = 108lb, and 2lb are 110lb, which, being the highest denomination, we write.

The answer, then, is 110lb 10℥ 4ʒ.

2. Multiply £27 13s. 6d. 2 far. by 2.

3. Mutliply £19 17s. 4d. 1 far. by 4.

4. Multiply 29 lb. 4 oz. 5 dwt. 7 gr. by 7.

5. Multiply 28 gal. 3 qt. 1 pt. 2 gi. by 9.

6. Multiply 69 bu. 2 pk. 5 qt. 1 pt. by 6.

7. Multiply 12 yd. 2 qr. 3 na. by 8.

8. Multiply 5 le. 2 m. 6 fur. 30 rd. by 9.

9. Multiply 96 yd. 2 ft. 10 in. by 5.

10. Multiply 5 w. 6 da. 20 h. 29 m. 43 sec. by 2.

11. Multiply 9 T. 14 cwt. 1 qr. 17 lb. 5 oz. 14 dr. by 8.

12. Multiply 8° 43′ 15″ by 12.

13. Multiply £14 16s. 8d. 3 far. by 16.

14.* Multiply 7 m. 6 fur. 30 rd. 4 yd. 2 ft. 7 in. by 52.

15. Multiply 9 m. 5 fur. 20 rd. 2 yd. 1 ft. 4 in. by 12.

16. How much will 8 silver watches cost, at £3 15s. 11d. each?

17. How much will a man who earns £4 7s. 9d. 3 far. per month, earn in a year?

18. I bought 16 pieces of cloth, each measuring 32 yd. 3 qr. 1 na. How many yards did I buy?

19. A man bought 18 boxes of sugar, each weighing 4 cwt. 3 qr. 15 lb. 8 oz. How much did the whole weigh?

20. A person owned a rectangular garden, around which he wished to build a fence. To measure it, he took a pole 1 rd. 1 yd. 2 ft. 8 in. long, and found that the length of the field was 12 times, and its width 10 times, the length of the pole. What length of fence would be required?

21. I bought 4 loads of coal, each weighing 1 T. 5 cwt. 2 qr. 21 lb. What did the whole weigh, the quarter being reckoned at 28 pounds. (See **18**, **b.**)

22. An English merchant bought 5 lots of iron, each weighing 6 T. 17 cwt. 3 qr. 17 lb. What was the weight of the whole? (See **18**, **b.**)

---

* In reducing the product of the yards, observe that there will be 2 rods for every 11 yards.

23. How far will a man travel in 9 days, if he travels 33 m. 5 fur. 27 rd. 5 yd. 2 ft. 10 in. per day?

24. What is the weight of a dozen silver table-spoons, each weighing 1 oz. 5 dwt. 17 gr.?

25. I bought in London 17 bags of coffee, each weighing 2 cwt. 1 qr. 22 lb., for £5 13s. 9d. per bag. What was the weight of the whole, and how much did I pay for it?

26. How much will 19 pieces of cloth cost, at 19 cents per nail, if each piece contains 24 yd. 3 qr. 1 na.?

SECTION X.

DIVISION.

52. Definitions and Explanations.

(a.) DIVISION is a process by which we ascertain how many times one given number is contained in another; or by which we ascertain what number is contained a given number of times in another given number.

ILLUSTRATIONS.—"24 = how many times 3?" and, "How many apples at 3 cents each can be bought for 24 cents?" are questions illustrating the first part of the definition (for they require us to find how many times 3 must be taken to equal 24); while "What is one-third of 24?" and "If 3 apples cost 24 cents, how many cents will 1 apple cost?" are questions illustrating the second part of the definition (for they require us to find what number must be taken 3 times to equal 24). Each question requires that 24 should be *divided* by 3.

(b.) The number to be divided is called the DIVIDEND; the number by which we divide is called the DIVISOR; and the result is called the QUOTIENT.

(c.) The REMAINDER, if there be one, will always be of the same denomination as the dividend.

NOTE. — Division is the reverse of Multiplication; for in Multiplication the factors are given and the product is required; while in Division the product and one factor are given, and the other factor is required. The dividend corresponds to the product, and the divisor and quotient to the factors.

(d.) We begin the division with the left-hand or highest denomination of the dividend. We first find how many times the divisor is contained in the smallest part of the dividend which will contain it, writing the result as the first figure of the quotient. We then reduce the remainder, if there be one, to the next lower denomination, and add to it the next figure of the dividend. We find how many times the divisor is contained in this sum, writing the quotient, and reducing the remainder as before, and so proceed till the division is completed.

1. $4767 = how many times $6?

SOLUTION. — $4767 contains $6 as many times as 4767 contains 6.

6)4767 — 3
794

We divide thus: 6 is contained in 47 hundreds, 7 hundreds times, with 5 hundreds remainder (for 7 hundreds times 6 = 42 hundreds, and 5 hundreds added are 47 hundreds). Writing 7 as the hundreds figure of the quotient, we reduce the 5 hundreds to tens, thus:

5 hundreds = 50 tens, and 6 tens added are 56 tens. 6 is contained in 56 tens 9 tens times, with 2 tens remainder (for 9 tens times 6 are 54 tens, and 2 tens added are 56 tens). Writing 9 as the tens figure of the quotient, we reduce the 2 tens to units, thus:

2 tens = 20 units, and 7 units are 27 units. 6 is contained in 27 units 4 units times, with 3 units remainder (for 4 units times 6 are 24 units, and 3 units added are 27 units). We write 4 as the units figure of the quotient. Hence the quotient is 794, and the remainder is 3.

NOTE. — The quotient really represents 794 times $6, or 794 units each equal to $6; and the remainder, 3, represents $3, and is an undivided part of the dividend.

2. What is $\frac{1}{6}$ of $4767?

SOLUTION. — $\frac{1}{6}$ of $4767 may be found by dividing 4767 by 6, as in the example last explained. The quotient is 794, and the remainder is 3; thus showing that $\frac{1}{6}$ of $4767 is $794, with a remainder of $3.

(e.) *Methods of Proof.* — 1st. Multiply the quotient by the divisor, and to the product add the remainder. The result should equal the dividend.

2d. Subtract the remainder from the dividend, and divide the difference thus found by the quotient. The result should equal the divisor.

(f.) The remainder may be made a part of the quotient, by writing it over the divisor so as to form a *fraction.*

ILLUSTRATIONS.—In the first of the above questions, the quotient would become 794$\frac{3}{8}$, *i. e.* 794$\frac{3}{8}$ times $6; while in the second it would become 794\frac{3}{8}$.

(g.) Whenever there is a remainder, the quotient may be carried out to Decimal Fractions, by reducing the remainder to tenths, then to hundredths etc. dividing at each step as already explained.

ILLUSTRATION.—In the above example, reducing the 3 units remainder to tenths, gives 3 units = 30 tenths, and 6 is contained in 30 tenths 5 tenths times. The quotient now takes the form 794.5, instead of 794$\frac{3}{8}$.

53. Examples and Practical Problems.

1. 795 ÷ 5.
2. 680 ÷ 7.
3. 258 ÷ 4.
4. 804 ÷ 3.
5. 8397 ÷ 6.
6. 39.76 ÷ 4.
7. 42.90 ÷ 5.
8. 8.642 ÷ 8.
9. .3579 ÷ 9.
10. 67.32 ÷ 3.
11. 2008 ÷ 6.
12. 7957 ÷ 9.
13. 90.09 ÷ 8.
14. 1.705 ÷ 7.
15. 5723 ÷ 4.
16. $\frac{1}{5}$ of 479.
17. $\frac{1}{3}$ of 586.
18. $\frac{1}{4}$ of 604.
19. $\frac{1}{7}$ of 925.
20. $\frac{1}{9}$ of 3867.
21. $\frac{1}{5}$ of 3006
22. $\frac{1}{8}$ of 6488.
23. $\frac{1}{7}$ of 32.58
24. $\frac{1}{6}$ of 5894.
25. $\frac{1}{8}$ of 30.69
26. $\frac{1}{9}$ of 467.3
27. $\frac{1}{8}$ of 2589.
28. $\frac{1}{4}$ of 30.11
29. $\frac{1}{6}$ of 4204.
30. $\frac{1}{7}$ of 9709.

31. How many barrels of flour at $8 per barrel can be bought for $3576?

REASONING PROCESS.—If 1 barrel can be bought for $8, as many barrels can be bought for $3576 as there are times $8 in $3576, which may be found by dividing 3576 by 8.

32. When 1 barrel of apples can be bought for $2, how many barrels can be bought for $578?

33. How many days will it take a man who earns $3 per day, to earn $198?

34. How many hours will it take a man who travels at the rate of 4 miles per hour, to travel 133 miles?

35. Joseph can learn 5 pages of history per day. How many days will it take him to learn 276 pages?

36. Edward sells 168 marbles at the rate of 7 marbles for 1 cent. How much does he receive for them?

37. When rice was worth 6 cents per pound, a person bought enough to come to 5783 cents. How many pounds did he buy?

38. If a man advances 3 feet at a step, how many steps must he take to advance 1785 feet?

39. A man drew from a bank $9474 in three-dollar bills. How many bills did he receive?

40. How many five-dollar bills will it take to pay a debt of $3675?

41. Among how many men can $5698 be divided, if each man receives $11?

42. How many bottles, each holding 3 pints, can be filled from 8753 pints of cider?

43. 5433 pints equal how many bushels, pecks, quarts, and pints?

SOLUTION.—Since 2 pints equal 1 quart, 5433 pints must equal as many quarts as there are times 2 in 5433, which are 2716 times, with 1 remainder. Hence 5433 pt. = 2716 qt. 1 pt.

Since 8 quarts equal 1 peck, 2716 quarts must equal as many pecks as there are times 8 in 2716, which are, etc.

Therefore, 5433 pints = 84 bu. 3 pk. 4 qt. 1 pt.

WRITTEN WORK.

2)5433 pt. — 1 pt.
8)2716 qt. — 4 qt.
4)339 pk. — 3 pk.
84 bu.

NOTE.—The reduction of numbers from a lower denomination to a higher is called Reduction Ascending. The last example and the seven which follow are illustrations of it.

44. 5975 nails equal how many yards, quarters, and nails?

45. 9645℈ equal how many ℔, ʒ, etc.?

46 3794 farthings equal how many pounds, shillings, pence, and farthings?

47. 1247 furlongs equal how many leagues, miles, and furlongs?

48. 5786 gills equal how many gallons, quarts, etc.?

49. 587 days equal how many months, weeks, and days, allowing 4 weeks to a month?

50. 645 inches equal how many yards, feet, and inches?

51. If 7 acres cost $5943, how much will 1 acre cost?

Reasoning Process.—If 7 acres cost $5943, 1 acre will cost $\frac{1}{7}$ of $5943, which, by dividing 5943 by 7, is found to be $849.

52. If 9 barrels of flour weigh 1764 pounds, how much does 1 barrel weigh?

53. Joseph can buy 588 marbles for 8 dimes. How many can he buy for 1 dime?

54. A man who owned 7359 square feet of land, divided it into 9 equal house-lots. How many square feet were there in each lot?

55. Arthur's travelling expenses during a journey of 8 days were $67.52. What were his average expenses per day?

56. In how many days can 11 men do a piece of work which 1 man can do in 137 days?

57. If 1 man can do a piece of work in 584 days, in how many days can 6 men do it?

58. How much will a peck of wheat cost at the rate of $1.876 per bushel?

59. If 2 bu. 1 pk. or 9 pk. of cranberries cost $10.71, how much will 1 peck cost?

60. If it takes a man 7231 seconds to walk 2 leagues and 1 mile, how many seconds will it take him to walk 1 mile?

61. How many bottles of ink at $.06 per bottle can be bought for $57?

Solution.—If one bottle of ink can be bought for 6 cents, as many bottles can be bought for $57, or 5700 cents, as there are times 6 in 5700, which (found by division) is 950 times. Hence, 950 bottles of ink can be bought.

Note.—In solving problems like these, both divisor and dividend should be reduced to the lowest denomination mentioned in either. In the above example, both were reduced to cents.

62. How many sheets of drawing paper, at $.08 per sheet, can be bought for $9.76?

63. How many pounds of soap, at $.09 per pound, can be bought for $65.97?

64. How many pounds of sugar, at $.08 per pound, can be bought for $114?

65. How many quills, at $.006 each, can be bought for $75?

66. How many steel pens, costing $.008 each, can be bought for $147?

54. Division by Factors.

(a.) When the divisor is the product of factors, we may find the required quotient by dividing by the factors of the divisor.

1. What is the quotient of 1752 ÷ 24?

Solution.—Since 24 = 8 × 3, we may find the required quotient by first dividing by 8 and then by 3.

8)1752
3)219 = quotient by 8.
73 = quotient by 8 × 3, or 24.

Dividing by 8 gives 219 for a quotient, *i. e.* 219 eights. Dividing this by 3 gives 73 for a quotient, *i. e.* 73 units each equal to 3 eights or to 24.

(b.) In the same way perform the following:

2. 8528 ÷ 16?
3. 3456 ÷ 18?
4. 14352 ÷ 48?
5. 47328 ÷ 32?
6. 56592 ÷ 27?
7. 30156 ÷ 28?
8. 25785 ÷ 45?
9. 79344 ÷ 72?

10. What is the quotient of 7391 ÷ 42?

2)7391 — 1
3)3695 — 2
7)1231 — 6
175

Solution. — Since 42 = 2 × 3 × 7, we may find the required quotient by dividing by 2, then by 3, and then by 7.

Dividing by 2 gives 3695 for a quotient and 1 for a remainder, *i. e.* 3695 twos and 1 unit remaining.

Dividing 3695 by 3 gives 1231 for a quotient, and 2 for a remainder, *i. e.* 1231 units each equal to 3 times 2 or to 6, and 2 twos remaining.

Dividing 1231 by 7 gives 175 for a quotient and 6 for a remainder, *i. e.* 175 units, each equal to 7 times 3 times 2, or to 42, and 6 sixes remaining.

The quotient, then, is 175, and the remainder is 1 + 2 times 2 + 6 times 3 times 2 = 41.

(c.) From the above considerations, we infer that the true remainder, after any division by factors, may be found by multiplying the remainder of each division after the first by the divisors of all the preceding divisions, and then adding these products to the first remainder.

(d.) Find the quotient and remainder in the following examples:

11. 5855 ÷ 36?
12. 8794 ÷ 28?
13. 3795 ÷ 42?
14. 2767 ÷ 14?
15. 3859 ÷ 81?
16. 2987 ÷ 51?
17. 5321 ÷ 16?
18. 6743 ÷ 25?
19. 5425 ÷ 36?
20. 3891 ÷ 27?

(e.) One of the most convenient applications of the division by factors is made when the divisor contains 10, 100, 1000, or some higher power of 10, as a factor.

21. What is the quotient of 4573 ÷ 800?

Solution. — Since 800 = 8 × 100, we may find the required quotient by dividing by 100 and then by 8, *i. e.* by removing the point two places to the left (see **17**) and dividing by 8.

Removing the point gives 45 for a quotient and 73 for a remainder. Dividing 45 by 8 gives 5 for a quotient and 5 for a remainder. The quotient, then, is 5, and the remainder is 5 times 100 + 73, *i. e.* 500 + 73 = 573.

(f.) Perform the following examples:

22. 5975 ÷ 600?
23. 3794 ÷ 20?
24 48697 ÷ 5000?
25. 32697 ÷ 50?
26. 67328 ÷ 900?
27. 69487 ÷ 1400?
28. 31754 ÷ 2100?
29. 168597 ÷ 1200?
30. 32754 ÷ 240?
31. 59867 ÷ 180?
32. 37642 ÷ 360?
33. 69483 ÷ 800?

(g) Had we wished for the complete quotient in the 21st example, we should have considered the first quotient to be 45.73, instead of 45 and 73 remainder. We should then have divided 45.73 by 8, giving $5.71\frac{5}{8}$, or 5.71625 for the full quotient.

(h.) Find the complete quotient in each of the examples under f.

55. Divisor a Large Number.

(a.) When the divisor is large, it is not always easy to determine the true quotient figure directly. In such cases, the number expressed by one or two of the left-hand figures of the divisor, may be selected as a sort of TRIAL DIVISOR; but to test the correctness of the quotient figure thus obtained, it will be necessary to find the product of the divisor by the quotient, and the remainder after subtracting that product from the dividend.

NOTE.—The product should be either equal to or less than the dividend, and the remainder should be less than the divisor.

1. What is the quotient of 537 ÷ 82?

$$\begin{array}{r} 82)537(6 \\ 492 \\ \hline 45 \end{array}$$

SOLUTION.—Writing the divisor on the left of the dividend, as opposite, we make 8 the trial divisor.* Since 8 is contained 6 times with a remainder in 53, we write 6 as the *probable* quotient figure. To see whether it be correct or not, we multiply the divisor by it. The product is 492, which, being less than the dividend, shows that the quotient figure is not too large. Subtracting 492 from 537 leaves a remainder of 45, which, being less than the divisor, shows that the quotient figure is not too small. Being neither too large nor too small, it must be correct. Hence, the quotient is 6 and the remainder is 45, or the complete quotient is $6\frac{45}{82}$.

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* For 82 is contained in 537 *about* the same number of times that 80 is, or that 8 is contained in 53.

(b.) Perform the following examples:

2. 239 ÷ 43?
3. 627 ÷ 81?
4. 518 ÷ 64?
5. 697 ÷ 79?
6. 416 ÷ 68?
7. 213 ÷ 47?
8. 438 ÷ 75?
9. 216 ÷ 45?
10. 715 ÷ 92?
11. 317 ÷ 53?
12. 3817 ÷ 617?
13. 1358 ÷ 421?
14. 3157 ÷ 835?
15. 3773 ÷ 937?
16. 6179 ÷ 875?
17. 3579 ÷ 498?
18. 3163 ÷ 369?
19. 43796 ÷ 8236?
20. 25794 ÷ 7167?
21. 36854 ÷ 5947?

56. Long Division.

(a.) When, in performing the division, we write only the divisor, dividend, quotient, and final remainder, the process is called SHORT DIVISION; but when, in addition to these, we write the products of the divisor by the successive figures of the quotient, and also the remainder of each subtraction, the process is called LONG DIVISION.

NOTE.—Short Division was illustrated in the articles from **52** to **54**, and Long Division was in part illustrated in **55**.

(b.) Long Division differs from Short Division in these respects, viz.: First. That having obtained a quotient figure, we multiply the divisor by it, *writing the product* under the part of the dividend considered.

Second. That we subtract this product from the part of the dividend considered, *writing the remainder*.

Third. That we *write* the next figure of the dividend after the remainder, to form the next partial dividend.

(c.) The essential difference, then, is that in Long Division more of the work is written than in Short Division.

What is the quotient of 31536 ÷ 497?

```
497)31536(63
    2982
    ----
     1716
     1491
     ----
      225
```

SOLUTION.—497 is so near 500, that we make 5 the trial divisor. Since 5 is contained 6 times in 31, we make 6 the first figure of the quotient, and infer that 497 is contained 6 tens times in 3153 tens. Multiplying 497 by 6 tens gives 2982 tens, which, subtracted from 3153 tens, leaves 171 tens.

171 tens = 1710 units, to which adding the 6 units

gives 1716 units to be next divided. Since 5 is contained 3 times in 17, we make 3 the next figure of the quotient, and infer that 497 is contained 3 units times in 1491 units. Multiplying 497 by 3 units gives 1491 units, which, subtracted from 1716 units leaves 225 units.

This gives 63 for the quotient and 225 for the remainder, or $63\frac{225}{497}$ for the complete quotient.

Note.—If it should be desirable to carry the quotient out in decimal fractions, we should reduce the 225 units to tenths thus:

225 units = 2250 tenths to be divided. Since 5 is contained 4 times in 22, we make 4 the next figure of the quotient, and infer that 497 is contained 4 tenths times in 2250 tenths. Multiplying 497 by 4 tenths gives 1988 tenths, which, subtracted from 2250 tenths, leaves 262 tenths.

262 tenths = 2260 hundredths, to be divided as before.

In the written work opposite, we have carried out the quotient to 4 places of decimal fractions, thus obtaining 63.4527 for the quotient, and .0081 for the remainder, or $63.4527\frac{81}{497}$ for the complete quotient.

$$\begin{array}{r}
497)31536.(63.4527 \\
2982 \\
\hline
1716. \\
1491 \\
\hline
225.0 \\
198\ 8 \\
\hline
26.20 \\
24\ 85 \\
\hline
1.350 \\
994 \\
\hline
.3560 \\
3479 \\
\hline
.0081
\end{array}$$

57. Examples and Practical Problems.

1. 2375 ÷ 52?
2. 3796 ÷ 43?
3. 4986 ÷ 72?
4. 5389 ÷ 91?
5. 2456 ÷ 33?
6. 4795 ÷ 49?
7. 1396 ÷ 37?
8. 4258 ÷ 46?
9. 2397 ÷ 68?
10. 675.4 ÷ 83?
11. 3586 ÷ 412?
12. 38.47 ÷ 212?
13. 5978 ÷ 613?
14. 2974 ÷ 391?
15. .6137 ÷ 196?
16. 87943 ÷ 45?
17. .62379 ÷ 83?
18. 25978 ÷ 127?
19. 36594 ÷ 41?
20. 63974 ÷ 25?
21. 39547 ÷ 62?
22. 542.96 ÷ 83?
23. 47698 ÷ 45?
24. 63897 ÷ 328?
25. 1.59468 ÷ 1274?
26. 294875 ÷ 3079?
27. 205879 ÷ 4983?
28. 62574 ÷ 99?
29. 37.9856 ÷ 5387?
30. 6257.94 ÷ 8215?

31. How many months will it take a man who earns $97 per month, to earn $2425 ?

32. A drover received $1363 for cattle, at $29 per head. How many cattle did he sell?

33. How many suits of clothes, at $32 each, can be bought for $1024?

34. If one acre will produce 19 bushels of wheat, how many acres will produce 1294 bushels?

35. A person bought 1224 yards of cloth, in pieces each containing 34 yards. How many pieces did he buy?

36. In how many hours will a cistern, capable of containing 32571 gallons, be filled through a pipe which discharges 185 gallons per hour?

37. How many barrels, each containing 196 pounds, can be filled from 23741 pounds of flour?

38. 937467 drams equal how many T. cwt. qr. lb. oz. and dr.?

39. 623458 grains equal how many lb. oz. dwt. and gr.?

40. 3517965 sq. in. equal how many sq. yd. sq. ft. and sq. in.?

41. 527943 cu. in. equal how many cu. ft. and cu. in.?

42. 379421 farthings equal how many £ s. d. and far.?

43. How many straw hats, at $.97 each, can be bought for $53.68? (See **53**, note after 61st problem)

44. If one passenger can be carried on a railroad for $1.41, how many can be carried for $31.02?

45. A man paid $28.625 for eggs, at $.125 per dozen. How many dozen did he buy?

46. How much will one horse cost if 14 horses cost $2534, the horses being of equal value?

47. A flour dealer exchanged 358 barrels of flour, worth $6.37 per bbl., for flour worth $7.83 per bbl. How many barrels did he receive?

48. I sold my house for $2846, receiving in payment a lot of land containing 37 acres, and $1378 in cash. How much ought the land to be worth per acre?

49. If 17 pounds of Young Hyson tea cost \$10.20, how much will 83 pounds cost?

50. I bought 38 cows at \$22 each, and sold one of them for \$35 and another for \$43. For how many dollars each must I sell the rest, to gain \$196 by the transaction?

SECTION XI.

58. COMPOUND DIVISION.

(a.) COMPOUND DIVISION is the division of Compound Numbers.

(b.) It is performed in accordance with the principles illustrated in Simple Division.

NOTE.—All questions in Compound Division are included in the second part of the definition of Division in **52, a.**

1. What is $\frac{1}{8}$ of £53 17s.?

SOLUTION.—$\frac{1}{8}$ of £53 = £6, with £5 remaining. But £5 = 5 times 20s. = 100s., to which adding the 17s. gives 117s. $\frac{1}{8}$ of 117s. = 14s., with 5s. remaining. But 5s. = 5 times 12d. = 60d., and $\frac{1}{8}$ of 60d. = 7d., with 4d. remaining. But 4d. = 16 far., and $\frac{1}{8}$ of 16 far. = 2 far. Hence, $\frac{1}{8}$ of £53 17s. = £6 14s. 7d. 2 far.

	£	s.	d.	far.
8)	53	17	0	0
	6	14	7	2

2. What is $\frac{1}{9}$ of 34 rd. 4 yd. 2 ft. 8 in.?

SOLUTION.—$\frac{1}{9}$ of 34 rd. = 3 rd., with 7 rd. remaining. But 7 rd. = 7 times $5\frac{1}{2}$ yd. = *$38\frac{1}{2}$ yd. = 38 yd. 1 ft. 6 in., to which adding the 4 yd. 2 ft. 8 in. gives 42 yd. 3 ft. 14 in. $\frac{1}{9}$ of 42 yd. = 4 yd., with 6 yd. remaining. But 6 yd. = 6 times 3 ft. = 18 ft., to which adding the 3 ft. gives 21 ft. $\frac{1}{9}$ of 21 ft. = 2 ft., with 3 ft. remaining. But 3 ft. = 36 in., to which adding the 14 in. gives 50 in. $\frac{1}{9}$ of 50 in. = 5 in., with 5 in. remainder, or $5\frac{5}{9}$ in.

Hence, $\frac{1}{9}$ of 34 rd. 4 yd. 2 ft. 8 in. = 3 rd. 4 yd. 2 ft. $5\frac{5}{9}$ in.

	rd.	yd.	ft.	in.
9)	34	4	2	8
	3	4	2	$5\frac{5}{9}$

---

* For 7 times 5 yd. = 35 yd., and 7 times $\frac{1}{2}$ yd. = $\frac{7}{2}$ yd. = $3\frac{1}{2}$ yd., which, added to 35 yd., gives $38\frac{1}{2}$ yd.

3. What is $\frac{1}{3}$ of £16 17s. 5d. 1 far.?

4. What is $\frac{1}{3}$ of 14 T. 5 cwt. 2 qr. 20 lb. 6 oz. 11 dr.?

5 What is $\frac{1}{8}$ of 51 T. 3 qr. 9 oz.?

6. What is $\frac{1}{5}$ of 64 lb. 11 oz. 15 dwt. 21 gr.?

7. What is $\frac{1}{3}$ of 5℔ 8℥ 5ʒ 2℈ 17 gr.?

8. What is $\frac{1}{11}$ of 437 bu. 3 pk. 5 qt. 1 pt.?

9 What is $\frac{1}{14}$ of 63 bu. 1 pk. 1 pt.?

10. What is $\frac{1}{5}$ of 33 gal. 2 qt. 1 pt. 3 gi.?

11. What is $\frac{1}{8}$ of 94 yd. 3 qr. 3 na.?

12. What is $\frac{1}{3}$ of 83 yd. 1 qr. 3 na.?

13. What is $\frac{1}{7}$ of 33 rd. 1 yd. 2 ft. 2 in.?

14. What is $\frac{1}{5}$ of 49 rd. 5 yd. 2 ft. 11 in.?

15. What is $\frac{1}{3}$ of 17 A. 1 R. 28 rd.?

16. What is $\frac{1}{12}$ of 17 sq. rd. 8 sq. yd. 137 sq. in.?

17. What is $\frac{1}{23}$ of 49 C. 3 cd. ft. 15 cu. ft. 813 cu. in.?

18. What is $\frac{1}{5}$ of 213 C. 5 cd. ft. 13 cu. ft. 1315 cu. in.?

19. What is $\frac{1}{16}$ of 75 wk. 3 da. 27 min. 18 sec.?

20. What is $\frac{1}{6}$ of £48 11d.?

21. What is $\frac{1}{9}$ of £87 13s. 5 d. 2 far.?

22. What is $\frac{1}{6}$ of 1 T. 1 cwt. 1 qr. 1 lb.?

23. What is $\frac{1}{19}$ of 27 T.?

24. What is $\frac{1}{13}$ of 12℔ 7ʒ?

25. What is $\frac{1}{29}$ of 38 m. 7 fur. 20 rd. 1 in.?

26. What is $\frac{1}{17}$ of 8 m.?

27. What is $\frac{1}{11}$ of 1 C. 4 cd. ft.?

28. What is $\frac{1}{7}$ of 48 rd. 3 yd. 7 in.?

29. What is $\frac{1}{16}$ of 11 A. 1 R. 29 sq. rd. 11 sq. in.?

30. What is $\frac{1}{27}$ of 39 m.?

59. Practical and Miscellaneous Problems.

1. If 8 acres of land cost £137 14s. 6d. 1 far., how much will 1 acre cost?

2. If 7 equal casks of wine contain 459 gal. 1 qt. 1 pt. 3 gi., how much does each cask contain?

3. I divided 10 A. 1 R. 13 sq. rd. 25 sq. yd. 7 sq. ft. 125 sq. in of land into 11 equal lots, how much will each lot contain?

4. A railroad 43 m. 5 fur. 23 rd. long, was divided into 4 equal sections. How long was each section?

5. 11 equal bars of silver weighed 31 lb. 5 oz. 16 dwt. 15 gr. What was the weight of each bar?

6. When 8 cwt. 2 qr. 20 lb. 14 oz. of iron can be bought for $9, how much can be bought for $1?

7. If 8 barrels of apples contain 22 bu. 3 pk., how much does 1 barrel contain?

8. A corn dealer bought 47 bu. 3 pk. 5 qt. of corn at one time, 85 bu. 2 pk. 6 qt. at another, and 65 bu. 1 pk. 7 qt. at another. He sold it in 4 equal lots. How much was there in each lot?

9. A man bought 7 stacks of hay, each containing 5 T. 17 cwt. 3 qr. 19 lb. He carried the hay to market in 21 equal loads. What was the weight of each load?

10. An apothecary mixed together 1℔ 5℥ 7ʒ 2℈ 15 gr. of one drug; 2℔ 11℥ 5ʒ 1℈ 18 gr. of another; 1℔ 5℥ of another; and 7℥ 5ʒ 1℈ 17 gr. of another. He then divided the whole into 20 equal portions. What was the weight of each portion?

11. A goldsmith who had 19 oz. 13 dwt. 15 gr. of gold dust, bought 5 lots, each weighing 13 oz. 9 dwt. 17 gr. He afterwards sold 1 lb. 3 oz. 5 gr. of it, and made the rest into 3 equal bars. What was the weight of each bar?

12. I gave 37 yards of cloth, at $4 per yard, for apples at $3 per barrel. How many barrels of apples did I obtain?

13. A man's annual income was $1500. His house rent was $275 per year; his family expenses were 3 times as much as his house rent, lacking $42.70; and his travelling and miscellaneous expenses were $83.25 less than $\frac{1}{3}$ of his family expenses. How much could he save per year?

14. A drover bought 27 head of cattle, at $38.25 per head. It cost him $25.38 to drive them to market, and he sold them at $42.35 per head. What did he gain on them?

15. I bought 17 cases of shoes, each case containing 50 pairs, at $1.97 per pair. It cost me 75 cents per case to have them carried to my store, and I sold them for $2.31 per pair. What did I gain on them?

16. If a man's income is $365 per year, and he spends $289.75 per year, how much will he save in 9 years?

17. I bought 127 acres of land at $19.50 per acre, and sold it at $27.25 per acre. How much did I gain?

18. A mechanic finds that on the average he earns $1.875 per day, and works 22 days per month. Now if his necessary expenses are $28.25 per month, how many years and months will it take him to lay up $1000?

19. A man who had $2625 bought 123 barrels of flour at $7 per barrel, and invested the rest of his money in flour at $6 per barrel How many barrels were there in the last lot?

20. If 9 pieces of cloth cost £15 17s. 10d. 3 far. per piece, and sell for £19 4s. 3d. per piece, what is the gain?

21. How many horse-shoes each weighing 12 oz. 5 dr. can be made from 9 bars of iron each weighing 19 lb. 8 oz.? (See note following the 61st problem in **53.**)

22. How many steps must a boy take in going to and returning from school, provided that he lives 2345 yards from the schoolhouse, and that his steps are 2 ft. 6 in. in length?

23. An Englishman gave £148 17s. 6d. for tea at 2s. 6d. per pound, and sold it for 3s. per pound. What did he gain on it?

24. A farmer bought 37 lb. of sugar at $.09 per pound; 14 lb. of coffee at $.17 per pound; 3 lb. of tea at $.45 per pound; 3 yd. of cloth at $2.63 per yard; a hat for $3.75; and a coat for $9.37; giving in payment 29 lb. of butter at $.19 per pound, and the rest in money. How much money did he pay?

25. How much will a pile of wood 84 ft. long, 8 ft. high, and 4 ft. wide, cost at $3.67 per cord?

26. How much will a lot of land, 80 rods long and 72 rods wide, cost at $87.30 per acre?

27. My garden is 229 ft. long and 137 ft. wide. Within the garden, but next to the outer edge, and extending completely around it, I have made a gravel-walk 3 ft. wide. How many square feet does the walk contain?

NOTE.—In solving such problems as the above, the pupil should draw a figure or plan of the surface to be measured.

28. A man who owns a garden 128 ft. long and 116 ft. wide, lays out a flower-bed 2 ft. wide, next the outer edge, and extending completely around it. How many square feet does the bed contain?

29. How many square feet are there in the four walls of a room 16 ft. long, 14 ft. wide, and 9 ft. high?

30. How many square feet are there in the top and walls of a hall 37 ft. long, 34 ft. wide, and 13 ft. high?

31. My orchard is 24 rods long and contains 3 acres. How wide is it? How much will it cost to build a fence round it at $1.75 per rod?

32. George has $157, William has $19 more than twice as much as George, Joseph has $69 more than George and William together, and Robert has $25 less than $\frac{1}{3}$ as much as Joseph. How many dollars has each of the boys?

33. A man who had 325 gal. 2 qt. 1 pt. of wine, sold 239 gal. 3 qt. 1 pt. at 25 cents per pint, and put the rest into bottles, each holding 1 pt. 3 gi. How many pints did he sell? How much did he receive for what he sold? How many bottles did he fill?

SECTION XII.

CONTRACTIONS AND ABBREVIATED PROCESSES.

60. To multiply by two or more Figures at one Operation.

(a.) We may multiply by two or more figures at one operation, by observing that the lowest denomination of the product will be found by multiplying the lowest denomination of the multiplicand by the lowest denomination of the multiplier; that the next denomination of the product will be found by multiplying the second of the multiplicand by the first of the multiplier, and the first of the multiplicand by the second of the multiplier, etc.

1. What is the product of 2476 × 48?

PRELIMINARY EXPLANATION.—We shall obtain the units of the product by multiplying 6 units by 8 units. We shall obtain the tens of the product by multiplying 7 tens by 8 units, and 6 units by 4 tens. We shall obtain the hundreds of the product by multiplying 4 hundreds by 8 units, and 7 tens by 4 tens. We shall obtain the other denominations in a similar manner. Hence the following solution.

$$\begin{array}{r} 2476 \\ 48 \\ \hline 118848 \end{array}$$

SOLUTION.—8 times 6 units are 48 units. 8 times 7 tens are 56 tens, and 4 tens (from the last product) are 60 tens, and 4 tens times 6 units or 24 tens are 84 tens; 8 times 4 hundreds are 32 hundreds, and 8 hundreds (from the last product) are 40 hundreds, and 4 tens times 7 tens or 28 hundreds are 68 hundreds. 8 times 2 thousands, etc.

Perform the following:

2. 3256 × 23.
3. 9742 × 54.
4. 1649 × 17.
5. 8264 × 19.
6. 93648 × 81.
7. 73274 × 123.
8. 59927 × 111.
9. 86975 × 426.

61. To multiply by 99, 999, etc.

(a.) Since 99 = 100 — 1, 999 = 1000 — 1, etc. it follows that 99 times a number must equal 100 times the number, minus the number. 999 times a number must equal 1000 times the number, minus the number. 98 times a number must equal 100 times the number, minus twice the number, etc.

1. What is the product of 67248 × 99?

Solution.—Since 99 = 100 — 1, the required product must equal 100 times 67248, minus 67248; which gives the written work in the margin.

$$\begin{array}{r}6724800\\67248\\\hline 6657552\end{array} = \text{Product.}$$

(b.) Find the following products:

2. 8496 × 999.
3. 34268 × 9999.
4. 4876 × 99.
5. 6251 × 98.
6. 84649 × 997.
7. 846723 × 999999.

62. To Multiply by any convenient Fractional Part of 100, 1000, etc.

(a.) It is obvious that the product of a number multiplied by $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$ of 10, must be $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$ of 10 times the number. The product of a number multiplied by $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$ of 100, must be $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$ of 100 times the number; and so of any fractional part of 10, 100, 1000, 10000, etc.

1. What is the product of 9473 × $166\frac{2}{3}$?

Solution.—Since $166\frac{2}{3} = \frac{1}{6}$ of 1000, the required product must equal $\frac{1}{6}$ of 1000 times 9473, or $\frac{1}{6}$ of 9473000, which is $1578833\frac{2}{6}$.

(b.) Find the following products:

2. 6438 × $12\frac{1}{2}$.
3. 8651 × 25.
4. 7327 × 125.
5. 4839 × $33\frac{1}{3}$.
6. 5479 × $16\frac{2}{3}$.
7. 8625 × $11\frac{1}{9}$.

63. One Part of the Multiplier a Factor of Another.

1. What is the product of 8693 × 328?

8693
328
69544
278176
2851304

Solution.—328 = 32 tens + 8 units, and 32 tens = 4 tens times 8. Hence, to find the required product, we first multiply by 8, then multiply that product by 4 tens, and add the two results together.

2. What is the product of 864727 × 25175?

864727
25175
21618175000
151327225
21769502225

Solution.—25175 = 25000 + 175, and 175 is 7 times 25. Hence, to find the required product, we first multiply by 25, writing the product as thousands, and then multiply this product by 7, writing the result as units, and add the two results together.

(c.) What is the product of—

3. 9783 × 427?
4. 6324 × 186?
5. 23463 × 2512½?
6. 63728 × 3612?
7. 54533 × 96486?
8. 37496 × 927?
9. 47938 × 19899?
10. 27645 × 54279?

64. Abbreviated Method for Long Division.

(a.) The labor of writing the successive products in Long Division may be avoided by subtracting each product when we obtain it.

1. What is the quotient of 54786 ÷ 217?

217)54786(252$\frac{102}{217}$
1138
536
102 = Rem.

Solution.—217 is contained in 547 twice. 2 times 7 are 14, which taken from 17 leaves 3. 2 times 1 are 2 and 1 are 3, which taken from 4 leaves 1. 2 times 2 are 4, which taken from 5 leaves 1. The first remainder is therefore 113, to which annexing the 8 gives 1138 as the number to be divided. 217 is contained in 1138 5 times. 5 times 7 are 35, which taken from 38 leaves 3. 5 times 1 are 5, and 3 are 8, which taken from 13 leaves 5, etc.

(b.) Find the following quotients:

2. 96348 ÷ 27.
3. 26953 ÷ 98.
4. 825649 ÷ 73.
5. 468251 ÷ 196.
6. 833443 ÷ 783.
7. 295654 ÷ 813.

65. To Divide by any convenient Fractional Part of 10, 100, 1000, etc.

(a.) Since 100 = 4 times 25, or 8 times 12½, it follows that the quotient of a number divided by 25 must be 4 times the quotient of the same number divided by 100; the quotient of a number divided by 12½ must be 8 times the quotient of the same number divided by 100. A similar thing will be true of any other fractional part of 10, 100, 1000, etc.

1. What is the quotient of 397463 ÷ 125?

Solution.—Since 1000 = 8 times 125, the quotient of a number divided by 125 must be 8 times the quotient of the same number divided by 1000. But 397463 ÷ 1000 = 397.463, and 8 times this result is 3179.704, which equals the quotient of 397463 ÷ 125.

Note.—The above form gives the complete quotient. The entire part of the quotient is found at the left of the decimal point: the remainder may be found by dividing the figures at the right of the decimal point by the number by which we multiplied the quotient of the division by 100 or 1000.

Thus, in the above example, the entire part of the quotient is 3179, and the remainder is ⅛ of 704, or 88.

(b.) Find the following quotients:

2. 8374 ÷ 125.
3. 2596 ÷ 16⅔.
4. 83742 ÷ 250.
5. 94163 ÷ 25.
6. 825349 ÷ 333⅓.
7. 64243 ÷ 12½.

SECTION XIII.

BILLS AND ACCOUNTS.

66. Bills.

(a.) When one person sells merchandise to another, it is customary for him to write out for the purchaser a statement of the articles sold, with their prices. Such a statement is called a Bill.

Note.—There are few persons, in any walk of life who are not at some time or other called on to pay a bill, or to make out or receipt one. Indeed the bill is the most common business paper. The student, then, will appreciate the importance of thoroughly mastering the following explanations.

1. On the 10th day of July, 1857, J. G. Lester, of Providence, sold to F. L. Gay, for cash (*i. e.* for ready money), 16 pounds of sugar, at 14 cents per pound; 3 pounds of tea at 68 cents per pound; and 1 barrel of flour for $9.50. Mr. Lester made the following bill:

1.

Providence, July 10, 1857.

Mr. F. L. Gay

Bought of J. G. Lester.

16 lb. Sugar @ 14 ct.	*$2.24*
3 lb. Tea " 68 "	*2.04*
1 bbl. Flour	*9.50*
	$13.78

Received Payment,

J. G. Lester.

(b.) In considering the above bill, we observe—1st, the Date, "Providence, July 10, 1857," which tells when and where the transactions took place.

2d, the Heading, "F. L. Gay bought of J. G. Lester," which shows the names of the parties and the nature of the transaction.

3d, the Statement of the articles bought, with their prices and amount.

4th, the **Receipt**, consisting of the **Signature** "J. G. Lester," written after the words "Received Payment."

Remarks.—1. The date is sometimes placed at the bottom of the bill.

2. The heading "*F. L. Gay to J. G. Lester, Dr.*" is much used instead of the preceding, and means that F. L. Gay is **Debtor** to J. G. Lester, *i. e.* that he *owes* him for the articles mentioned in the bill.

3. The words "*Received Payment*" would have no value unless Mr. Lester had written his name under them, or authorized some one to do it for him.

4. If D. S. Stone had received the money for Mr. Lester, he would have receipted the bill by writing after the words "Received Payment" some form similar to the following:

"D. S. Stone for J. G. Lester," or
"J. G. Lester by D. S. Stone."

(c.) The following bills furnish further illustrations of this subject. Let the class find the amount of each.

2.

Dedham, July 1, 1857

Town of Dedham
To C. A. Brigham, Dr

For 3 months' services as Teacher of Grammar-School in District No. 8, at $75 per month.............. $225

Received Payment,
C. A. Brigham,
by T. Baker

3.

Providence, July 24, 1857

Mr. Francis Peavey

To G. T. Gladding & Co Dr.

June 3.	For 3 yd. Broadcloth	@	$5.29
" 17.	" 7 " Cassimere	"	1.62
July 6.	" 3 Silk Cravats	"	1.45
" 8.	" 3 ps. Kid Gloves	"	.93
			Cr.
July 20.	By Cash		$18.00

Balance due G. T. G. & Co.

Note.—The date of the last bill, July 24, 1857, shows when it was made out. The dates at the left show when each transaction took place. The "Credit Entry," "July 20. By Cash $18.00," shows that on the 20th of July Mr. Peavey paid $18.00 towards the bill. Opposite the words "Balance due G. F. G. & Co." should be written the sum still due on the bill.

If Mr. Peavey should pay this balance Aug. 1st, the receipt would be written thus:

"Aug. 1, 1857. Received Payment,
"G. F. Gladding & Co."

Note.—When the bill is receipted, Mr. Peavey should keep it as evidence that he has paid it in full.

(e.) Make out bills for each of following transactions:

4. Suppose that you are a grocer, and sell to some person in your neighborhood, for cash, 7 gallons of molasses at 39 cents per gallon; 1 box soap containing 28 lb. at $.09 per lb.; 1 cheese weighing 18 lb. at $.09 per lb.; and 31 lb. butter at 21 cents per lb.

5. Suppose that you are a farmer, and sell some person 67 bu. potatoes at $.40 per bu.; 49 bu. Indian corn at $.75 per bu.; 27 bu. rye at $1.50 per bu.; 15 bu. wheat at $2.12 per bu.; and 58 bbl. apples at $1.87 per bbl.; and in payment receive 3 cows at $24 each; 1 plough worth $15.75; and the balance in cash.

6 Sell one of your companions the following articles for cash, and make out the bill:

6 doz. Warren's Common-School Geography, at $11.25 per doz
6 doz. Warren's Physical Geography, at $11.75 per doz.
9 doz. Colburn's First Book of Arithmetic, at $2.87 per doz.
6 doz. Colburn's Common-School Arithmetic, at $5.50 per doz.
6 doz. Colburn's Arithmetic and Applications, at $9 per doz.
8 doz. Leach's Complete Speller, at $2.40 per doz.
9 doz. Greene's Introduction to English Grammar, at $2.88 per doz.
4 doz. Greene's Analysis, at $5.25 per doz.
3 copies Webster's Quarto Dictionary, at $5.50 each.
5 copies Brande's Encyclopedia, at $3.87 per copy, and 1 set, 14 volumes, American Encyclopedia, at $2.12 per volume.

67. Accounts.

Note.—Every careful man, whether he does much or little business, will keep a record of his business transactions. The necessity of doing this in a convenient and systematic manner, gives rise to the science and art of ACCOUNTS, or of BOOK-KEEPING—the essential principles of which ought to be so far familiar to all, as to enable them to keep at least some simple form of accounts. We have, therefore, arranged the following described transactions so that, while they furnish problems requiring a practical application of arithmetical principles, they will serve to illustrate a form of book-keeping adapted to that large class of farmers and mechanics who do but little business except for cash, and hence have few entries to make. Each pupil should suppose that his own transactions are here described, and that he wishes to determine the exact state of his affairs Oct. 1, 1857, and to ascertain whether he has gained or lost money during the preceding month. His work should be written out with care and system, whether he adopts the forms of the accountant, or regards the problems merely as arithmetical. Should the teacher prefer the latter, the explanations preceding the problems may be omitted.

(a.) Accounts are written statements of business transactions, so arranged as to show at any time the exact state of our affairs with each individual with whom we have dealings.

(b.) Book-Keeping is the science of accounts.

(c.) Every statement of a transaction written in an account-book is called an Entry.

(d.) We debit or charge a man with what he owes us or receives from us on account.

(e.) We credit a man with what we owe him or receive from him on account.

(f.) We may keep an account with articles of property, as though they were individuals, by charging them with what they cost us, and crediting them with what they bring us in. The most important of this class of accounts is the Cash Account, which should be kept by every one.

(g.) In keeping a cash account, CASH is regarded as an individual whom we charge with the money in our possession when we open the account, and with all which we afterwards receive, and whom we credit with all the money we pay out.

Illustration.—If I have $100, and afterwards receive $50, and pay out $40, I should debit Cash for $100 and for $50, and credit Cash for $40. The difference, $110, between the sums of the debit and credit entries, shows the amount of money I have on hand.

(h.) In the simplest form of accounts, but one book is used. Two pages, facing each other, are given to each account—the left-hand page being devoted to the debit entries, and the right-hand page to the credit entries. The pages are ruled, and the entries written as illustrated on pages 96 and 97, which contain George Brown's account, and a part of Cash Account.

Note.—The lines at the foot of Cash Account, and the entries "By Cash to Balance" and "To Balance from old acc't," are designed merely to show how the account should be closed. (See pages 96 and 97.)

1. On the first of September, 1857, you take an inventory, *i. e.* a careful account of all your property, and find that you have the following: Cash, $150; 50 bu. potatoes, valued at 60 cents per bu.; and 8 bbl. apples, valued at $2.25 per bbl. Nothing is due you and you owe nothing.

Note.—This inventory should be entered on the first page of your account-book. It should be headed "Inventory of the assets and liabilities of A. B. (inserting your own name), Sept. 1, 1857." Then should be written, "I own Cash $150," etc.

You afterwards engage in the following transactions:

2. Sept. 2. Sell George Brown 6 bu. potatoes at 65 cents per bu.; and 1 bbl. apples for $2.37.

Note.—When nothing is said of payment, the articles are supposed to be bought and sold on account.

Suggestion.—The pupil may if he prefers it, substitute the names of his companions instead of the names here given; but care must be taken to make the same substitution for each name every time it occurs.

3. Sept. 2. Sell Henry Gay 12 bu. potatoes at 63 cents per bu.; and 5 bbl. apples at $2.37 per bbl.

4. Sept. 3. Buy of Henry Gay 6 lb. beef at 12 cents per lb.

5. Sept. 4. Sell for cash 2 bbl. apples at $2.33 per bbl.

6. Sept. 5. Buy of Edward Morris 1 pair of children's shoes for 87 cents; 1 pair of women's gaiter-boots for $1.75.

7. Sept. 5. Buy of Smith & Jones 6 lb. brown sugar at 12 cents per lb.; 6 lb. white sugar at 14 cents per lb.; and ½ lb. Young Hyson tea at 64 cents per lb.

8. Sept. 5. Charge Charles French for 3 days' labor at $1.62 per day.

9. Sept. 5. Buy for cash 15 bbl. apples at $2.25 per bbl., and pay cash for incidental expenses, 50 cents.

10. Sept. 5. Buy of Henry Gay 9 lb. sausages at 13 cents per lb.

11. Sept. 7. Charge Alfred Baker for 1 day's labor $1.75, and sell him 25 bu. potatoes at 65 cents per bu.

12. Sept. 8. Edward Morris has mended your boots, for which he charges $1.00.

13. Sept. 9. Charge Oliver Ellis for 2 days' labor at $1.75 per day, and buy of him 1 pair of pants for $5.25, and 1 vest for $3.37.

14. Sept. 9. Hire a horse and carriage of George Brown for $2.

15. Sept. 10. Buy for cash 5 tons of hay at $16.75 per ton.

16. Sept. 10. Sell Edward Morris 2 bbl. apples at $2.35 per bbl. and 7 bu. potatoes at 60 cents per bu.

17. Sept. 11. Buy of Charles French 30 bu. Dover potatoes at 58 cents per bu.; 45 bu. Chenango potatoes at 56 cents per bu.; 100 lb. butter at 19 cents per lb.; and 124 lb. cheese at 8 cents per lb.; paying $15 in cash.*

18. Sept. 12. Buy of Henry Gay 7 lb. corned beef at 12 cents per pound.

19. Sept. 12. Sell for cash 2 tons hay at $19.63 per ton; receive cash for 3 days' labor at $1.75 per day; and sell for cash 25 bu. Dover potatoes at 60 cents per bu.; and 1 bbl. apples for $2.30.

20. Sept. 12. Buy of Smith & Jones 4 lb. old Java coffee at 20 cents per lb.; 3 lb. bar soap at 8 cents per lb.; 2 gal. molasses at 63 cents per gallon; and sell them 1 bbl. apples for $2.30; and 5 bu. Dover potatoes at 60 cents per bu.

21. Sept. 14. Charge Charles French with 2 days' labor at $1.75 per day, and buy of him 3 tons of hay at $16.62 per ton.

22. Sept. 14. Sell Henry Gay 27 lb. butter at 23 cents per lb., and 1 cheese weighing 26 lb. at 9 cents per lb.

23. Sept. 15. Buy for cash 1 plough for 6.50, and 1 cultivator for $5.62.

24. Sept. 15. Sell for cash 1 bbl. apples for $2.25.

25. Sept. 16. Charge Smith & Jones for 2 days' labor at $1.75 per day; and buy of them 3 qt. sperm oil at $1.56 per gal.; and 1 ham weighing 13 lb. at 15 cents per lb.

26. Sept. 16. Buy of Edward Morris 1 harness for $33.75, and sell him 25 cwt. of hay at 90 cents per cwt.

27. Sept. 16. Sell for cash 53 lb. butter at 22 cents per lb., and 40 lb. cheese at 9 cents per lb.

28. Sept. 17. Sell your harness for cash, $35.

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* This requires an entry both in French's account and on cash account.

29. Sept. 18. Sell Edward Morris 10 bu. Chenango potatoes at 58 cents per bu.

30. Sept. 18. Charge Alfred Baker for 2 days' labor at $1.62 per day, and sell him 2 tons 10 cwt. of hay at $18.50 per ton.

31. Sept. 19. Sell Oliver Ellis 10 bu. Chenango potatoes at 54 cents per bu.

32. Sept. 19. Buy of Henry Gay 9 lb. mutton at 12 cents per lb., and 25 lb. lard at 13 cents per lb.

33. Sept. 21. Sell Charles French a plough for $8 and a cultivator for $6.75.

34. Sept. 21. Pay cash to Dr. Howe for services as physician, $1.50; and buy for cash 12 bu. peaches at $1 per bu.; and 5 bu. pears at $1.25 per bu.

35. Sept. 23. Sell Alfred Baker 3 pk. peaches at 33 cents per pk., and 2 pk. pears at 35 cents per pk.

36. Sept. 23. Sell Oliver Ellis 3 pk. peaches at 33 cents per pk.; 2 pk. pears at 35 cents per pk.; 8 lb. butter at 23 cents per lb.; and 16 lb. cheese at 10 cents per lb.

37. Sept. 23. Sell Smith & Jones 2 bu. 3 pk. peaches at $1.20 per bu.; and 1 bu. 2 pk. pears at $1.40 per bu.

38. Sept. 23. Receive cash for peaches and pears peddled out, $6.83.

39. Sept. 24. Sell for cash 3 bu. 1 pk. peaches at $1.20 per bu., and pay cash for a dress-coat $20.

40. Sept. 26. Buy of Smith & Jones 12 lb. rice at 7 cents per lb., and 12 lb. sugar at 13 cents per lb.

41. Sept. 26. Receive cash for labor $3.15, and also $5 from George Brown on account.

42. Sept. 28. Buy of Edward Morris 1 pair thick boots for yourself, $3, and 1 pair thick boots for your son, $2.

43. Sept. 28. Buy of Henry Gay 5 lb. beef at 11 cents per lb., and 7 lb. sausages at 14 cents per lb.

44. Sept. 29. Pay Charles French cash on account $10, and charge him $10 for labor.

45. Sept. 29. Sell George Brown 6 lb. butter at 23 cents per lb., and 16 lb. cheese at 10 cents per lb.

46. Sept. 30. Settle Henry Gay's account, and also George Brown's, with cash. How much was paid in each case, and by whom?

NOTE.—The method of making these entries, and of drawing lines to show that the accounts are closed, may be seen in the specimen pages. Entries are also required in cash account.

47. Oct. 1. Find the answers to the following questions, in the case of each person with whom you have an account: How much does he owe you? How much do you owe him? What is the balance, and in whose favor? Close the account by passing balance to new account.

NOTE.—The proper method of doing this is to find the difference of the Debit and Credit Entries on each man's account. This difference shows the balance which you owe each person, or the balance which he owes you. If you owe him, the proper entry will be on the Debit page: "To Balance carried to new account." If he owes you, it will be on the Credit page: "By Balance carried to new account."

The sums of the debit and credit columns (which will now be equal) should be written in their appropriate places, and lines should be drawn as before explained. The new account should at once be opened by debiting or crediting the person for this balance. He should be debited when the balance is due you, and credited when it is due him.

The new entry will always be on the page opposite to that on which the closing entry was made. This is illustrated in the specimen account on pages 96 and 97 of this book.

48. How much cash has been in your possession during the past month? How much have you paid out? How much, then, ought now to be in your possession? Now close Cash Account by passing balance to new account.

NOTE.—In real life, it would be necessary to count the cash you have on hand, and see if it corresponds with the amount indicated by your cash account. If it does, you may know that your cash account has been kept correctly. If it does not, there has been an error, either in the cash account or in paying out or receiving money, or else you have in some way lost some money.

Persons doing much business balance their cash account every day, but those doing but little may not balance it more than once a week or month. The practice of balancing it frequently tends not only to cultivate careful habits of receiving and paying out money, but renders it more easy to detect any errors which may have been made.

49. Fill the blanks in the following statements with the appropriate figures, and then find the answers to the subsequent questions.

Oct. 1, 1857. On examining the *property in your possession*, you find that you have on hand —

Cash (The amount may be found from Cash Account).
25 bu. Chenango Potatoes @ $.56 .... $
11 bbl. Apples @ $2.25 .... $
2 lb. Cheese @ $.09........ .... $
12 lb. Butter @ $.20 .... $
2 T. 5 cwt. Hay @ $.90 .... $

By examining your *accounts*, you find that —

Smith & Jones owe you........................ $
Alfred Baker owes you......................... $
Oliver Ellis owes you.......................... $

You owe Charles French....................... $
You owe Edward Morris......................... $

50. What is the value of all the property you have in your possession?

51. How much money is due you in all?

52. What, then, is the total amount of your assets, *i. e.* of all the property in your possession, and of all the money due you?

53. What is the amount of your liabilities, *i. e.* of all the debts which you owe?

54. How much were you really worth on the first of October, *i. e.* how much did your assets exceed your liabilities?

55. Have you, then, gained or lost money during the last month, and how much?

SPECIMEN ACCOUNTS.

1857.		Cash.	Dr.	
Sept.	1	To Cash on hand-------	$150	00
"	4	" 2 bbl. Apples,		
		@ $2.33-----------	4	66
"	12	" 2 tons Hay,		
		@ $19.63----------	39	26
"	"	" 3 days' Labor,		
		@ $1.75-----------	5	25
"	"	" 25 bu. Dover		
		Potatoes, @ $.60----	15	00
"	"	" 1 bbl. Apples---------	2	30
			216	47
Sept.	12	To Balance from old acc't	83	47

1857.		George Brown.	Dr.	
Sept.	2	To 6 bu. Potatoes,		
		@ $.65-------------	3	90
"	"	" 1 bbl. Apples--------	2	37
"	29	" 6 lb. Butter,		
		@ $.23-------------	1	38
"	"	" 16 lb. Cheese,		
		@ $.10-------------	1	60
			9	25

SPECIMEN ACCOUNTS.

1857.	Cash.	Cr.	
Sept. 5	By 15 bbl. Apples @ $2.25	$33	75
" "	" Sundries		50
" 10	" 5 tons Hay, @ $16.75	83	75
11	" Cash paid Charles French on acc't	15	00
12	Balance to new acc't	83	47
		$216	47

1857.	George Brown.	Cr.	
Sept. 9	By Horse & Carriage	$2	00
" 26	" Cash	5	00
" 30	" Cash to Balance	2	25
		$9	25

SECTION XIV.

FACTORS, MULTIPLES, AND DIVISORS.

68. Definitions and Explanations.

(a.) A FACTOR of any given number is such an entire number as taken an entire number of times will produce the given number; or the FACTORS of a number are such entire numbers as multiplied together will produce it.

ILLUSTRATIONS.—6 and 2, 4 and 3, 12 and 1, or 3, 2, and 2, are factors of 12, because $12 = 6 \times 2 = 4 \times 3 = 12 \times 1 = 3 \times 2 \times 2$.

Again: 2 is a factor of 2, 4, 6, 8, etc.; 3 is a factor of 3, 6, 9, etc.

(b.) A DIVISOR of a number is any entire number which will exactly divide it.

ILLUSTRATIONS.—1, 2, 3, 4, 6, and 12, are divisors of 12, for each will divide it without a remainder.

(c.) Every divisor of a number is also a factor of it, and every factor of a number is a divisor of it.

(d.) A MULTIPLE of a number is any number which contains it as a factor.

ILLUSTRATION.—12 is a multiple of 1, 2, 3, 4, 6, and 12, because it contains each of them as a factor.

(e.) Every number is a divisor and a factor of all its multiples, and a multiple of all its factors and divisors.

(f.) A PRIME NUMBER is one which has no factor except itself and unity.

(g.) A COMPOSITE NUMBER is one which has other factors besides itself and unity.

ILLUSTRATIONS.—1, 3, 5, 7, 11, etc. are prime numbers, and 4, 6, 8, 9, 10, etc. are composite.

(h.) Two numbers are PRIME TO EACH OTHER when they have no *common* factors.

ILLUSTRATIONS.—4 and 9 are prime to each other; but 6 and 9 are not, because they have the common factor 3.

(i.) A number is divided into factors when any factors which will produce it are found.

ILLUSTRATIONS.—12 = 2 × 2 × 3; 30 = 2 × 3 × 5.

(j.) The product of a number taken any number of times as a factor is called a POWER of that number.

ILLUSTRATION.—8, which is the product of 2 × 2 × 2, *i. e.* of 2 taken 3 times as a factor, is the third power of 2.

(k.) We may indicate the power of a number by writing a little figure called an EXPONENT above it and a little to the right.

ILLUSTRATIONS.—"3^3" is read 3 *to the third power*, and equals 3 × 3 × 3, or 27; "2^5" is read 2 *to the fifth power*, and equals 2 × 2 × 2 × 2 × 2.

(l.) The second power of a number is sometimes called its SQUARE, and the third power its CUBE.

ILLUSTRATIONS.—8, or 2^3, is the *cube* of 2; 25, or 5^2, is the *square* of 5.

69. Properties of Numbers.

(a.) A number which is a factor of another number must also be a factor of any multiple of that other number.

ILLUSTRATION.—4 is a factor of 12, and hence of 24, 36, 48, or any other multiple of 12.

(b.) If each of two numbers is divisible by a third number, both their sum and their difference must be divisible by that third number.

ILLUSTRATION.—Both 12 and 30 are divisible by 6. Hence, their sum, 42, and their difference, 18, are also divisible by 6.

(c.) If one of two numbers is divisible by a third number, and the other is not, neither their sum nor their difference will be divisible by that third number.

ILLUSTRATION.—12 is divisible by 4 and 21 is not; hence, neither their sum, 33, nor their difference, 9, is divisible by 4.

(d.) If neither of two numbers is divisible by a third, their sum and their difference may or may not be divisible by that third number.

ILLUSTRATIONS. — Neither 10 nor 18 are divisible by 4; yet both their sum, 28, and their difference, 8, are divisible by 4.

Again: neither 11 nor 25 are divisible by 4; yet their sum, 36, is, and their difference, 14, is not divisible by 4.

(e.) If a number is divisible by each of two numbers which are prime to each other, it must also be divisible by their product.

ILLUSTRATION. — A number which is divisible by 4 and 9, must be divisible by 36.

(f.) If a number is divisible by each of two numbers which have a common factor, it may or may not be divisible by their product.

ILLUSTRATIONS. — 80 is divisible by 4 and 10, and by their product, 40; 140 is divisible by 4 and 10, but not by their product, 40.

(g.) A number which is not divisible by another will not be divisible by any multiple of that other number.

ILLUSTRATION. — A number which is not divisible by 2, is not divisible by 4, 6, 8, 10, or any other multiple of 2.

(h.) A number which is divisible by any composite number, is also divisible by all the factors of that composite number.

ILLUSTRATION.—A number which is divisible by 36, is divisible by 2, 3, 4, 6, 9, 12, and 18, the factors of 36.

(i.) A number is divisible by 2 or 5, when its right-hand figure is thus divisible.

ILLUSTRATIONS. — 470 is divisible by 2 and by 5, because the right-hand figure is zero; 538 is divisible by 2 because 8 is.

(j.) A number is divisible by 4, 20, 25, or 50, when the number expressed by its two right-hand figures is thus divisible.

ILLUSTRATION. — 1724 is divisible by 4 because 24 is; but is not divisible by 20, 25, or 50, because 24 is not.

(k.) A number is divisible by 8, 40, 125, 250, or 500, when the number expressed by its three right-hand figures is thus divisible.

ILLUSTRATION. — 54750 is divisible by 125 and 250 because 750 is; but is not divisible by 8, 40, or 500, because 750 is not.

(l.) A number is divisible by 3 or by 9, when the sum of its digit figures is thus divisible.

Illustrations.—The sum of the digit figures of 2574 is $2 + 5 + 7 + 4 = 18$, which is divisible by 9; hence, 2574 is divisible by 9. The sum of the digits of 8571 is $8 + 5 + 7 + 1 = 21$, which is divisible by 3, but not by 9; hence, 8571 is divisible by 3, but not by 9.

(m.) A number is divisible by 11 when the sums of its alternate digits are alike, or their difference is a multiple of 11.

Illustrations.—In 857934, the sum of the first set of alternate digits is $4 + 9 + 5 = 18$; the sum of the second set is $3 + 7 + 8 = 18$. These sums being alike, the number is divisible by 11. In 8295716, the sum of the first set of alternate digits is $6 + 7 + 9 + 8 = 30$; the sum of the second set is $1 + 5 + 2 = 8$. Their difference, $30 - 8 = 22$, being a multiple of 11, the number is divisible by 11. In 716896, the sum of the first set of alternate digits is $6 + 8 + 1 = 15$; the sum of the second set is $9 + 6 + 7 = 22$. Their difference, $22 - 15 = 7$, not being a multiple of 11, the number is not divisible by 11.

70. Exercises in Factoring Numbers.

1. What are the prime factors of 72?

Solution.—$72 = 8 \times 9$, which, since $8 = 2^3$ and $9 = 3^2$, gives $72 = 2^3 \times 3^2$.

Find the prime factors of the following numbers:

2. 36.
3. 24.
4. 81.
5. 63.
6. 27.
7. 48.
8. 84.
9. 96.
10. 35.
11. 99.
12. 54.
13. 30.

14. What are the prime factors of 3564?

Solution.—It is readily seen that this number is divisible by 4, 9, and 11, and hence (see **69**, e) by their product, 396. Dividing by 396 gives 9 for a quotient. Hence $3564 = 4 \times 9 \times 11 \times 9$, or since $4 = 2^2$, and $9 = 3^2$, $3564 = 2^2 \times 3^2 \times 11 \times 3^2 = 2^2 \times 3^4 \times 11$.

15. What are the prime factors of 381191?

Solution.—We first observe that 381191 is not divisible by 2, 3, 5, or 11, and hence that it is not divisible by any multiple of these numbers. We have, then, to try other prime numbers, beginning, for convenience, with the smallest. By actual trial, we find that 7 and 13 each leave a remainder after division; but that 17 is contained in it 22423 times. We have now to find the factors of 22423. It can contain no factor less than 17, for the original number contained none. We therefore try 17, and find that it is contained 1319 times. We have, therefore, to find the factors of 1319. Beginning with 17, we try in succession the prime numbers 17, 19, 23, 29, 31, and 37, each of which leaves a remainder. But dividing by 37 gives 35 for the entire part of the quotient, which, being less than the divisor, 37, shews that 1319 is a prime number.[*] Hence, the prime factors required are $17 \times 17 \times 1319 = 17^2 \times 1319$.

Find the prime factors of—

16. 144.
17. 132.
18. 169.
19. 205.
20. 896.
21. 275.
22. 141.
23. 222.
24. 341.
25. 175.
26. 512.
27. 732.
28. 527.
29. 931.
30. 621.
31. 507.
32. 625.
33. 659.
34. 841.
35. 793.
36. 1001.
37. 1728.
38. 2156.
39. 3125.
40. 6157.
41. 1431.
42. 8375.
43. 6561.
44. 2310.
45. 4641.
46. 5368.
47. 8145.
48. 8283.
49. 1972.
50. 2512.
51. 2021.
52. 2737.
53. 7056.
54. 6075.
55. 4374.

71. Greatest Common Divisor.

(**a.**) A Divisor of a number is any number which will exactly divide it.

(**b.**) A Common Divisor of two or more numbers is any number which is a divisor of each of them.

(**c.**) The Greatest Common Divisor of two or more numbers is the largest number which is a divisor of each of them.

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[*] This conclusion depends on the fact that the larger the divisor is, the smaller will be the quotient. Hence, any number larger than 37 must give a quotient less than that obtained by 37, *i. e.* less than 35, and as we have found that no number less than 37 is a factor of 1319, it follows that none greater than 37 can be.

(d.) From these definitions, and the principles previously illustrated, it is obvious that —

1st. A Divisor of a number is the product of prime factors found in that number.

2d. A Common Divisor of two or more numbers is the product of prime factors which are common to all those numbers.

3d. The Greatest Common Divisor of two or more numbers is the product of *all* the prime factors *common* to those numbers.

(e.) To find the Greatest Common Divisor of two or more numbers, we factor one of the numbers, and then see which of its prime factors, if any, are factors of all the other given numbers. The product of the prime factors thus found is the Common Divisor sought.

1. What is the greatest common divisor of 36, 54, and 60?

SOLUTION. — $36 = 2^2 \times 3^2$. Of these factors, 54 contains only 2 and 3^2; and of these last, 60 contains only 2 and 3. Hence, the only common factors are 2 and 3, and the greatest common divisor must be 2×3, or 6.

2. What is the greatest common divisor of 46, 69, and 72?

SOLUTION. — $46 = 2 \times 23$. But 2 cannot be a factor of the common divisor, because it is not a factor of 69, and 23 cannot be, because it is not a factor of 72. Hence, the only factor common to all the numbers is 1, and 1 is the greatest common divisor.

3. What is the greatest common divisor of 378, 504, 567, and 756?

SOLUTION. — $504 = 2^3 \times 3^2 \times 7$. But 2 cannot be a factor of the greatest common divisor, because it is not a factor of 567. Next considering 3^2 and 7, we find that they are factors of all the numbers. Hence, the greatest common divisor must be $3^2 \times 7$, or 63.

What is the greatest common divisor of —

4. 12 and 18?
5. 15 and 25?
6. 24 and 36?
7. 30 and 42?
8. 51 and 102?
9. 28 and 42?

10. 132 and 144?
11. 49 and 72?
12. 35 and 70?
13. 78 and 91?
14. 36 and 72?
15. 40 and 75?
16. 84 and 97?
17. 63 and 98?
18. 12 and 25?
19. 17 and 69?
20. 150 and 210?
21. 143 and 176?
22. 462 and 693?
23. 511 and 748?
24. 412 and 596?
25. 218 and 654?
26. 48, 84, 108, and 114?
27. 75, 63, 100, and 40?
28. 81, 120, and 141?
29. 46, 115, 161, and 184?
30. 78, 156, 169, and 195?
31. 288, 432, 504, and 576?
32. 327, 436, 545, and 654?
33. 2592, 3888, 5184, and 7776?

72. Method for Large Numbers.

(a.) The greatest common divisor of two numbers is the same as the greatest common divisor of the smaller number, and the remainder left after dividing the larger number by the smaller.

ILLUSTRATIONS.—The greatest common divisor of 36 and 102 is the same as the greatest common divisor of 36, and the remainder, 30, which is left after dividing 102 by 36.

Again: the greatest common divisor of this remainder, 30, and the smaller number, 36, is the same as the greatest common divisor of 30 and the remainder, 6, which is left after dividing 36 by 30. Now, as 6 divides 30, it is the greatest common divisor of 30 and 36, and also of the two original numbers.

1. What is the greatest common divisor of 3473 and 8909?

```
3473)8909(2
     6946
     ----
     1963)3473(1
          1963
          ----
          1510 = 2 × 5 × 151.
151)1963(13
    151
    ---
     453
     453
     ---
```

SOLUTION.—As the factors of these numbers are not readily recognized, we divide the greater by the less, which gives 2 for a quotient and 1963 for a remainder. We have now to find the greatest common divisor of 1963 and 3473. As the factors of 1963 are not readily recognized, we divide 3473 by it, which gives 1 for a quotient and 1510 for a remainder. Hence, we have now o find the greatest common divisor of 1510 and 1963. But 1510 = 2 × 5 × 151, of which factors only 151 is a factor of 1963. Hence, 151 is the greatest common divisor required.

(b.) What is the greatest common divisor of —

2. 221 and 507?
3. 1073 and 1537?
4. 3233 and 3551?
5. 1693 and 3869?
6. 9991 and 10961?
7. 833 and 1547?
8. 413 and 931?
9. 887 and 973?
10. 1147 and 1829?
11. 1309 and 3003?

73. To find the Least Common Multiple.

(a.) A MULTIPLE of a number is any number which contains it as a factor.

(b.) A COMMON MULTIPLE of two or more numbers is any number which contains them all as factors.

(c.) The LEAST COMMON MULTIPLE of two or more numbers is the least number which contains them all as factors.

(d.) From these definitions, and the principles previously established, it follows that —

1st. A multiple of a number must contain all the prime factors of that number.

2d. A common multiple of two or more numbers must contain all the prime factors of those numbers.

3d. The least common multiple of two or more numbers must be the least number that contains all the prime factors of those numbers.

(e.) To find the least common multiple of two or more numbers, we multiply one of them by such prime factors of the other numbers as it does not contain.

1. What is the least common multiple of 270 and 504?

SOLUTION. — The least common multiple of 270 and 504 is the least number which contains all the prime factors of 270 and 504.

$$270 = 2 \times 3^3 \times 5.$$
$$504 = 2^3 \times 3^2 \times 7.$$

We must have all the prime factors of 270, or, which is the same thing, 270 itself. We must also have all the prime factors of 504, which are $2^3 \times 3^2 \times 7$. But as in 270 we have already taken the 3's and one of the 2's, we shall only have to introduce the two remaining 2's and the 7, *i. e.* 2^2 or 4, and 7. Hence, the least common multiple of 270 and 504 is $270 \times 4 \times 7 = 7560$.

NOTE. — Had we first taken the factors of 504, or 504 itself, the work would have taken the form, $504 \times 3 \times 5 = 7560$.

What is the least common multiple of —

2. 12 and 18?
3. 15 and 21?
4. 4 and 6?
5. 8 and 9?
6. 18 and 36?
7. 15 and 25?
8. 24 and 36?
9. 12 and 24?
10. 54 and 72?
11. 48 and 54?
12. 16 and 27?
13. 49 and 98?
14. 8 and 10?
15. 14 and 21?
16. 144 and 132?
17. 15 and 28?
18. 105 and 189?
19. 91 and 169?
20. 84 and 126?
21. 24 and 25?

22. What is the least common multiple of 21, 24, 28, and 35?

SOLUTION. — The least common multiple of 21, 24, 28, and 35, is the smallest number which contains all the prime factors of these numbers.

$21 = 3 \times 7.$ $28 = 2^2 \times 7.$

$24 = 3 \times 2^3.$ $35 = 5 \times 7.$

Beginning with 35, we must have all its prime factors, or, which is the same thing, 35 itself. We must also have all the prime factors of 28, which are $2^2 \times 7$. But as we have already taken 7, we have only to introduce the 2^2 or 4, giving 4×35. We must also have the prime factors of 24, which are $2^3 \times 3$. As we have already taken 2^2, we have only to introduce 2×3, or 6, giving $6 \times 4 \times 35$. We must also have the prime factors of 21, which are 3×7. But as in taking 35 we took the factor 7, and in taking 6 we took the factor 3, we have no new factor to introduce. Hence, the least common multiple required is $6 \times 4 \times 35$, or 840.

NOTE. — We might have begun with any of the other numbers instead of 35, but it is usually most convenient to begin with the largest number. The careful student will doubtless discover abbreviated methods of finding the least common multiple. He may adopt any method by which he can find the required factors.

(f.) What is the least common multiple of—

23. 4, 6, 8, and 10?
24. 3, 5, 9, and 10?
25. 5, 7, 9, and 12?
26. 2, 4, 6, 9, and 18?
27. 8, 12, 15, and 20?
28. 2, 3, 4, 5, and 6?
29. 5, 6, 7, 8, and 9?
30. 6, 7, 8, 9, and 10?
31. 7, 8, 9, 10, and 11?
32. 8, 9, 10, 11, and 12?
33. 18, 24, 30, and 36?
34. 9, 11, 14, and 16?
35. 3, 5, 7, 11, and 13?
36. 28, 42, and 70?
37 25, 50, 75, 125, and 150?
38. 42, 49, and 56?
39. 4, 8, 16, 32, and 64?
40. 54, 81, 108, and 135?
41. 39, 51, 52, and 68?
42. 60, 76, 114, and 120?

SECTION XV.

FRACTIONS.

74. Definitions and Explanations.

(a.) Such parts as are obtained by dividing any unit whatever into equal parts are called Fractional Parts, and the numbers expressing them are called Fractions.

(b.) Halves are such fractional parts as are obtained by dividing a unit into two equal parts.

(c.) Thirds are such fractional parts as are obtained by dividing a unit into three equal parts.

(d.) In like manner we may define fourths, fifths, sixths, sevenths, etc.

(e.) The following definitions are sometimes more convenient in their application than the preceding.

(f.) Halves are equal parts of such kind that two of them are equal to a unit.

(g.) In like manner we may define thirds, fourths, fifths, etc.

(h.) Fractional parts are generally expressed by drawing a line above the number which shows how many of them must be taken to equal a unit.

Illustrations.—We may express fourths by writing the figure 4 with a line above it, thus, $\overline{4}$. In like manner, $\overline{9}$ means *ninths*, $\overline{12}$ = *twelfths*, etc.

(j.) The number which thus shows the kind or denomination of fractional parts is called a Denominator.

(k.) To show how many fractional parts are taken or considered, a figure called the Numerator is written above the denominator.

Illustration.—To express 3 sevenths, we write the numerator 3 above the denominator $\overline{7}$, thus, $\frac{3}{7}$.

(l.) Write each of the following fractions in figures:

1. Three-fifths.
2. Five-ninths.
3. Twelve-fifths.
4. Six-seventeenths.
5. Twenty forty-firsts.
6. Fifty-six eighths.

7. What does the fraction $\frac{3}{5}$ express?

Answer.—Three-fifths expresses the value of three such parts as are obtained by dividing a unit into five equal parts.

(m.) In the same manner explain each of the following:

8. $\frac{2}{3}$.
9. $\frac{4}{7}$.
10. $\frac{9}{10}$.
11. .9
12. $\frac{7}{100}$.
13. .07
14. .36
15. $\frac{5}{17}$.

(n.) Explain the preceding fractions after the following

Model.—Three-fifths expresses the value of three equal parts such that five of them would equal a unit.

(o.) A Mixed Number is a whole number and a fraction.

Illustration.—$5\frac{2}{3}$, which is read "five and two-thirds," is a mixed number.

75. Recapitulation.

(a.) A Fraction is a number which expresses one or more fractional parts.

(b.) Fractional Parts are such parts as are obtained by dividing a unit into equal parts, or—

(c.) Fractional Parts are equal parts of such kind that a certain number of them would equal a unit.

(d.) The unit from which the fractional parts are supposed to be obtained is called the Unit of the Fraction.

Illustrations.—In $\frac{4}{5}$, which means $\frac{4}{5}$ of 1, the unit of the fraction is the abstract unit one.

In $\frac{2}{3}$ of a foot, it is *one foot.*
In $\frac{1}{3}$ of 12, it is the *number twelve.*
In $\frac{3}{4}$ of 8 bu., it is the quantity 8 *bushels.*
In $\frac{2}{7}$ of $\frac{5}{6}$, it is the *fraction* $\frac{5}{6}$.

(e.) The Denominator of a fraction is the number which shows into how many equal parts the unit of the fraction is supposed to be divided, or—

(f.) The Denominator of a fraction is the number which shows how many parts must be taken to equal the unit of the fraction.

(g.) The Numerator of a fraction is the number which shows how many fractional parts are taken or considered.

(h.) The Numerator and Denominator are called the Terms of the fraction.

(i.) A fraction is in its Lowest Terms when its numerator and denominator are the smallest entire numbers which will express its value.

(j.) A Mixed Number is a whole number and a fraction considered together.

76. Classification of Fractions.

(a.) Fractions may be Vulgar, Decimal, or Duodecimal.

(b.) Vulgar Fractions, or Common Fractions, are those in which the numerator and denominator are both written, as $\frac{4}{7}$, $\frac{3}{8}$.

(c.) Decimal Fractions are those whose denominator is some power of ten, and, instead of being written, is indicated by the position of the decimal point, as .5, .07.

(d.) Duodecimal Fractions are those whose denominator is some power of 12, and, instead of being written, is indicated by marks or accents placed over the numerator. (See 102.)

(e.) Fractions may be either PROPER OR IMPROPER.

(f.) A PROPER FRACTION is one whose numerator is less than its denominator, as $\frac{3}{4}$, .05.

(g.) An IMPROPER FRACTION is one whose numerator is either equal to or greater than its denominator, as $\frac{4}{4}$, $\frac{9}{8}$, 1.7.*

(h.) Fractions may be SIMPLE, COMPLEX, or COMPOUND.

(i.) A SIMPLE FRACTION is one which has but one numerator and one denominator, each of which is a whole number, as .6, $\frac{3}{8}$.

(j.) A COMPLEX FRACTION is one which has a fraction in one or both its terms, as $\frac{2\frac{1}{2}}{3}$, $\frac{4\frac{2}{3}}{5\frac{3}{7}}$, $\frac{.9}{4}$, $\frac{.07}{1.5}$, .03$\frac{1}{2}$.

(k.) A COMPOUND FRACTION is a fraction of a fraction, as .9 of 1.7, $\frac{2}{3}$ of $\frac{4}{7}$.

77. Fractional Operations Illustrated.

(a.) Fractions may be added, subtracted, multiplied, or divided, as whole numbers are.

ILLUSTRATIONS. — $\frac{3}{4} + \frac{2}{4} = \frac{5}{4}$, just as 3 pk. + 2 pk. = 5 pk, or as 3 + 2 = 5. But $\frac{3}{4} + \frac{2}{8}$ no more equal $\frac{5}{4}$ or $\frac{5}{8}$, than 3 pk. + 2 qt. = 5 pk. or 5 qt.

$\frac{17}{20} - \frac{9}{20} = \frac{8}{20}$, just as 17 dwt. — 9 dwt. = 8 dwt., or as 17 — 9 = 8. But $\frac{17}{20} - \frac{9}{24}$ no more equal $\frac{8}{20}$ or $\frac{8}{24}$, than 17 dwt. — 9 gr. = 8 dwt., or 8 gr.

8 times $\frac{3}{7} = \frac{24}{7}$, just as 8 times 3 days = 24 days, or as 8 times 3 = 24.

$\frac{15}{16} + \frac{3}{16} = 5$, just as 15 oz. + 3 oz. = 5, or as 15 + 3 = 5.

78. Reduction of Whole and of Mixed Numbers to Improper Fractions.

(a.) Every whole and every mixed number can be reduced to an improper fraction.

1. Reduce 7$\frac{2}{3}$ to thirds.

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* This may be read as the improper fraction $\frac{17}{10}$, or as the mixed number 1$\frac{7}{10}$ (see **12**, a and c). It is important that the pupil should be familiar with both methods of reading it.

Solution.—Since $1 = 3$ thirds, 7 must equal 7 times $\frac{3}{3}$, or $\frac{21}{3}$, to which adding the $\frac{2}{3}$, gives $\frac{23}{3}$. Hence, $7\frac{2}{3} = \frac{23}{3}$.

(b.) When the number to be reduced is larger than the denominator of the required fraction, it may be convenient to modify the above solution, as in the following example.

2. Reduce $479\frac{3}{8}$ to eighths.

Solution.—Since $1 = 8$ eighths, 479 must equal 479 times 8 eighths, which is equivalent to 8 times 479 eighths, or to 3832 eighths, to which adding the 3 eighths gives 3835 eighths. Hence, $479\frac{3}{8} = \frac{3835}{8}$.

Note.—The above questions are really questions in Reduction Descending, See page 59, note.

(c.) It will be seen that, practically, we multiply the entire number by the given denominator, add the given numerator to the product, and write the result over the given denominator.

(d.) Reduce—

3. $4\frac{5}{9}$ to ninths.
4. $8\frac{6}{7}$ to sevenths.
5. 12 to thirds.
6. 5 to tenths.
7. $24\frac{3}{7}$ to sevenths.
8. $523\frac{5}{23}$ to twenty-thirds.
9. 622 to thirteenths.
10. $684\frac{1}{2}$ to halves.
11. $4\frac{9}{10}$ to tenths.
12. 4.9 to tenths.
13. $7\frac{5}{100}$ to hundredths.
14. 7.05 to hundredths.
15. 473 to thousandths.
16. 12.15 to hundredths.
17. $6\frac{24}{25}$ to twenty-fifths.
18. $537\frac{1}{1000}$ to thousandths.

79. Reduction of Improper Fractions to Whole or to Mixed Numbers.

(a.) Every improper fraction may be reduced either to a whole or to a mixed number.

Note.—It can be reduced to a whole number when its numerator is a multiple of its denominator, and to a mixed number in all other cases.

1. Reduce $\frac{625}{7}$ to a whole or to a mixed number.

Solution.—Since 7 sevenths $= 1$, 625 sevenths must equal as many ones as there are times 7 sevenths in 625 sevenths, or as there are times 7 in 625, which are $89\frac{2}{7}$ times. Hence $\frac{625}{7} = 89\frac{2}{7}$.

Note.—The preceding example is really an example in Reduction Ascending. See page 69, note.

(b.) To reduce an improper fraction to a whole or to a mixed number, we divide its numerator by its denominator.

(c.) Reduce the following to whole or to mixed numbers.

2. $\frac{96}{12}$.
3. $\frac{124}{9}$.
4. $\frac{836}{5}$.
5. $\frac{729}{11}$.
6. $\frac{486}{13}$.
7. $\frac{524}{10}$.
8. $\frac{926}{30}$.
9. $\frac{445}{17}$.
10. $\frac{8324}{1000}$.
11. $\frac{708}{10}$.
12. $\frac{659}{47}$.
13. $\frac{8007}{100}$.
14. $\frac{479}{3}$.
15. $\frac{624}{8}$.
16. $\frac{5207}{100}$.

80. Miscellaneous Problems involving Fractions.

1 What is the product of 9 times $48\frac{6}{7}$?

SOLUTION.—9 times $48\frac{6}{7}$ = 9 times 48, + 9 times $\frac{6}{7}$. 9 times $\frac{6}{7} = \frac{54}{7}, = 7\frac{5}{7}$, and 9 times 48 = 432, to which adding the $7\frac{5}{7}$ gives $439\frac{5}{7}$. Hence 9 times $48\frac{6}{7} = 439\frac{5}{7}$.

NOTE.—Compare the above question with: "What is the product of 9 times 48 wk. 6 da.?"

(a.) Find the value of each of the following:

2. 7 times $86\frac{2}{3}$.
3. 5 times $96\frac{3}{4}$.
4. 13 times $27\frac{5}{16}$.
5. 18 times $49\frac{3}{8}$.
6. 24 times $127\frac{8}{11}$.
7. 63 times $142\frac{7}{9}$.
8. 81 times $336\frac{25}{27}$.
9. 96 times $721\frac{43}{51}$.

10. $87\frac{2}{3}$ = how many times $2\frac{1}{3}$?

SOLUTION.—$87\frac{2}{3} = \frac{263}{3}$; $2\frac{1}{3} = \frac{7}{3}$; and $\frac{263}{3}$ contains $\frac{7}{3}$ as many times as 263 contains 7, which is $37\frac{4}{7}$ times. Hence $87\frac{2}{3} = 37\frac{4}{7}$ times $2\frac{1}{3}$.

NOTE.—Compare the above with: "87 yd. 2 ft. = how many times 2 yd 1 ft.?"

11. $9\frac{3}{4}$ = how many times $2\frac{1}{4}$?
12. $8\frac{4}{5}$ = how many times $2\frac{1}{5}$?
13. $29\frac{1}{2}$ = how many times $3\frac{1}{2}$?
14. $56\frac{5}{7}$ = how many times $2\frac{3}{7}$?
15. $248\frac{1}{3}$ = how many times $2\frac{2}{3}$?
16. $169\frac{1}{8}$ = how many times $1\frac{1}{8}$?

17. What is the sum of $\frac{5}{9} + \frac{7}{9} + \frac{6}{9} + \frac{8}{9} + \frac{5}{9}$?

ANSWER.—$\frac{31}{9} = 3\frac{4}{9}$.

Note. — Compare the above with: "5 sq. ft. + 7 sq. ft. + 6 sq. ft. + 8 sq. ft. + 5 sq. ft. = 31 sq. ft. = 3 sq. yd. 4 sq. ft."

18. $\frac{6}{7} + \frac{3}{7} + \frac{5}{7} + \frac{4}{7}$.
19. $\frac{2}{5} + \frac{3}{5} + \frac{4}{5} + \frac{6}{5}$.
20. $\frac{12}{13} + \frac{5}{13} + \frac{9}{13} + \frac{10}{13}$.
21. $\frac{17}{20} + \frac{13}{20} + \frac{15}{20} + \frac{19}{20}$.

22. What is the sum of $4\frac{7}{8} + 9\frac{6}{8} + 7\frac{5}{8}$?

Note. — Compare this example with: "What is the sum of 4 pk. 7 qt. + 9 pk. 6 qt. + 7 pk. 5 qt.?"

23. $8\frac{4}{5} + 3\frac{2}{5} + 6\frac{4}{5}$.
24. $9\frac{5}{11} + 8\frac{7}{11} + 12\frac{10}{11}$.
25. $6\frac{19}{20} + 4\frac{13}{20} + 5\frac{16}{20}$.
26. $5\frac{8}{17} + 9\frac{12}{17} + 6\frac{13}{17}$.
27. $\frac{19}{20} - \frac{15}{20}$.
28. $\frac{89}{100} - \frac{36}{100}$.
29. $\frac{375}{825} - \frac{149}{825}$.
30. $\frac{426}{753} - \frac{317}{753}$.

31. What is the value of $143\frac{5}{12} - 68\frac{7}{12}$?

Note. — Compare the last example with: "What is the value of 143 lb. 5 oz. — 68 lb. 7 oz Troy Weight?"

32. $19\frac{1}{4} - 16\frac{3}{4}$?
33. $12 - 7\frac{2}{3}$?
34. $53\frac{5}{6} - 14\frac{1}{6}$?
35. $327 - \frac{1}{327}$?
36. $1 - \frac{18}{443}$?
37. $624\frac{3}{25} - 136\frac{21}{25}$?

38. How much will 625 yards of cloth cost at $\frac{3}{4}$ of a dollar per yard?

Solution. — Since 1 yard costs 3 fourths of a dollar, 625 yards will cost 625 times 3 fourths of a dollar, equal to three times 625 fourths of a dollar, which, found by multiplying 625 by 3, is 1875 fourths, equal (by **79**) to $468\frac{3}{4}$ dollars.

Note. — Compare the preceding example with: "How many bushels of grain will 625 bags contain, if each bag contains 3 pecks?"

39. How much will 47 books cost at $\frac{3}{8}$ of a dollar each?

40. What will 575 lb. of coffee cost at $\frac{3}{16}$ of a dollar per lb.?

41. What will 796 bushels of potatoes cost at $\frac{7}{8}$ of a dollar per bushel?

42. How much will 9 horses cost at \$$165\frac{2}{3}$ each?

43. If a boat sails $8\frac{7}{13}$ miles per hour, how many miles will it sail in 24 hours?

44. What will a cargo of coal containing 134 tons cost at \$$5.87\frac{1}{2}$ per ton?

45. How many square feet are there in a gravel-walk $329\frac{11}{12}$ ft. long and 5 ft. wide?

46. How many casks each holding $2\frac{3}{4}$ bushels can be filled from 273 bushels of grain?

Solution. — Since 1 cask can be filled from $2\frac{3}{4}$ bushels, as many casks can be filled from 273 bushels as there are times $2\frac{3}{4}$ in 273, to find which we must reduce both to fourths. $2\frac{3}{4} = \frac{11}{4}$, $273 = \frac{1092}{4}$, and $\frac{11}{4}$ is contained in $\frac{1092}{4}$ as many times as 11 is contained in 1092, which is $99\frac{3}{11}$ times. Hence $99\frac{3}{11}$ casks can be filled.

Note. — Compare the preceding problem with: "How many casks each containing 2 bu. 3 pk. can be filled from 273 bu. of grain?"

47. How many coats each containing $2\frac{1}{4}$ yards can be made from 45 yards of cloth?

48. If a man can earn $\$4\frac{4}{5}$ per day, how many days will it take him to earn $\$17\frac{3}{5}$?

49. A man who had $\$147\frac{1}{2}$ exchanged it for quarter-eagles. How many did he obtain?

50. How many nails are there in a piece of cloth $643\frac{1}{4}$ inches long?

81. Multiplication and Division of the Numerator.

(a.) From the nature of fractions, it follows that multiplying the numerator of a fraction must multiply, and dividing the numerator must divide, the number of parts considered, without affecting their size, and must therefore multiply or divide the fraction.

1. What is the effect of multiplying the numerator of $\frac{5}{22}$ by 3?

Answer. — Multiplying the numerator of the fraction $\frac{5}{22}$ by 3 gives $\frac{15}{22}$ for a result, which expresses three times as many parts each of the same size as before, and is therefore 3 times as large. Hence, multiplying the numerator of the fraction $\frac{5}{22}$ by 3 multiplies the fraction by 3.

What is the effect of multiplying the numerator of —

2. $\frac{4}{9}$ by 2?
3. $\frac{8}{9}$ by 5?
4. $\frac{12}{95}$ by 6?
5. $\frac{3}{17}$ by 8?
6. $\frac{7}{13}$ by 9?
7. $\frac{3}{20}$ by 6?

8. What is the effect of dividing the numerator of $\frac{12}{17}$ by 4?

Answer.—Dividing the numerator of the fraction $\frac{12}{17}$ by 4 gives $\frac{3}{17}$ for a result, which expresses $\frac{1}{4}$ as many parts each of the same size as before, and is therefore $\frac{1}{4}$ as large. Hence, dividing the numerator of $\frac{12}{17}$ by 4 divides the fraction by 4.

Note.—The first of the above solutions is equivalent to: "3 times $\frac{5}{22}$ = $\frac{15}{22}$, just as 3 times 5 apples = 15 apples;" and the second is equivalent to: "$\frac{1}{4}$ of $\frac{12}{17}$ = $\frac{3}{17}$, just as $\frac{1}{4}$ of 12 apples = 3 apples." They are necessary in this form, as a preparation for the exercises which follow.

What is the effect of dividing the numerator of—

9. $\frac{14}{15}$ by 7?
10. $\frac{16}{17}$ by 4?
11. $\frac{27}{32}$ by 9?
12. $\frac{48}{93}$ by 8?
13. $\frac{25}{27}$ by 5?
14. $\frac{18}{11}$ by 6?

82. Multiplication and Division of the Denominator.

(**a.**) From the nature of fractional parts, it follows that—

1st. The larger the number of fractional parts into which a unit is divided, or which it takes to equal that unit, the smaller each part will be.

2d. The smaller the number of fractional parts into which a unit is divided, or which it takes to equal that unit, the larger each part will be.

3d. Multiplying the number of fractional parts into which a unit is divided, or which it takes to equal that unit, divides each part.

4th. Dividing the number of fractional parts into which a unit is divided, or which it takes to equal that unit, multiplies each part.

Illustrations.—Halves are larger than thirds, fourths, or fifths, because it takes a less number to equal a unit. Thirds are larger than fourths, fifths, or sixths, because it takes a less number of them to equal a unit.

1. What is the effect of multiplying the denominator of the fraction $\frac{3}{4}$ by 2?

Answer.—Multiplying the denominator of the fraction $\frac{3}{4}$ by 2 gives $\frac{3}{8}$ for a result, which expresses the same number of parts each $\frac{1}{2}$ as large as before. Hence, multiplying the denominator of $\frac{3}{4}$ by 2 divides the fraction by 2.

(b.) What is the effect of multiplying the denominator of—

2. $\frac{2}{3}$ by 4?
3. $\frac{7}{8}$ by 4?
4. $\frac{3}{4}$ by 7?
5. $\frac{11}{12}$ by 3?
6. $\frac{7}{11}$ by 2?
7. $\frac{5}{12}$ by 3?

8. What is the effect of dividing the denominator of $\frac{7}{9}$ by 3?

ANSWER.—Dividing the denominator of the fraction $\frac{7}{9}$ by 3 gives $\frac{7}{3}$ for a result, which expresses the same number of parts each 3 times as large as before. Hence, dividing the denominator of $\frac{7}{9}$ by 3 multiplies the fraction by 3.

What is the effect of dividing the denominator of—

9. $\frac{3}{4}$ by 2?
10. $\frac{7}{8}$ by 4?
11. $\frac{5}{12}$ by 6?
12. $\frac{3}{14}$ by 7?
13. $\frac{5}{24}$ by 8?
14. $\frac{2}{27}$ by 9?

83. Multiplication and Division of both Terms.

1. What is the effect of multiplying both terms of the fraction $\frac{2}{3}$ by 4?

ANSWER.—Multiplying both terms of the fraction $\frac{2}{3}$ by 4 gives $\frac{8}{12}$ for a result, which expresses 4 times as many parts each $\frac{1}{4}$ as large as before. Hence, the value of the fraction is not altered, or $\frac{2}{3} = \frac{8}{12}$.

What is the effect of multiplying both terms of—

2. $\frac{1}{2}$ by 8?
3. $\frac{5}{6}$ by 2?
4. $\frac{2}{5}$ by 5?
5. $\frac{5}{7}$ by 6?
6. $\frac{2}{3}$ by 9?
7. $\frac{7}{11}$ by 3?

8. What is the effect of dividing both terms of the fraction $\frac{9}{15}$ by 3?

ANSWER.—Dividing both terms of the fraction $\frac{9}{15}$ by 3 gives $\frac{3}{5}$ for a result, which expresses $\frac{1}{3}$ as many parts each 3 times as large as before. Hence, the value of the fraction is not altered, or $\frac{9}{15} = \frac{3}{5}$.

What is the effect of dividing both terms of—

9. $\frac{6}{8}$ by 2?
10. $\frac{3}{6}$ by 3?
11. $\frac{8}{10}$ by 2?
12. $\frac{10}{15}$ by 5?
13. $\frac{14}{21}$ by 7?
14. $\frac{12}{16}$ by 4?

15. What is the effect of multiplying the numerator of a fraction by 5, and the denominator by 6?

ANSWER.—Multiplying the numerator of a fraction by 5 and the denominator by 6 gives for a result $\frac{5}{6}$ of the original fraction, for it gives 5 times as many parts each $\frac{1}{6}$ as large as before.

What is the effect of multiplying the numerator of a fraction—

16. By 2 and the denominator by 9?
17. By 7 and the denominator by 5?
18. By 13 and the denominator by 4?
19 By 12 and the denominator by 8?
20. By 20 and the denominator by 11?

84. Recapitulation and Inferences.

(a.) Multiplying the numerator multiplies the fraction by multiplying the number of parts considered, without affecting their size.

(b.) Dividing the numerator divides the fraction by dividing the number of parts considered, without affecting their size.

(c.) Multiplying the denominator divides the fraction by dividing each part, without affecting the number of parts considered.

(d.) Dividing the denominator multiplies the fraction by multiplying each part, without affecting the number of parts considered.

(e.) A fraction may be multiplied either by multiplying the numerator or by dividing the denominator.

(f.) A fraction may be divided either by dividing the numerator or by multiplying the denominator.

(g.) Multiplying both numerator and denominator of a fraction both multiplies and divides the fraction.

(h.) Dividing both numerator and denominator of a fraction, both divides and multiplies the fraction.

(i.) Multiplying both numerator and denominator of a fraction by the *same number*, both multiplies and divides the fraction by that number, and therefore does not alter its value.

(j.) Dividing both numerator and denominator of a fraction by the *same number*, both divides and multiplies the fraction by that number, and therefore does not alter its value.

85. Reduction to Lowest Terms, and Cancellation.

(a.) A fraction is in its LOWEST TERMS when its numerator and denominator are the smallest entire numbers which will express its value.

(b.) When a fraction is reduced to its lowest terms, its numerator and denominator will be prime to each other.

(c.) A fraction may be reduced to its lowest terms by dividing both numerator and denominator by all their common factors, or, which is the same thing, by their greatest common divisor.

1. Reduce $\frac{24}{36}$ to its lowest terms.

1st Solution.—Dividing both terms of $\frac{24}{36}$ by their greatest common divisor, 12, gives $\frac{2}{3}$ for a result.

2d Solution.—Dividing both terms of $\frac{24}{36}$ by their common factor, 4, gives $\frac{6}{9}$ for a result, both terms of which contain the factor 3. Dividing them by 3 gives $\frac{2}{3}$ for a result, the terms of which are prime to each other. Hence, $\frac{24}{36}$ reduced to its lowest terms $= \frac{2}{3}$, as before.

(d.) The numbers by which we divide in reducing fractions to lower terms, are said to be cancelled.

Illustrations.—In the first solution, we cancelled the factor 12. In the second, we cancelled the factor 4 and then the factor 3.

(e.) Reduce the following fractions to their lowest terms:

2. $\frac{4}{6}$.
3. $\frac{9}{12}$.
4. $\frac{15}{18}$.
5. $\frac{15}{21}$.
6. $\frac{30}{40}$.
7. $\frac{16}{48}$.
8. $\frac{54}{56}$.
9. $\frac{12}{25}$.
10. $\frac{48}{72}$.
11. $\frac{35}{105}$.
12. $\frac{91}{143}$.
13. $\frac{323}{391}$.
14. $\frac{931}{969}$.
15. $\frac{209}{715}$.
16. $\frac{374}{561}$.
17. $\frac{132}{143}$.
18. $\frac{2261}{2737}$.
19. $\frac{7293}{8151}$.

(f.) A fraction is sometimes expressed by the factors of its numerator and denominator.

Illustration.—The fraction $\frac{8 \times 15}{9 \times 16}$, which may be read, "8 times 15 divided 9 times 16," or, "The fraction having 8 times 15 for a numerator and 9 times 16 for a denominator."

(g.) Such fractions should be reduced to their lowest terms before multiplying the factors together.

20 Reduce $\frac{8 \times 15}{9 \times 16}$ to its lowest terms.

Solution.—Cancelling 8 from the 8 of the numerator and from the 16 of the denominator gives 1 in place of the former and 2 in place of the latter.

Cancelling 3 from the 15 of the numerator and from the 9 of the denominator gives 5 in place of the former and 3 in place of the latter. As no further cancellation can be made, we multiply the remaining factors together, which gives $\frac{1 \times 5}{3 \times 2} = \frac{5}{6}$ for the result.

The following is a convenient form of writing the work:

$$\frac{\overset{1}{\cancel{8}} \times \overset{5}{\cancel{15}}}{\underset{3}{\cancel{9}} \times \underset{2}{\cancel{16}}} = \frac{1 \times 5}{3 \times 2} = \frac{5}{6}.$$

(h.) Reduce the following to their lowest terms:

21. $\frac{7 \times 9}{8 \times 12}$.
22. $\frac{10 \times 12}{8 \times 15}$.
23. $\frac{4 \times 7}{3 \times 14}$.
24. $\frac{5 \times 9}{6 \times 15}$.
25. $\frac{7 \times 8 \times 9}{14 \times 12 \times 18}$.
26. $\frac{8 \times 9 \times 10}{25 \times 24 \times 27}$.
27. $\frac{9 \times 10 \times 11}{15 \times 55 \times 3}$.
28. $\frac{15 \times 17 \times 24}{51 \times 48 \times 30}$.
29. $\frac{21 \times 16 \times 32 \times 49}{24 \times 28 \times 35 \times 36}$.
30. $\frac{17 \times 13 \times 36 \times 28}{38 \times 40 \times 51 \times 39}$.
31. $\frac{10 \times 11 \times 12 \times 9 \times 8}{24 \times 27 \times 60 \times 33 \times 55}$.
32. $\frac{15 \times 63 \times 12 \times 77}{33 \times 40 \times 56 \times 144}$.
33. $\frac{144 \times 18 \times 75 \times 132}{150 \times 1728 \times 96 \times 38}$.
34. $\frac{299 \times 77 \times 374 \times 262}{121 \times 713 \times 324 \times 561}$.

86. To find a Fractional Part of a Number.

1. What is $\frac{5}{6}$ of 715?

1st Solution.—$\frac{5}{6}$ of 715 equals 5 times $\frac{1}{6}$ of 715. $\frac{1}{6}$ of 715, found by dividing 715 by 6, is $119\frac{1}{6}$, and 5 times $119\frac{1}{6}$ is $595\frac{5}{6}$. Hence, $\frac{5}{6}$ of 715 = $595\frac{5}{6}$.

2d Solution.—$\frac{5}{6}$ of 715 = $\frac{1}{6}$ of 5 times 715. 5 times 715 = 3575, and $\frac{1}{6}$ of 3575 is $595\frac{5}{6}$. Hence, $\frac{5}{6}$ of 715 = $595\frac{5}{6}$, as before.

(a.) As $\frac{5}{6}$ of 715 equals 5 times $\frac{1}{6}$ of 715, or $\frac{1}{6}$ of 5 times 715, so $\frac{5}{6}$ of any number equals 5 times $\frac{1}{6}$ of that number, or $\frac{1}{6}$ of 5 times that number; .07 of any number equals 7 times .01 of that number, or .01 of 7 times that number, etc. Hence —

(b.) To get a fractional part of a number, we either divide tha number by the denominator of the fraction and multiply the quotient by the numerator, or we multiply the number by the numerator of the fraction and divide the product by the denominator.

(c.) What is —

2. $\frac{4}{5}$ of 1728?
3. $\frac{3}{7}$ of 2946?
4. $\frac{5}{8}$ of 4243?
5. $\frac{6}{11}$ of 6181?
6. $\frac{4}{7}$ of 1359?
7. $\frac{3}{8}$ of 6453?
8. $\frac{13}{35}$ of 4284?
9. $\frac{22}{25}$ of 1235?
10. $\frac{8}{21}$ of 1235?

11. What is $4\frac{2}{3}$ times 389?

SOLUTION. — $4\frac{2}{3}$ times 389 = 4 times 389 + $\frac{2}{3}$ of 389 = 1556 + $259\frac{1}{3}$ $1815\frac{1}{3}$.

(d.) What is —

12. $5\frac{7}{8}$ times 957?
13. $2\frac{4}{7}$ times 879?
14. $9\frac{3}{4}$ times 745?
15. $8\frac{1}{2}$ times 1347?
16. $26\frac{2}{3}$ times 1856?
17. $37\frac{9}{16}$ times 7431?
18. $19\frac{7}{17}$ times 3886?
19. $48\frac{3}{11}$ times 7932?

20. What is .009 of 32.45?

1ST SOLUTION.—.009 of 32.45 equals 9 times .001 of 32.45. .001 of 32.45, found by removing the decimal point 3 places to the left (see 14), is .03245, and 9 times .03245 = .29205.

2D SOLUTION.—.009 of 32.45 = .001 of 9 times 32.45. 9 times 32.45 = 292.05, and .001 of this, found by removing the decimal point 3 places further to the left, is .29205, as before.

(e.) What is —

21. .05 of 375?
22. .13 of 87.3?
23. .016 of 245.03?
24. .25 of .25?
25. .623 of 4.86?
26. .001 of .001?
27. 1.3[*] of 34.2?
28. 2.5 of 25?
29. 2.03 of 832?
30. 1.54 of .761?
31. 3.7 of .896?
32. 2.003 of 2.003?

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[*] The 1.3 should be read as an improper fraction, as should all other numbers similarly situated.

87. Reduction of Compound Fractions to Simple Ones.

(a.) To reduce a compound fraction to a simple one is merely to find a fractional part of a fraction.

1. Reduce the compound fraction $\frac{9}{10}$ of $\frac{5}{12}$ to a simple fraction, *i. e.* find $\frac{9}{10}$ of $\frac{5}{12}$.

SOLUTION. — $\frac{9}{10}$ of $\frac{5}{12}$ may be found by writing $\frac{5}{12}$, and then making 9 a factor of the numerator and 10 a factor of the denominator (see **83**, ex. 15) cancelling and reducing as in the written work below:

$$\frac{9}{10} \text{ of } \frac{5}{12} = \frac{\cancel{5} \times \overset{3}{\cancel{9}}}{\underset{4}{\cancel{12}} \times \underset{2}{\cancel{10}}} = \frac{3}{8}.$$

2. Reduce $\frac{11}{21}$ of $\frac{49}{54}$ of $1\frac{5}{22}$ to a simple fraction.

SOLUTION. — Writing $1\frac{5}{22}$, or its equal, $\frac{27}{22}$, as the number of which the fractional part is to be obtained, we find $\frac{49}{54}$ of $\frac{27}{22}$ by making 49 a factor of the numerator and 54 a factor of the denominator; and $\frac{11}{21}$ of this result by making 11 a factor of the numerator and 21 a factor of the denominator. The written work would take the following form:

$$\frac{11}{21} \text{ of } \frac{49}{54} \text{ of } 1\frac{5}{22} = \frac{\cancel{27} \times \overset{7}{\cancel{49}} \times \cancel{11}}{\underset{2}{\cancel{22}} \times \underset{2}{\cancel{54}} \times \underset{3}{\cancel{21}}} = \frac{7}{12}.$$

(b.) These solutions show that, to reduce a compound fraction to a simple one, we first reduce the mixed numbers, if there are any, to improper fractions, then make all the numerators of the compound fraction factors of the new numerator, and all the denominators factors of the new denominator, and then cancel and reduce.

(c.) Reduce each of the following compound fractions to simple ones.

3. $\frac{3}{4}$ of $\frac{4}{5}$.
4. $\frac{7}{8}$ of $\frac{21}{32}$.
5. $\frac{15}{16}$ of $\frac{24}{25}$.
6. $\frac{3}{5}$ of $\frac{4}{9}$.
7. $\frac{36}{49}$ of $\frac{35}{48}$.
8. $\frac{3}{5}$ of $\frac{9}{10}$.
9. $\frac{17}{18}$ of $\frac{27}{24}$.
10. $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$.
11. $\frac{5}{6}$ of $\frac{8}{9}$ of $\frac{18}{25}$.
12. $\frac{3}{7}$ of $\frac{5}{9}$ of $2\frac{1}{3}$.
13. $\frac{3}{5}$ of $\frac{10}{11}$ of $\frac{8}{9}$.
14. $\frac{7}{4}$ of $\frac{8}{14}$ of $\frac{2}{5}$.
15. $\frac{4}{7}$ of $\frac{21}{32}$ of $2\frac{3}{5}$.
16. $\frac{6}{7}$ of $\frac{21}{4}$ of $\frac{8}{9}$.

17. $\frac{9}{10}$ of $\frac{8}{9}$ of $\frac{7}{8}$ of $\frac{6}{7}$ of $\frac{5}{6}$.
18. $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{8}{9}$ of $\frac{9}{10}$ of $\frac{11}{12}$.
19. $\frac{7}{8}$ of $\frac{13}{14}$ of $\frac{24}{25}$ of $7\frac{1}{2}$.
20. $\frac{8}{9}$ of $\frac{4}{5}$ of $\frac{3}{5}$ of $\frac{7}{8}$ of $4\frac{1}{6}$.
21. $\frac{8}{11}$ of $\frac{8}{9}$ of $\frac{3}{5}$ of $1\frac{3}{5}$.
22. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{7}{8}$ of $\frac{8}{9}$ of $\frac{9}{10}$.

88. Reduction of Vulgar Fractions to Decimals.

(a.) To reduce a vulgar fraction to an equivalent decimal, we have only to get the part of 1 which the given fraction indicates, carrying out the work to the required number of decimal places.

(b.) Practically, the reduction is made by annexing zeroes to the numerator and dividing by the denominator.

Note.—In most cases it will be sufficiently exact to carry out the division to 3 or 4 decimal places, but sometimes it is necessary to carry it out much further.

1. Reduce $\frac{5}{14}$ to a decimal fraction.

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14)5.0000).3571 3/7
   ------
     80
    ----
     100
    -----
       20
```

Solution.—$\frac{5}{14}$, or $\frac{5}{14}$ of $1 = \frac{1}{14}$ of $5 = \frac{1}{14}$ of 50 tenths $= \frac{1}{14}$ of 500 hundredths $= \frac{1}{14}$ of 5000 thousandths, etc. which may be found as in the written work opposite. Hence, $\frac{5}{14} = .3571 +$.

(c.) Reduce the following to decimal fractions, carrying out the division, when only approximate values can be obtained, to 6 places of decimals.

2. $\frac{9}{16}$.
3. $\frac{10}{27}$.
4. $\frac{14}{25}$.
5. $\frac{8}{19}$.
6. $\frac{17}{32}$.
7. $\frac{5}{6}$.
8. $\frac{2}{3}$.
9. $\frac{2}{625}$.
10. $\frac{3}{40}$.
11. $\frac{7}{20}$.
12. $\frac{13}{14}$.
13. $\frac{21}{26}$.

89. Fractional Parts of Denominate Numbers.

1. What is the value of $\frac{7}{12}$ of a mile in units of lower denominations, *i. e.* in fur. rd. yd. etc.?

1st Solution.—$\frac{7}{12}$ of a mile $= \frac{1}{12}$ of 7 miles, which is found by dividing *7 m.* 0 fur. 0 rd. 0 yd. 0 ft. 0 in. by 12, as in compound division.

2nd Solution.—Since 1 m. = 8 fur., $\frac{7}{12}$ of a mile must equal $\frac{7}{12}$ of 8 fur. = $4\frac{2}{3}$ fur. But $\frac{2}{3}$ of a fur. = $\frac{2}{3}$ of 40 rd. = $26\frac{2}{3}$ rd. But $\frac{2}{3}$ of a rd. = $\frac{2}{3}$ of $5\frac{1}{2}$ yd. = $\frac{2}{3}$ of $\frac{11}{2}$ yd. = $3\frac{2}{3}$ yd. But $\frac{2}{3}$ of a yd. = $\frac{2}{3}$ of 3 ft. = 2 ft. Hence, $\frac{7}{12}$ of a mile = 4 fur. 26 rd. 3 yd. 2 ft.

3d Solution.—There must be 8 times as many furlongs as miles, or in $\frac{7}{12}$ of a mile, 8 times $\frac{7}{12}$ fur. = $4\frac{2}{3}$ fur. But there must be 40 times as many rods as furlongs, or in $\frac{2}{3}$ fur. 40 times $\frac{2}{3}$ rods = $26\frac{2}{3}$ rods. But there must be $5\frac{1}{2}$ or $\frac{11}{2}$ times as many yards as rods, or in $\frac{2}{3}$ of a rod $\frac{11}{2}$ times $\frac{2}{3}$ yd. = $3\frac{2}{3}$ yd. But there must be three times as many feet as yards, or in $\frac{2}{3}$ of a yard, 3 times $\frac{2}{3}$ ft. = 2 ft. Hence, $\frac{7}{12}$ of a mile = 4 fur. 26 rd. 3 yd. 2 ft.

(a.) Reduce each of the following to units of a lower denomination:

2. $\frac{5}{9}$ of £1.
3. $\frac{2}{3}$ of 1 gal.
4. $\frac{6}{7}$ of 1 wk.
5. $\frac{4}{5}$ of 1 cord.
6. $\frac{8}{11}$ of 1 acre.
7. $\frac{5}{11}$ of 1 ton.
8. $\frac{17}{22}$ of 1 mile.
9. $\frac{43}{54}$ of 1 degree.

10. Reduce .5135 of a pound Troy to units of a lower denomination.

Solution.—There must be 12 times as many ounces as pounds, or, in this case, 12 times .5135 of an ounce = 6.1620 oz. = 6.162 oz. But there must be 20 times as many pennyweights as ounces, or, in .162 of an ounce, 20 times .162 of a pennyweight = 3.240 dwt. = 3.24 dwt. But there must be 24 times as many grains as pennyweights, or, in .24 of a pennyweight, 24 times .24 of a grain = 5.76 gr.

Hence, .5135 of a lb. = 6 oz. 3 dwt. 5.76 gr.

$$\begin{array}{r} .5135 \\ 12 \\ \hline 6.1620 \\ 20 \\ \hline 3.240 \\ 24 \\ \hline 5.76 \end{array}$$

Note.—It will be seen that, practically, the reductions in the last example, as well as in those immediately preceding it, were made by multiplying the fractional part of each denomination by the number which expresses how many units of the next lower denomination are equal to a unit of the denomination of the fraction.

(b.) Reduce each of the following to lower denominations:

11. .4595 of a £.
12. .3572 of a ton.
13. .076 of a week.
14. .00276 of a league.
15. .1824 of a rood.
16. .00568 of a cord.
17. .475 of a bushel.
18. .396 of a ℔.
19. .0005 of a mile.
20. .1876 of an acre.

90. One Number a Fractional Part of Another.

1. What part of 3 is 1?

ANSWER.—1 is $\frac{1}{3}$ of 3, because 3 times 1 = 3, or because 1 taken three times = 3.

(a.) What part—

2. Of 5 is 1?
3. Of 6 is 1?
4. Of 9 is 1?

5. What part of 12 is 9?

ANSWER.—Since 1 is $\frac{1}{12}$ of 12, 9 must be $\frac{9}{12}$ of 12 = $\frac{3}{4}$ of 12.

(b.) What part—

6. Of 7 is 4?
7. Of 10 is 6?
8. Of 10 is 9?
9. Of 9 is 10?

10. What part of 4 lb. is 3 lb.?

ANSWER.—3 lb. is the same part of 4 lb. that 3 is of 4, which is $\frac{3}{4}$.

11. What part of $\frac{6}{13}$ is $\frac{9}{13}$?

ANSWER.—$\frac{9}{13}$ is the same part of $\frac{6}{13}$ that 9 is of 6, which is $\frac{9}{6} = \frac{3}{2}$.

(c.) What part—

12. Of 19 gr. is 3 gr.?
13. Of 25℥ is 5℥?
14. Of $\frac{25}{13}$ is $\frac{5}{12}$?
15. Of $\frac{12}{18}$ is $\frac{9}{18}$?
16. Of .64 is .48?
17. Of .5 is .3?

18. What part of 9 lb. 7 oz. 13 dwt. is 1 lb. 3 oz. 15 dwt. 17¼ gr.?

SOLUTION.—Reducing both quantities to the lowest denomination mentioned, *i. e.* to fourths of a grain, we have 9 lb. 7 oz. 13 dwt. = $\frac{222048}{4}$ of a grain; 1 lb. 3 oz. 15 dwt. 17¼ gr. = $\frac{30309}{4}$ of a grain, and $\frac{30309}{4}$ is the same part of $\frac{222048}{4}$ that 30309 is of 222048, which is $\frac{30309}{222048}$, or $\frac{10103}{74016}$. Hence, 1 lb. 3 oz. 15 dwt. 17¼ gr. is $\frac{10103}{74016}$ of 9 lb. 7 oz. 13 dwt.

(d.) The above solutions show that we may find what part one compound number is of another of similar denominations, by reducing both to the lowest denomination mentioned in either, and making the value of the number of which the fractional part is required the denominator, and the value of the other number the numerator. The fraction thus obtained should be reduced to its lowest terms.

(e.) What part—

20. Of 3 dwt. 17 gr. is 5 dwt. 9 gr.?
21. Of 4 qt. 1 pt. is 3 qt.?
22. Of 3 da. is 1 wk.?
23. Of £7 is 7s.?
24. Of 3 qt. is 1 pk.?
25. Of 2 qr. 23 lb. is 2 qr. 22 lb.?
26. Of $17\frac{13}{47}$ is $12\frac{38}{47}$?
27. Of $4\frac{1}{2}$ is 3?
28. Of $\frac{3}{7}$ is 1?
29. Of 6 is .06?
30. Of .08 is 1?
31. Of 1.4 is 6.2?
32. Of £1 13s. 3d. 1 far. is 8d. 2 far.?
33. Of 3 bu. 2 pk. $5\frac{1}{3}$ qt. is 1 bu.?
34. Of 1 m. 6 fur. $4\frac{3}{4}$ rd. is 1 m. 2 fur. 19 rd.?
35. Of 1 wk. is 5 da. 3 h. 15 min. 37.3 sec.?
36. Of 1 da. 13 h. 19 min. 24.7 sec. is 1 wk.?
37. Of 1 T. is 13 cwt. 2 qr. 17 lb. 6 oz. $5\frac{3}{4}$ dr.?

(f.) The following method is often better than the preceding, when we wish to find what part a compound number is of a *unit* of a higher denomination.

38. What part of 1 mile is 4 fur. 26 rd. 3 yd. 2 ft.?

1st Solution.—Since 1 ft. = $\frac{1}{3}$ of a yd., 2 ft. must equal $\frac{2}{3}$ of a yard, to which adding the 3 yd. gives $3\frac{2}{3}$ yd. or $\frac{11}{3}$ yd. Since 1 yd. = $\frac{2}{11}$ of a rod,* $\frac{11}{3}$ of a yard must equal $\frac{11}{3}$ of $\frac{2}{11}$ of a rod = $\frac{2}{3}$ of a rod, to which adding the 26 rd. gives $26\frac{2}{3}$ rd. = $\frac{80}{3}$ rd. Since 1 rod = $\frac{1}{40}$ of a furlong $\frac{80}{3}$ rd. must equal $\frac{80}{3}$ of $\frac{1}{40}$ of a fur. = $\frac{2}{3}$ of a fur., to which adding the 4 fur. gives $4\frac{2}{3}$ fur. = $\frac{14}{3}$ fur. Since 1 fur. = $\frac{1}{8}$ of a mile, $\frac{14}{3}$ fur. must equal $\frac{14}{3}$ of $\frac{1}{8}$ of a mile = $\frac{7}{12}$ of a mile. Hence, 4 fur. 26 rd. 3 yd. 2 ft. = $\frac{7}{12}$ of a mile.

2nd Solution.—There must be $\frac{1}{3}$ as many yards as feet, or, in 2 ft. $\frac{1}{3}$ of 2 yd. = $\frac{2}{3}$ of a yard. But there must be $\frac{2}{11}$ as many rods as yards, or, in $3\frac{2}{3}$ yd., there must be $\frac{2}{11}$ of $3\frac{2}{3}$ rods = $\frac{2}{3}$ of a rod. But there must be $\frac{1}{40}$ as many furlongs as rods, or, etc.

Note.—This reduction is the opposite of that explained in 89.

(g.) What part—

39. Of £1 is 11s. 1 d. $1\frac{1}{3}$ far.?
40 Of 1 gal. is 2 qt. 1 pt. $1\frac{1}{3}$ gi.?
41. Of 1 wk. is 6 da.?
42. Of 1 cord is 6 cd. ft. 6 cu. ft. $691\frac{1}{5}$ cu. in.?

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* For 1 rod = $5\frac{1}{2}$ yd. = $\frac{11}{2}$ of a yd., and 1 yd. or $\frac{2}{2}$ of a yd. is the same part of $\frac{11}{2}$ that 2 is of 11, which is $\frac{2}{11}$.

43. Of 1 acre is 2 R. 36 sq. rd. 11 sq. yd.?
44. Of 1 ton is 9 cwt. 9 lb. 1 oz. $7\frac{3}{11}$ dr.?
45. Of 1 mile is 6 fur. 7 rd. 1 yd. 1 ft. 6 in.?
46. Of 1 degree is 47′ $46\frac{2}{3}$″?

(h.) Should it be required to give the answers in a decimal form, it will only be necessary to reduce the vulgar fractions obtained as above to decimal fractions.

(i.) The process illustrated in the following solution may also be employed.

47. What part of 1 lb. is 6 oz. 3 dwt. 5.76 gr.?

SOLUTION.—There must be $\frac{1}{24}$ as many pennyweights as grains, or, in 5.76 gr. there must be $\frac{1}{24}$ of 5.76 dwt. = .24 of a dwt., to which adding the 3 dwt. gives 3.24 dwt. But there must be $\frac{1}{20}$ as many ounces as pennyweights, or, in 3.24 dwt., $\frac{1}{20}$ of 3.24 oz. = .162 oz., to which adding the 6 oz. gives 6.162 oz. But there must be $\frac{1}{12}$ as many pounds as ounces, or, in 6.162 oz., $\frac{1}{12}$ of 6.162 lb. = .5135 of a lb. Hence, 6 oz. 3 dwt. 5.76 gr. = .5135 of a lb.

NOTE.—The most convenient form of writing the work for the last solution, is to write the numbers to be reduced in a vertical column, the smallest denomination uppermost, as indicated below.

$$\begin{array}{l}
24.)\underline{5.76}\text{ gr.} \\
20)\underline{3.24}\text{ dwt.} = 3\text{ dwt. }5.76\text{ gr.} \\
12)\underline{6.1620}\text{ oz.} = 6\text{ oz. }3\text{ dwt. }5.76\text{ gr.} \\
\quad\;\; .5135\text{ lb.} = 6\text{ oz. }3\text{ dwt. }5.76\text{ gr.}
\end{array}$$

(j.) What decimal part—

48. Of £1 is 9s. 2d. 1.12 far.?
49. Of 1 ton is 7 cwt. 14 lb. 6 oz. 6.4 dr.?
50. Of 1 week is 12 h. 46 min. 4.8 sec.?
51. Of 1 league is 2 rd. 3 yd. 1 ft. 8.6208 in.?
52. Of 1 rood is 7 sq. rd. 8 sq. yd. 8 sq. ft. 84.384 sq. in.?
53. Of 1 cord is 1256.32512 cu. in.?
54. Of 1 bu. is 1 pk. 7 qt. 1.6 gi.?
55. Of 1℔ is 4℥ 6ʒ .96 gr.?
56. Of 1 mile is 2 ft. 7.68 in.?
57. Of 1 acre is 30 sq. rd. 4 sq. ft. 51.264 sq. in.?

91. To multiply by a Fraction.

(a.) Multiplying a number by 1 gives the number itself for a product. Hence, multiplying a number by $\frac{3}{4}$ of 1, or $\frac{3}{4}$, must give $\frac{3}{4}$ of the number for a product; multiplying a number by .23 must give .23 of the number for a product; and, generally—

(b) To multiply a number by a fraction is merely to obtain such a part of it as the fraction indicates.

(c.) Hence, to multiply by a fraction, we have only to multiply by the numerator of the fraction and divide by the denominator.

1. What is $\frac{7}{8}$ times 254?

Solution.—$\frac{7}{8}$ times 254 equals $\frac{7}{8}$ of 254, which, found as explained in 86, is $222\frac{1}{4}$.

Note.—The fraction in each answer should be reduced to its lowest terms.

(d.) What is the product of—

2. $\frac{5}{6}$ times 783?
3. $\frac{2}{3}$ times 4.86?
4. $\frac{3}{5}$ times 6.345?
5. $\frac{4}{9}$ times 868?
6. $\frac{5}{11}$ times 7.92?
7. $\frac{4}{13}$ times 6.07?
8. $\frac{8}{9}$ times 4.158?
9. $\frac{3}{7}$ times 943?

10. What is the product of $\frac{8}{9} \times \frac{11}{12}$?

Solution.—$\frac{8}{9}$ multiplied by $\frac{11}{12} = \frac{11}{12}$ of $\frac{8}{9}$, which is found by writing the fraction $\frac{8}{9}$ and then making 11 a factor of the numerator and 12 a factor of the denominator, as in the written work below.

$$\frac{8}{9} \times \frac{11}{12} = \frac{\overset{2}{\cancel{8}} \times 11}{9 \times \underset{3}{\cancel{12}}} = \frac{22}{27}.$$

11. What is the product of $3\frac{1}{2} \times 2\frac{2}{3} \times 1\frac{5}{7}$?

Solution.—$3\frac{1}{2} \times 2\frac{2}{3} \times 1\frac{5}{7} = \frac{7}{2} \times \frac{8}{3} \times \frac{12}{7}$. But $\frac{7}{2}$ multiplied by $\frac{8}{3} = \frac{8}{3}$ of $\frac{7}{8}$, and this product multiplied by $\frac{12}{7} = \frac{12}{7}$ of $\frac{8}{3}$ of $\frac{7}{2}$. Hence the following written work:

$$3\frac{1}{2} \times 2\frac{2}{3} \times 1\frac{5}{7} = \frac{\cancel{7} \times 8 \times \overset{2}{\cancel{12}}}{\cancel{2} \times \cancel{3} \times \cancel{7}} = 16.$$

(e.) The above solutions show that to multiply vulgar fractions together is the same thing as to reduce compound fractions to simple ones. (See **87.**)

Find the following products:

12. $\frac{5}{6} \times \frac{7}{8}$?
13. $\frac{1}{2} \times \frac{1}{2}$?
14. $\frac{2}{3} \times \frac{2}{3}$?
15. $\frac{1}{2} \times \frac{1}{3}$?
16. $\frac{3}{5} \times \frac{3}{4}$?
17. $3\frac{2}{3} \times \frac{7}{8}$?
18. $4\frac{2}{3} \times 5\frac{1}{7}$?
19. $6\frac{3}{7} \times 4\frac{5}{8}$?
20. $\frac{7}{8} \times \frac{2}{3} \times \frac{5}{6}$?
21. $\frac{4}{9} \times \frac{10}{11} \times \frac{8}{15}$?
22. $\frac{7}{8} \times \frac{5}{9} \times 5\frac{1}{6}$?
23. $3\frac{1}{5} \times \frac{11}{12} \times \frac{2}{3}$?
24. $\frac{34}{39} \times \frac{52}{47} \times 2\frac{13}{17}$?
25. $4\frac{1}{5} \times 3\frac{2}{3} \times 7\frac{1}{2}$?
26. $8\frac{7}{11} \times 5\frac{3}{7} \times 2\frac{1}{5}$?
27 $27\frac{4}{5} \times 6\frac{3}{8} \times 4\frac{1}{2}$?
28. $\frac{4}{7}$ of $\frac{5}{9} \times \frac{14}{15}$ of $\frac{27}{32}$?
29. $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{5}{9} \times \frac{3}{4}$ of $\frac{7}{8}$?
30. $\frac{5}{6}$ of $2\frac{2}{5} \times \frac{3}{8}$ of $4\frac{1}{2} \times 5\frac{1}{3} \times \frac{8}{9}$?
31. $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4} \times \frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$?
32. $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{6}{7} \times 1\frac{1}{2} \times 1\frac{1}{4} \times 1\frac{1}{6}$?

92. To multiply by a Decimal Fraction.

$$\begin{array}{r} 8.43 \\ 23.7 \\ \hline 5901 \\ 2529 \\ 1686 \\ \hline 199.791 \end{array}$$

1. What is 23.7 × 8.43?

1st Solution. — 23.7 times 8.43 = 23.7* of 8.43, which, found by multiplying 8.43 by 23.7 and removing the point one place further to the left (86, 20th example) is 199.791.

$$\begin{array}{r} 23.7 \\ 8.43 \\ \hline 711 \\ 948 \\ 1896 \\ \hline 199.791 \end{array}$$

2d Solution. — 3.7 multiplied by 8.43 = 8.43* of 23.7, which, found by multiplying 23.7 by 8.43 and removing the point two places further to the left, is 199.791.

Note. — The slight difference in the above solutions of the last example results from the different reading of the sign of multiplication. The student should not be confined to either form, but should be prepared to use the one that is most convenient in the example he is considering.

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* When mixed decimal numbers are to be multiplied together, the multiplier should in the solution be read as an improper fraction. The multiplicand may be read either as a mixed number or as an improper fraction. Thus in the first of the above solutions, "23.7 of 8.43" should either be read $\frac{237}{10}$ of $\frac{843}{100}$, or $\frac{237}{10}$ of $8\frac{43}{100}$; while in the second "8.43 of 23.7" should either be read $\frac{843}{10}$ of $\frac{237}{10}$, or $\frac{843}{10}$ of $23\frac{7}{10}$.

(a.) From the above solutions, we infer that, to multiply by a decimal fraction, we multiply as though it were a whole number, and remove the decimal point in the product as many places further to the left than it is in the multiplicand as there are places of decimal fractions in the multiplier. The product, then, will contain as many places of decimal fractions as there are in both the multiplier and multiplicand.

(b.) Find each of the following products:

2. .3 × .3?
3. 4.5 × .007?
4. .25 × .25?
5. .48 × 6.3?
6. .006 × .03?
7. .005 × .005?
8. .878 × 1.06?
9. 3243 × 1.04?
10. 8 × .3 × 7?
11. .08 × .03 × .07?
12. 8 × .3 × .07?
13. 653 × .4 × 9.5?
14. .25 × .25 × .25?
15. 12.5 × .5 × 1.6?
16. 3.07 × 2.05 × 40.3?
17. 14 × 1.4 × .14?
18. .06 × .7 × .005 × 600?
19. 3.4 × .007 × 8.03 × 50?
20. 500 × .005 × .05 × 5000?
21. 375.0623 × 6.423 × 50.0329?

93. Practical Problems.

1. How much will $\frac{5}{12}$ of a ton of iron cost at $54.25 per ton?

SOLUTION.—If 1 ton of iron costs \$54.25, $\frac{5}{12}$ of a ton will cost $\frac{5}{12}$ of \$54.25, which, found as before explained, is \$22.60$\frac{5}{12}$ = \$22.604 +.

NOTE.—It will often, if not usually, be found best to write the work for these problems in form for cancelling.

How much will—

2. $\frac{3}{4}$ of a barrel of flour cost at \$11.28 per bbl.?
3. $\frac{5}{6}$ of a bag of coffee weigh if a bag weighs 149 lb.?
4. $\frac{7}{8}$ of a pound of tea cost at $\frac{5}{9}$ of a dollar per lb.?
5. $\frac{4}{5}$ of an acre of land cost at \$85$\frac{4}{9}$ per acre?
6. .9 of a cord of wood cost at \$6.54 per cord?
7. .73 of a hogshead of wine at \$137.50 per hhd.?

8. .297 of a stack of hay weigh if the whole weighs 4,36 tons?

9. $13\frac{2}{3}$ rods of fence cost at $\frac{3}{8}$ of a dollar per rod?

10. A lot $17\frac{3}{5}$ rods long and $16\frac{9}{11}$ rd. wide cost at $2 per rod?

11. A block 8.3 ft. long, 2.9 ft. wide, and 12 ft. high, cost at $1.53 per ft.?

12. $\frac{4}{7}$ of a sq. rd. cost at $\frac{8}{9}$ of a dollar per sq. yd.?

13. $5\frac{2}{3}$ rd. of wall cost at $\$1\frac{4}{11}$ per yd.?

14. George has $148.75, Joseph has $\frac{3}{5}$ as much money as George, and Edward has $\frac{2}{3}$ as much as Joseph. How many dollars does each have?

15. A man bought a house for $3483, and sold it so as to gain .75 of the cost? What was his gain?

16. A man who owned an acre of land sold .3 of it at the rate of $124.74 per acre, and the remainder at the rate of $137.96 per acre? How much did he receive for each lot and for both?

17. I bought .7 of a ton of coal at $8.43 per ton, 1.16 cords of wood at $7.39 per cord, and 5.5 baskets of charcoal at $.35 per basket. What was the amount of my purchase?

18. If it takes $1\frac{4}{5}$ bushels of seed to sow an acre of land, how many bushels will it take to sow $3\frac{1}{8}$ acres?

19. How many square feet are there in the four walls of a room 16 ft. long, $15\frac{3}{4}$ feet wide, and $8\frac{2}{3}$ ft. high?

20. If 12 lb. of chocolate cost $3\frac{3}{4}$, how much will $7\frac{2}{3}$ lb. cost?

SUGGESTION. — 1 lb. will cost $\$\frac{15}{4 \times 12}$, and $7\frac{2}{3}$ lb. will cost $\$\frac{15 \times 23}{4 \times 12 \times 3}$

21. If 18 acres of land are worth $\$1745\frac{5}{8}$, how much are $8\frac{5}{8}$ acres worth?

22. 3 men bought a cargo of coal for $875.625. The first took $\frac{2}{9}$ of it, the second took twice as much as the first, and the third took the remainder. How many dollars ought each to pay?

23. For how much must I sell a lot of goods costing $3244 in order to gain .15 of the cost?

24. For how much must I sell a lot of goods costing $838.54 in order to lose .27 of their cost?

25. I bought $2\frac{3}{4}$ yards of broadcloth at $5.83 per yard, $5\frac{2}{3}$ yards of cassimere at $1.65 per yard, and $6\frac{1}{2}$ yards of doeskin at $1.90 per yard, giving in payment a hundred-dollar bill. How much change ought I to receive?

26. I sold $8\frac{5}{8}$ tons of hay at $19.37 per ton, receiving in payment $8\frac{3}{8}$ cwt. of sugar at $8.69 per cwt., and the rest in money. How much did I receive in money?

27. In a cornfield there are 87 rows of corn, each row containing 138 hills. Now, allowing that each hill produces 5 ears of corn, and that each ear contains $1\frac{2}{3}$ gills, how many bushels of corn will the field contain?

28. In a certain house there are 25 windows, each window containing 12 panes of glass, each pane $16\frac{1}{2}$ inches long and $11\frac{3}{4}$ inches wide. How many square inches of glass are there in all?

29. I bought 4 loads of hay, the first weighing $1\frac{7}{8}$ tons, the second $1\frac{3}{8}$ tons, the third 2 tons, and the fourth $1\frac{5}{8}$ tons. How many tons did I buy, and what did it all cost, at $18\frac{3}{4}$ per ton?

94. To find a Number from its Fractional Part.

1. 36 is $\frac{1}{8}$ of what number?

1st Solution.—If 36 is $\frac{1}{8}$ of some number, $\frac{8}{8}$ of that number must be 8 times 36, or 288. Hence, 36 is $\frac{1}{8}$ of 288.

2nd Solution.—36 is $\frac{1}{8}$ of 8 times 36, or of 288.

(a.) Of what number—

2. Does $87 = \frac{1}{6}$?
3. Does $527 = .1$?
4. Does $5.06 = .001$?
5. Does $\frac{14}{15} = \frac{1}{18}$?

6. 42 equals $\frac{12}{13}$ of what number?

Solution.—If $42 = \frac{12}{13}$ of some number, $\frac{1}{13}$ of that number must equal $\frac{1}{12}$ of 42, which is $\frac{42}{12}$, and $\frac{13}{13}$ of the number must equal 13 times this result, which, found by making 13 a factor of the numerator and reducing, is $45\frac{1}{2}$. Hence, 42 equals $\frac{12}{13}$ of $45\frac{1}{2}$.

$$\frac{\overset{7}{\cancel{42}} \times 13}{\underset{2}{\cancel{12}}} = 45\tfrac{1}{2}.$$

7. $\frac{15}{16} = \frac{25}{36}$ of what number?

$$\frac{\overset{3}{\cancel{15}} \times \overset{9}{\cancel{36}}}{\underset{4}{\cancel{16}} \times \underset{5}{\cancel{25}}} = 1\tfrac{7}{20}.$$

Solution.—If $\frac{15}{16} = \frac{25}{36}$ of some number, $\frac{1}{36}$ of that number must equal $\frac{1}{25}$ of $\frac{15}{16}$, which is expressed by making 25 a factor of the denominator; and $\frac{36}{36}$ of the number must equal 36 times the last result, which is expressed by making 36 a factor of the numerator, as in the written work in the margin. Hence, $\frac{15}{16} = \frac{25}{36}$ of $1\frac{7}{20}$.

(b.) Of what number—

8. Does $84 = \frac{7}{11}$?
9. Does $593 = \frac{5}{6}$?
10. Does $487 = \frac{21}{43}$?
11. Does $863 = .9$?
12. Does $47.85 = .05$?
13. Does $.56 = .56$?
14. Does $5.84 = .012$?
15. Does $\frac{2}{3} = \frac{3}{4}$?
16. Does $8\frac{1}{2} = \frac{4}{9}$?
17. Does $3\frac{5}{6} = \frac{10}{11}$?
18. Does $\frac{3}{4}$ of $\frac{6}{7} = \frac{5}{7}$?
19. Does $\frac{5}{9} = \frac{5}{6}$?
20. Does $\frac{4}{5}$ of $6\frac{1}{2} = \frac{13}{14}$?
21. Does $\frac{8}{11}$ of $\frac{9}{10} = \frac{5}{6}$?

22. If $\frac{18}{25}$ of an acre of land cost \$149.88, what will an acre cost?

$$\$\frac{\overset{24.98}{\cancel{149.88}} \times 25}{\underset{3}{\cancel{18}}} = \$\frac{624.50}{3} = \$208.16\tfrac{2}{3}.$$

1st Solution.—If $\frac{18}{25}$ of an acre cost \$149.88, $\frac{1}{25}$ of an acre will cost $\frac{1}{18}$ of \$149.88, which is $\$\frac{149.88}{18}$, and $\frac{25}{25}$ of an acre will cost 25 times the last result, which may be expressed by making 25 a factor of the numerator, as in the written work in the margin.

How much will—

23. A bale of cotton weigh if $\frac{3}{4}$ of a bale weighs 327 lb.?
24. A cargo of coal cost if $\frac{5}{8}$ of it costs \$1528.35?
25. A load of wood cost if $\frac{7}{16}$ of it costs \$9.73?
26. A hogshead of molasses cost if $\frac{4}{11}$ of a hhd. costs \$17.25?
27. A barrel of flour cost if $1\frac{3}{5}$ barrel costs \$12.50?
28. A load of hay cost if $2\frac{2}{11}$ loads costs $\$35\frac{1}{3}$?
29. A barrel of pork cost if $4\frac{4}{5}$ bbl. costs \$72.96?
30. $4\frac{3}{5}$ yards of cloth cost if $4\frac{2}{7}$ yd. costs $\$5\frac{5}{8}$?
31. $\frac{6}{7}$ of a yard cost if $\frac{5}{8}$ of a yard costs $\$1\frac{5}{27}$?

32. How much will a man earn in $7\frac{3}{10}$ days if he earns \$15 in $4\frac{1}{5}$ days?

33. If $3\frac{1}{3}$ bushels of corn are worth as much as $5\frac{3}{5}$ bushels of oats, how many bushels of oats are $8\frac{4}{7}$ bushels of corn worth?

34. If 6 lb. of coffee can be bought for \$$\frac{5}{6}$, how many pounds can be bought for \$$3\frac{1}{2}$?

35. I owned $\frac{4}{5}$ of a vessel and sold $\frac{3}{5}$ of my share for \$4728. What was the whole vessel worth at that rate?

36. A prize was to be divided between the captain and the first, second, and third mates of a vessel, in such a manner that the first mate was to have $\frac{4}{5}$ as much as the captain, the second mate $\frac{3}{4}$ as much as the first mate, and the third mate $\frac{4}{5}$ as much as the second mate. Now, allowing that the share of the third mate was \$1000, what was the share of each of the others?

37. If $\frac{7}{8}$ of a yard of cloth is worth as much as $3\frac{1}{2}$ pecks of salt at $\frac{3}{16}$ of a dollar per peck, how many dollars are $15\frac{1}{5}$ yards of cloth worth?

95. To divide by a Fraction.

(a.) From the nature of division, it is obvious that—

1st. With the same dividend, the larger the divisor the smaller will be the quotient, and the smaller the divisor the larger will be the quotient.

2d. The quotient of any number divided by 1 equals the number itself. Hence, dividing by a number greater than 1 must give for a quotient a number less than the dividend, and dividing by a number less than 1 must give for a quotient a number greater than the dividend.

ILLUSTRATIONS.—The quotient of 24 ÷ 12 is 2, but the quotient of 24 ÷ $\frac{1}{2}$ of 12, or 6, is 2 times 2, or 4, and the quotient of 24 ÷ $\frac{1}{3}$ of 12, or 4, is 3 times 2, or 6.

In like manner, since the quotient of 8 ÷ 1 is 8, the quotient of 8 ÷ $\frac{1}{2}$ is 2 times 8, or 16 and the quotient of 8 ÷ $\frac{1}{3}$ is 3 times 8, or 24.

1. What is the quotient of $7 \div \frac{1}{3}$?

Solution. — Since the quotient of $7 \div 1$ is 7, the quotient of $7 \div \frac{1}{3}$ must be 3 times 7, or 21.

2. What is the quotient of $\frac{2}{5} \div \frac{1}{6}$?

Solution. — Since the quotient of $\frac{2}{5} \div 1$ is $\frac{2}{5}$, the quotient of $\frac{2}{5} \div \frac{1}{6}$ must be 6 times $\frac{2}{5}$, or $2\frac{2}{5}$.

(b.) From these examples, it is evident that the quotient of a number divided by $\frac{1}{5} = 5$ times the number; divided by $\frac{1}{7} = 7$ times the number; and, generally, that —

(c.) The quotient of a number divided by a fraction having 1 for its numerator equals the product of that number multiplied by the denominator of the fraction.

(d.) What is the quotient of —

3. $3 \div \frac{1}{2}$?
4. $4 \div \frac{1}{7}$?
5. $9 \div \frac{1}{8}$?
6. $\frac{5}{9} \div \frac{1}{4}$?
7. $\frac{2}{3} \div \frac{1}{12}$?
8. $\frac{3}{8} \div \frac{1}{6}$?
9. $8 \div .1$?
10. $6.3 \div .01$?
11. $46 \div .001$?

12. What is the quotient of $7 \div \frac{4}{5}$?

Solution.—Since the quotient of 7 divided by $\frac{1}{5}$ is 5 times 7, the quotient of 7 divided by $\frac{4}{5}$ must equal $\frac{1}{4}$ of 5 times 7, or $\frac{5}{4}$ of $7 = 8\frac{3}{4}$.

13. What is the quotient of $\frac{8}{9} \div \frac{2}{3}$?

Solution.—Since the quotient of $\frac{8}{9}$ divided by $\frac{1}{3}$ is 3 times $\frac{8}{9}$, the quotient of $\frac{8}{9}$ divided by $\frac{2}{3}$ must equal $\frac{1}{2}$ of 3 times $\frac{8}{9} = \frac{3}{2}$ of $\frac{8}{9} = 1\frac{1}{3}$.

(e.) What is the quotient of —

14. $\frac{3}{8} \div \frac{5}{6}$?
15. $\frac{2}{7} \div \frac{4}{9}$?
16. $\frac{5}{6} \div \frac{11}{12}$?
17. $\frac{14}{15} \div \frac{7}{8}$
18. $\frac{2}{3} \div \frac{3}{5}$?
19. $\frac{11}{12} \div \frac{7}{9}$?
20. $1\frac{2}{3} \div 2\frac{1}{7}$?
21. $4\frac{1}{2} \div 5\frac{1}{4}$?
22. $3\frac{3}{4} \div 1\frac{7}{8}$?

(f.) From the foregoing, it appears that the quotient of a number divided by $\frac{2}{3} = \frac{1}{2}$ of 3 times the number, or $\frac{3}{2}$ of the number; that the quotient of a number divided by $\frac{3}{5} = \frac{1}{3}$ of 5 times the number, or $\frac{5}{3}$ of the number; and, generally, that —

(g.) The quotient of a number divided by a fraction equals the product of that number multiplied by the fraction inverted.

(h.) Hence, to find the quotient of a number divided by a fraction, we multiply that number by the fraction inverted.

23. What is the quotient of $\frac{4}{7} \div \frac{5}{9}$?

SOLUTION.—The quotient of $\frac{4}{7} \div \frac{5}{9}$ = the product of $\frac{4}{7} \times \frac{9}{5} = \frac{36}{35} = 1\frac{1}{35}$.

24. What is the quotient of $3\frac{1}{2} \div \frac{7}{8}$ of $\frac{12}{13}$?

SOLUTION.—The quotient of $3\frac{1}{2}$ or of $\frac{7}{2}$ divided by $\frac{12}{13}$ must equal the product of $\frac{7}{2}$ multiplied by $\frac{13}{12}$, and the quotient of $\frac{7}{2} \div \frac{7}{8}$ of $\frac{12}{13}$ must equal $\frac{8}{7}$ of this last result, which gives the written work below:

$$3\frac{1}{2} \div \frac{7}{8} \text{ of } \frac{12}{13} = \frac{\cancel{7} \times 13 \times \overset{2}{\cancel{8}}}{\cancel{2} \times \underset{3}{\cancel{12}} \times \cancel{7}} = \frac{13}{3} = 4\frac{1}{3}.$$

(i.) Perform the following examples:

25. $\frac{1}{2} \div \frac{2}{3}$?

26. $2\frac{2}{3} \div 3\frac{1}{5}$?

27. $4\frac{1}{2} \div 5\frac{1}{4}$?

28. $5\frac{3}{5} \div 2\frac{5}{8}$?

29. $8\frac{1}{3} \div 2\frac{6}{7}$?

30. $5\frac{5}{6} \div 1\frac{1}{6}$?

31. $6\frac{3}{5} \div 8\frac{7}{8}$?

32. $4\frac{3}{8} \div 5\frac{7}{12}$?

33. $2\frac{17}{18} \div 2\frac{5}{24}$?

34. $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{7}{8} \div \frac{5}{6}$ of $\frac{7}{8}$ of $\frac{9}{10}$?

35. $\frac{2}{3}$ of $\frac{4}{9}$ of $\frac{18}{24} \div \frac{5}{7}$ of $\frac{6}{9}$ of $\frac{24}{25}$?

36. $\frac{8}{9}$ of $\frac{7}{8}$ of $\frac{5}{6} \div \frac{10}{11}$ of $\frac{8}{15}$ of $\frac{21}{20}$?

37. $\frac{1}{9}$ of $\frac{18}{11} \times \frac{6}{7}$ of $\frac{14}{5} \div 2\frac{3}{4} \times \frac{4}{5}$ of $\frac{10}{12}$?

38. $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{9}{10}$ = how many times $\frac{2}{3}$?

39. $\frac{3}{5}$ of $\frac{7}{9} \div 2\frac{4}{5}$ = how many times 6?

40. $\frac{5}{8}$ of $\frac{6}{7} \div \frac{15}{16}$ of $\frac{13}{14}$ = how many times 9?

41. A man owned $4\frac{4}{5}$ acres of land which he divided into house-lots, each containing $\frac{2}{3}$ of an acre. How many house-lots did he obtain?

42. When $\frac{3}{4}$ of a dollar will buy 1 yard of cloth, how many yards will \$$4\frac{1}{2}$ buy?

43. When $2\frac{1}{2}$ yards of cloth cost \$1, how many dollars will $8\frac{1}{3}$ yards cost?

44. How many bushels of grain at $\frac{3}{4}$ of a dollar per bushel can be bought for 7 quarter-eagles?

45. A man exchanged $7\frac{1}{2}$ barrels of apples worth \$$2\frac{3}{4}$ per barre for wheat worth \$$1\frac{1}{12}$ per bushel. How many bushels of wheat did he receive?

46. A man purchased 16 baskets of peaches each containing $\frac{7}{8}$ of a bushel, and sold them at the rate of $\frac{5}{8}$ of a bushel for a dollar. How much did he receive for them?

47. A man who had \$50 bought 17 bushels of salt at $\frac{7}{12}$ of a dollar per bushel, and paid the rest of his money for apples at \$2$\frac{1}{3}$ per bbl. How many barrels of apples did he buy?

48. A man who had \$100 paid $\frac{5}{6}$ of his money for broadcloth at \$2$\frac{1}{3}$ per yard. How many yards did he buy?

49. I own a lot of land 48$\frac{1}{2}$ rods long and 16$\frac{2}{3}$ rods wide. How many dollars will it cost to build a fence completely around it, provided that the fence costs $\frac{3}{4}$ of a dollar per yard?

96. To divide by a Decimal Fraction.

(**a.**) Since dividing by any fraction is equivalent to multiplying by that fraction inverted, it follows that to divide by .023 is the same thing as to multiply by $\frac{1000}{23}$, which may be done by removing the point in the dividend three places to the right and then dividing by 23, or by dividing by 23 and then removing the point in the quotient three places to the right. As a similar thing is true of any other decimal fraction, it follows that —

(**b.**) To divide by a decimal fraction, we may perform the division as though the divisor were a whole number, and then remove the point in the quotient as many places to the right as there are decimal places in the divisor. The quotient will thus contain as many places of decimal fractions as there are in the dividend more than in the divisor.

NOTE. — In case the dividend does not contain as many places of decimal fractions as the divisor, it will be well to annex ciphers enough to make as many, before beginning to divide.

1. What is the quotient of 21.201 ÷ 3.7?

SOLUTION. — The quotient of 21.201 divided by 3.7 equals the product of 21.201 multiplied by $\frac{10}{37}$, which, found by dividing by 37 and removing the decimal point one place to the right, is 5.73.

(c.) What is the quotient of—

2. 25 ÷ 5?
3. 25 ÷ .5?
4. 25 ÷ .05?
5. 25 ÷ .005?
6. 2.5 ÷ 5?
7. 2.5 ÷ .5?
8. 2.5 ÷ .05?
9. 2.5 ÷ .005?
10. .25 ÷ 5?
11. .25 ÷ .5?
12. .25 ÷ .05?
13. .25 ÷ .005?
14. .025 ÷ 5?
15. .025 ÷ .5?
16. .025 ÷ .05?
17. .025 ÷ .005?
18. 48 ÷ 1.6?
19. 6 ÷ .001?
20. .048 ÷ .24?
21. .24 ÷ .048?
22. 16 ÷ .0016?
23. 8.72 ÷ 400?
24. 9.28 ÷ 1.16?
25. .001 ÷ .25?
26. .6 of .24 of 4.8 ÷ .012 of 3.6 of .002?
27. .5 of .05 of .005 ÷ .005 of .05 of .5?
28. 1.7 of 62.5 of .07 ÷ 5.1 of .125 of 4.9?
29. 7.2 of 1.47 of .0064 of .003 ÷ 1800 of .0049 of .128 of 9000?
30. .02 of 1.5 of .075 of 8.7 ÷ .3 of 1.25 of .0029?

97. Complex Fractions.

(a.) A Complex Fraction is one which has a fraction in one or both its terms.

(b.) Complex fractions may be regarded as indicating that the numerator is to be divided by the denominator.

1. Reduce $\frac{3\frac{3}{5}}{5\frac{1}{7}}$ to a simple fraction.

Solution.—$\frac{3\frac{3}{5}}{5\frac{1}{7}} = 3\frac{3}{5} \div 5\frac{1}{7} = \frac{18 \times 7}{5 \times 36} = \frac{7}{10}$.

(c.) In the same manner reduce the following:

2. $\frac{2\frac{1}{3}}{2}$.
3. $\frac{5}{6\frac{1}{4}}$.
4. $\frac{\frac{2}{3}}{\frac{4}{7}}$.
5. $\frac{3\frac{2}{5}}{4\frac{4}{7}}$.
6. $\frac{.32}{8.1}$.
7. $\frac{7.5}{9.6}$.
8. $\frac{.44}{.0016}$
9. $\frac{5}{.005}$.

(d.) Complex fractions may also be reduced by multiplying both terms by such a number as will give a whole number in place of each.

10. Reduce $\frac{2\frac{3}{4}}{3\frac{5}{6}}$ to a simple fraction.

Solution. — If $2\frac{3}{4}$ be multiplied by 4 or any multiple of 4, and $3\frac{5}{6}$ be multiplied by 6 or any multiple of 6, the result in each case will be a whole number. Hence, if both terms of the given fraction be multiplied by some multiple of both 4 and 6, the resulting fraction will be a simple one. Multiplying by 12, the least common multiple of 4 and 6 gives $\frac{2\frac{3}{4}}{3\frac{5}{6}} = \frac{33}{46}$.

(e.) In the same manner reduce the following:

11. $\frac{6\frac{2}{3}}{7\frac{1}{2}}$.

12. $\frac{25\frac{4}{9}}{31\frac{1}{6}}$.

13. $\frac{43\frac{2}{3}}{50\frac{5}{7}}$.

14. $\frac{28\frac{3}{5}}{42\frac{3}{7}}$.

15. $\frac{3.5}{.28}$

16. $\frac{6.3}{2.7}$.

17. $\frac{1.44}{7}$.

18. $\frac{.006}{.08}$.

19. $.3\frac{1}{2}$.

20. $.05\frac{3}{4}$.

21. $.0073\frac{4}{5}$.

22. $6.58\frac{5}{9}$.

23. Reduce $\frac{2\frac{6}{7} \times 3\frac{7}{9} \times 5\frac{7}{13}}{4\frac{4}{35} \times 8\frac{8}{9} \times 10\frac{1}{5}}$ to a simple fraction.

Solution. — $\frac{2\frac{6}{7} \times 3\frac{7}{9} \times 5\frac{7}{13}}{4\frac{4}{35} \times 8\frac{8}{9} \times 10\frac{1}{5}} = \frac{20 \times 34 \times 72}{7 \times 9 \times 13} + \frac{144 \times 80 \times 51}{35 \times 9 \times 5} =$ $\frac{20 \times 34 \times 72 \times 35 \times 9 \times 5}{7 \times 9 \times 13 \times 144 \times 80 \times 51} = \frac{25}{156}$.

(f.) Reduce the following fractions:

24. $\frac{4\frac{7}{8} \times 3\frac{1}{3} \times 6\frac{4}{5}}{5\frac{1}{3} \times 9\frac{2}{7}}$.

25. $\frac{15\frac{3}{8} \times 7\frac{6}{17} \times 9}{7\frac{4}{17} \times 24 \times 1\frac{2}{13}}$.

26. $\frac{12 \times \frac{8}{9} \times 9\frac{2}{15}}{7\frac{1}{2} \times 8\frac{4}{5} \times 6}$.

27. $\frac{14\frac{2}{7} \times 18\frac{4}{5} \times 6\frac{4}{9}}{7\frac{7}{9} \times 100 \times 16}$

28. $\frac{.025 \times .75 \times 4.8}{.001 \times 62.5 \times .9}$.

29. $\frac{.009 \times .08 \times 600}{48 \times .06 \times .54 \times .036}$.

30. $\frac{.03 \times .7 \times .008 \times .0005}{3 \times 40 \times .25 \times 4.9}$.

31. $\frac{.0125 \times 6.4 \times 7000 \times .0081}{.00096 \times 500 \times 5.6 \times .000001}$.

98. Other changes in the Terms of a Fraction.

1. Reduce $\frac{5}{8}$ to an equivalent fraction having 25 for its numerator.

Solution. — Observing that the proposed numerator, 25, is 5 times the given numerator, 5, we multiply both terms of $\frac{5}{8}$ by 5, which gives $\frac{5}{8} = \frac{25}{40}$.

2. Reduce $\frac{6}{7}$ to an equivalent fraction having 15 for a numerator.

Solution.— Observing that the proposed numerator, 15, is $\frac{15}{6}$, or $\frac{5}{2}$ of the given numerator, 6, we multiply both terms of $\frac{6}{7}$ by $\frac{5}{2}$, or by $2\frac{1}{2}$, which gives $\frac{6}{7} = \frac{15}{17\frac{1}{2}}$.

(a.) In the fractions written below substitute an appropriate denominator in place of x.

3. $\frac{8}{9} = \frac{16}{x}$.
5. $\frac{6}{11} = \frac{3}{x}$.
7. $\frac{14}{17} = \frac{42}{x}$.
4. $\frac{4}{17} = \frac{12}{x}$.
6. $\frac{12}{13} = \frac{9}{x}$.
8. $\frac{5}{13} = \frac{125}{x}$.

9. Reduce $\frac{5}{8}$ to twenty-fourths.

Solution. — Observing that the proposed denominator, 24, is 3 times the given denominator, 8, we multiply both terms of $\frac{5}{8}$ by 3, which gives $\frac{5}{8} = \frac{15}{24}$.

10. Reduce $\frac{5}{8}$ to sixths.

Solution.—Observing that the proposed denominator, 6, is $\frac{3}{4}$ of the given denominator, 8, we multiply both terms of $\frac{5}{8}$ by $\frac{3}{4}$, which gives $\frac{5}{8} = \frac{3\frac{3}{4}}{6}$.

Note. — The following form of solution is sometimes adopted instead of the preceding:

Since $1 = \frac{24}{24}$, $\frac{5}{8}$ must equal $\frac{5}{8}$ of $\frac{24}{24}$, which is $\frac{15}{24}$.

Since $1 = \frac{6}{6}$, $\frac{5}{8}$ must equal $\frac{5}{8}$ of $\frac{6}{6} = \frac{3\frac{3}{4}}{6}$.

(b.) Reduce —

11. $\frac{4}{5}$ to twentieths.
12. $\frac{6}{7}$ to twenty-eighths.
13. $\frac{5}{9}$ to thirty-sixths.
14. $\frac{7}{12}$ to ninths.
15. $\frac{8}{15}$ to tenths.
16. $\frac{6}{17}$ to fifty-firsts.
17. $\frac{3}{4}$ and $\frac{2}{3}$ to twelfths.
18. $\frac{3}{4}$ and $\frac{1}{4}$ to twenty-eighths.
19. $\frac{3}{8}$ and $\frac{5}{8}$ to eighths.
20. $\frac{2}{3}$ and $\frac{3}{4}$ to twelfths.
21. $\frac{4}{7}$ and $\frac{5}{8}$ to fifty-sixths.
22. $\frac{7}{8}$ and $\frac{11}{12}$ to seventy-seconds.

99. Reduction to a Common Denominator.

(a.) Fractions have a COMMON DENOMINATOR when their denominators are alike.

ILLUSTRATIONS.—$\frac{5}{9}$ and $\frac{4}{9}$ have the common denominator, 9. $\frac{5}{6}$ and $\frac{3}{7}$ do not have a common denominator.

(b.) Fractions having different denominators can be reduced to equivalent fractions having a common denominator. This is called REDUCING THEM TO A COMMON DENOMINATOR, and is illustrated in the last six examples of the preceding article.

(c.) In reducing fractions to a common denominator —

1st. Reduce the compound and complex fractions, if any, to simple ones.

2d. Reduce the simple fractions to their lowest terms, except when to do it would increase the labor of reducing to a common denominator.

3d. Select a convenient number for the common denominator. Usually it will be best to select the least common multiple of the denominators.

4th. Multiply both terms of each fraction by the number which gives the common denominator as the denominator of the resulting fraction.

1. Reduce $\frac{3}{4}$, $\frac{4}{5}$, $\frac{11}{12}$, $\frac{13}{15}$, and $\frac{18}{20}$ to a common denominator.

Solution. — The least common multiple of the given denominators, 4, 5, 12, 15, and 20, is 60. Hence, we select 60 as the common denominator.

Then, since 60 = 15 times 4, we multiply both terms of $\frac{3}{4}$ by 15, which gives $\frac{3}{4} = \frac{45}{60}$.

Since 60 = 12 times 5, we multiply both terms of $\frac{4}{5}$ by 12, which, etc.

Hence, $\frac{3}{4} = \frac{45}{60}$, $\frac{4}{5} = \frac{48}{60}$, $\frac{11}{12} = \frac{55}{60}$, $\frac{13}{15} = \frac{52}{60}$, $\frac{19}{20} = \frac{57}{60}$.

Note. — Many select the product of the denominators for a common denominator. This method usually involves larger numbers than the preceding, as will be seen in the following —

2nd Solution. — The product of the given denominators is 72000, which we select for the common denominator. To obtain this, we multiplied 4, the denominator of the first fraction, by all the other denominators, *i. e.* by 5, 12, 15, and 20. Hence, we must multiply the numerator by the same numbers, which gives $\frac{3}{4} = \frac{54000}{72000}$. We multiplied the denominator of the second fraction by all the other denominators, *i. e.* by 4, 12, 15, and 20; hence, we must multiply the numerator by the same numbers, which gives $\frac{4}{5} = \frac{57600}{72000}$, etc.

(d.) Reduce to a common denominator —

2. $\frac{1}{2}$ and $\frac{1}{4}$.
3. $\frac{2}{3}$ and $\frac{1}{2}$.
4. $\frac{3}{4}$ and $\frac{2}{3}$.
5. $\frac{5}{6}$ and $\frac{3}{4}$.
6. $\frac{7}{8}$ and $\frac{7}{10}$.
7. $\frac{8}{9}$ and $\frac{7}{12}$.
8. $\frac{4}{5}$ and $\frac{7}{14}$.
9. $\frac{28}{32}$ and $\frac{9}{12}$.
10. $\frac{16}{24}$ and $\frac{24}{36}$.
11. $\frac{4}{5}$, $\frac{2}{3}$, and $\frac{1}{2}$.
12. $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$.
13. $\frac{11}{12}$, $\frac{13}{14}$, and $\frac{8}{21}$.
14. $\frac{5}{24}$, $\frac{17}{36}$, and $\frac{11}{18}$.
15. $\frac{2\frac{1}{2}}{5}$, $\frac{1\frac{1}{4}}{3\frac{3}{4}}$, and $\frac{7}{12}$.
16. $\frac{1\frac{1}{2}}{2\frac{1}{3}}$, $\frac{1}{1\frac{5}{9}}$, and $\frac{3}{4\frac{1}{5}}$.
17. $\frac{12}{16}$, $\frac{3\frac{1}{2}}{4\frac{2}{3}}$, and $\frac{2\frac{1}{10}}{2\frac{4}{5}}$.
18. $\frac{.5}{.21}$, $\frac{6}{1.4}$, and $\frac{.005}{35}$.
19. $\frac{3}{4}$, .9, .7$\frac{1}{2}$, and $\frac{5}{6}$.
20. $\frac{2}{9}$ of $\frac{12}{13}$, $\frac{9}{14}$ of $\frac{48}{60}$, and $\frac{3}{5}$ of $\frac{7}{12}$.
21. .6, .75, 1.0289, and .000004.
22. .05$\frac{2}{3}$, .7$\frac{3}{4}$, .07, and .09$\frac{7}{12}$.

100. Addition and Subtraction of Fractions.

(a.) Fractions, like other numbers, must be of the same denomination in order to be added or subtracted.

(**b.**) To be of the same denomination, they must be fractions of the same unit, and also have a common denominator.

Illustrations. — $\frac{5}{6}$ and $\frac{3}{8}$ cannot be added in their present form, any more than can 5 shillings and 3 pence. $\frac{11}{12}$ of a gallon and $\frac{11}{12}$ of a quart cannot be added, in their present form, any more than can 11 gallons and 11 quarts.

(**c.**) Hence, to add or subtract fractions, it is necessary to reduce compound and complex fractions to simple ones, and fractions of different units (as, $\frac{3}{4}$ of a bushel and $\frac{3}{4}$ of a quart) to fractions of the same unit, and then to reduce the fractions thus obtained to a common denominator. The addition or subtraction can then be made by adding or subtracting the numerators, as illustrated in **77**.

1. What is the sum of $\frac{5}{6}$ of $\frac{3}{4} + \frac{4\frac{1}{3}}{6\frac{1}{2}} + \frac{11}{12} + \frac{7}{18}$?

Solution. — Reducing the compound and complex fractions to simple ones, we have $\frac{5}{6}$ of $\frac{3}{4} = \frac{5}{8}$ and $\frac{4\frac{1}{3}}{6\frac{1}{2}} = \frac{2}{3}$. Hence, the problem becomes $\frac{5}{8} + \frac{2}{3} + \frac{11}{12} + \frac{7}{18}$.

Selecting 72 as the common denominator, and reducing the fractions to seventy-seconds, gives $\frac{45}{72} + \frac{48}{72} + \frac{66}{72} + \frac{28}{72} = \frac{187}{72} = 2\frac{43}{72}$.

2. What is the sum of $\frac{4}{15}$ of £1 + $2\frac{1}{2}$ shillings?

Solution. — $\frac{4}{15}$ of £1 = $5\frac{1}{3}$ s., to which adding the $2\frac{1}{2}$ shillings gives $5\frac{2}{6}$ s. + $2\frac{3}{6}$ s. = $7\frac{5}{6}$ s. $\frac{5}{6}$ of a shilling = 10 d. Hence, $\frac{4}{5}$ of £1 + $2\frac{1}{2}$ s. = 7 s. 10 d.

3. What is the value of $784\frac{11}{18} - 236\frac{23}{30}$?

Solution. — Selecting 90 as the common denominator, $784\frac{11}{18} - 236\frac{23}{30} = 784\frac{55}{90} - 236\frac{69}{90} = 547\frac{38}{45}$.

$$\begin{array}{rrr} & \text{ones.} & \text{90ths.} \\ 784\frac{11}{18} = & 784 & 55 \\ 236\frac{23}{30} = & 236 & 69 \\ \hline & 547 & 76 = 547\frac{38}{45}. \end{array}$$

Note. — The written work in the margin shows the resemblance of the above to compound subtraction.

(d.) Perform the following operations:

4. $\frac{3}{4} + \frac{1}{2}$.

5. $\frac{1}{2} + \frac{1}{3}$.

6. $\frac{2}{3} + \frac{2}{5}$.

7. $\frac{5}{6} + \frac{3}{4}$.

8. $\frac{7}{8} - \frac{3}{4}$.

9. $\frac{8\frac{2}{3}}{10\frac{1}{2}} + \frac{5\frac{1}{3}}{6\frac{2}{5}}$.

10. $\frac{9}{10} - \frac{3}{4}$.

11. $\frac{7}{12} - \frac{4}{15}$.

12. $\frac{5}{7} - \frac{5}{8}$.

13. $4\frac{2}{3} + 3\frac{3}{4}$.

14. $5\frac{1}{2} + 2\frac{5}{6}$.

15. $\frac{5\frac{1}{4}}{9\frac{1}{3}} - \frac{1\frac{5}{6}}{7\frac{6}{7}}$.

16. $\frac{9}{10} -- \frac{8}{9}$.

17 $\frac{7.2}{8} - \frac{.6}{.72}$.

18. $\frac{.25}{.075} + \frac{.075}{.25}$

19. $\frac{.2}{.03} - \frac{.02}{.3}$.

20. $\frac{4.5}{9} + \frac{3.75}{7.5}$.

21. $\frac{2\frac{2}{3}}{4.5} + \frac{1.3}{2\frac{3}{5}}$.

22. $\frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6}$.

23. $1\frac{1}{2} + 2\frac{1}{3} + 3\frac{1}{4}$.

24. $5\frac{1}{2} + 3\frac{1}{4} + 2\frac{1}{8} + \frac{1}{16}$.

25. $\frac{7}{12} + \frac{5}{9} + 2\frac{1}{8} + \frac{13}{18}$.

26. $\frac{7}{8}$ of $\frac{20}{21} - \frac{4}{9}$.

27. $\frac{6}{7} + \frac{7}{8} - \frac{19}{28}$.

28. $2\frac{3}{4} + \frac{7}{8} - \frac{5}{12}$.

29. $\frac{10}{11} + .7 + \frac{9}{55}$.

30. $.4\frac{1}{2} + 4\frac{1}{2} + .3$.

31. $\frac{5}{6} + \frac{7}{8} - \frac{8}{9}$.

32. $1 - \frac{1}{2} - \frac{1}{4} - \frac{1}{8} - \frac{1}{16} - \frac{1}{32} - \frac{1}{64}$.

33. $1 - \frac{1}{3} - \frac{1}{9} - \frac{1}{27} - \frac{1}{243} - \frac{1}{729}$.

34. $24\frac{3}{5} + 15\frac{1}{2} + 17\frac{2}{3} - 6\frac{5}{9} - 3\frac{3}{8} + 6\frac{8}{15}$.

35. $\frac{11}{12}$ of $\frac{16}{27}$ of $\frac{81}{88} + \frac{13\frac{1}{8}}{15} - \frac{2\frac{2}{3}}{5\frac{1}{2}}$ of $\frac{8\frac{1}{4}}{7\frac{1}{9}}$.

36. $\frac{47}{221} + \frac{89}{323} + \frac{197}{399}$.

37. $6.03 + 2\frac{5}{8} - 4.6\frac{4}{5} - .083 + \frac{5}{6} + \frac{.9}{2\frac{3}{4}} + .04\frac{2}{3}$.

38. $\frac{7}{8}$ of a £ + $\frac{5}{9}$ of a shilling.

39. $\frac{5}{12}$ of a mile + $\frac{1}{6}$ of a yard.

40. $\frac{4}{5}$ of a gallon + $\frac{2}{3}$ of a quart.

41. $\frac{7}{8}$ of a lb. + $\frac{3}{8}$ of an oz. + $\frac{5}{8}$ of a dwt.

42. $\frac{7}{9}$ of a ℔ − $\frac{2}{5}$ of an ℥ + $\frac{4}{7}$ of a ʒ.

43. $\frac{25}{32}$ of a bushel + $\frac{15}{16}$ of a pk. − $\frac{24}{49}$ of a qt. − $\frac{1}{49}$ of a pt.

44. $\frac{2}{7}$ of an acre $- \frac{24}{25}$ of a sq. rd. + $\frac{2.5}{4\frac{1}{3}}$ sq. yd. − $\frac{17.5\frac{2}{3}}{4.6}$ sq. in.

101. Miscellaneous Problems.

1. A merchant bought 4 pieces of cloth, the first containing $27\frac{1}{2}$ yd., the second $28\frac{2}{3}$ yd., the third $28\frac{5}{6}$ yd., and the fourth $27\frac{5}{12}$ yd. How many yards did they all contain?

2. I bought three yards of cloth at $\$3\frac{4}{5}$ per yd., 2 pairs of shoes at $1\frac{7}{8}$ per pair, and a hat for $\$3\frac{5}{6}$, giving in payment a twenty-dollar bill. How much change ought I to receive?

3. I bought $\frac{3}{4}$ of a bushel of meal at $\frac{5}{6}$ of a dollar per bu., giving in payment a quarter-eagle. How many dollars and cents ought I to receive back?

4. A lady went a shopping, and after paying $\$14\frac{1}{2}$ for a dress, $\$8\frac{3}{4}$ for a shawl, $\$7\frac{5}{6}$ for a bonnet, $\$1\frac{1}{3}$ for a collar, and $\frac{1}{6}$ of a dollar for incidental expenses, she had $\$5\frac{3}{4}$ left. How much money had she at first?

5. A, B, C & D together own a ship. A owns $\frac{1}{8}$ of her, B $\frac{2}{9}$, C $\frac{5}{12}$, and D the remainder. What part does D own?

6. A person on a journey travelled $\frac{1}{15}$ of the distance the first day, $\frac{1}{10}$ of it the second, $\frac{1}{6}$ the third, $\frac{1}{12}$ the fourth, $\frac{5}{20}$ the fifth, $\frac{5}{18}$ the sixth, and the rest the seventh. What part of the whole distance did he travel the seventh day?

7. How much more will $7\frac{2}{3}$ bu. of corn cost at $\frac{5}{8}$ of a dollar per bu., than $9\frac{3}{4}$ bu. of oats at $\frac{5}{12}$ of a dollar per bu.?

8. How much will it cost to build a rod of wall at $\frac{4}{15}$ of a dollar per foot?

9. How many solid feet in a block of stone $4\frac{1}{2}$ ft. long, $1\frac{2}{3}$ ft. thick, and $1\frac{4}{5}$ ft. wide; and how much will it cost at $\$2\frac{4}{9}$ per cubic foot?

10. If the sum of two fractions is $\frac{1}{2}$, and one of them is $\frac{2}{9}$, what is the other?

11. George has $\frac{5}{8}$ of a dollar. How much more money must he get, to be able to buy 2 knives at $\frac{7}{16}$ of a dollar each?

12. A boy who had a quarter-eagle bought 3 books with it, and received in change a quarter-dollar, 4 half-dimes, 7 three-cent pieces, and 4 cents. How much did the books cost each, they being of equal value?

13. What is the least common multiple of 18, 24, 36, 42, 54, and 60?

14. How many days will it take a man who earns $\$1\frac{2}{3}$ per day to earn $100?

15. If a man can earn $\$1\frac{3}{4}$ per day, and a boy can earn $\frac{1}{3}$ of a dollar per day, how much can 1 man and 2 boys earn in a week, there being 6 working days in a week?

16. If a man can earn $1 in $\frac{3}{4}$ of a day, how much can he earn in a week?

17. A farmer cuts $\frac{5}{6}$ of a ton of hay from $\frac{2}{3}$ of an acre of land. How much will he cut from a field containing $7\frac{1}{5}$ acres, the crop being uniform?

18. A young man who had inherited a large estate, spent $\frac{5}{8}$ of it in dissipation, when he had $2582.52 left. How much had he spent in dissipation, and what was the original value of the estate?

19. After spending $\frac{1}{5}$ and $\frac{1}{3}$ of my money, I had $13.58 left. How much money had I at first?

20. What number multiplied by $3\frac{1}{5}$ equals $1\frac{3}{7}$ times the quotient obtained by dividing 21 by 9?

21. An estate of $8539.16 was divided among 2 sons and 3 daughters, in such a way that each son had twice as much as each daughter. What was the share of each?

22. If $18000 is to be divided between two persons in such a way that the first will have three times as much as the second, what will be the share of each?

23. When pears are worth three times as much per bushel as apples, a person bought a bushel of pears and a bushel of apples for $5. What was the price of each per bushel?

24. A person bought a yard of silk and a yard of muslin for $\$2\frac{1}{4}$. Now, allowing that the muslin was worth only $\frac{2}{7}$ as much per yard as the silk, how many dollars was each worth per yard?

25. How much will it cost to build 23.86 miles of railroad at $23463.75 per mile?

26. How much will 7.5 yards of ribbon cost at $.125 per yard?

27. A man who owned a cotton factory sold $\frac{1}{6}$ of it to A, $\frac{2}{9}$ of what remained to B, $\frac{3}{5}$ of what remained to C, and the rest to D. How many dollars ought each man to pay for his share, if the factory was worth $50000?

28. What will $2\frac{7}{8}$ tons of English bar-iron cost at £1 13s. 6d. per ton?

29. Mr. Ball's farm contained 67 A. 3 R. 15 sq. rd., and I bought $\frac{3}{5}$ of it. How many A. R. and sq. rd. did I buy?

30. George started from home to go to school, a distance of $\frac{4}{7}$ of a mile. After walking $\frac{5}{6}$ of a furlong, he met James. How many furlongs, rods, etc. were the boys from the school-room when they met?

31. I bought a gold pen and case for $\frac{7}{9}$ of a £, a bottle of ink for $\frac{4}{5}$ of a shilling, and a box of wafers for $3\frac{3}{8}$ pence. What was the amount of my purchase?

32. I bought $3\frac{1}{2}$ yd. of ribbon at $\frac{3}{7}$ of a £ per yd., and $5\frac{2}{3}$ yd. at $\frac{7}{8}$ of a shilling per yd. What was the amount of my purchase?

33. How many miles of furrow will be turned in ploughing an acre, if the furrows are $\frac{1}{15}$ of a foot wide?

34. What is the value of $2\frac{2}{3}$ acres of land at $\$46\frac{5}{7}$ per acre?

35. How much will a square rod of land cost at $\$1.35\frac{5}{9}$ per square yard?

36. What must be paid for a piece of cloth containing $7\frac{1}{12}$ yards at $\$2\frac{1}{15}$ per yard?

37. How much will a nail of cloth cost at $\frac{2}{15}$ of a dollar per inch?

38. I bought $8\frac{4}{5}$ bushels of wheat at $\$1\frac{1}{3}$ per bushel, and sold it at $\$1\frac{4}{7}$ per bushel. How much did I gain by the transaction?

39. What will be gained by purchasing $15\frac{3}{4}$ barrels of apples at \$$1\frac{3}{4}$ per barrel, and selling them at \$$2\frac{5}{12}$ per bbl.?

40. I bought $27\frac{2}{3}$ bbl. of apples at \$$1.62\frac{1}{2}$ per bbl. It cost me \$$.18\frac{3}{4}$ per bbl. to have them carried to market, and I sold them for \$$2.16\frac{2}{3}$ per bbl. What did I gain on them?

41. If Charles can do a piece of work in 9 days, and Joseph can do it in 10 days, what part of it can both together do in 1 day? How many days will it take both to do it?

42. If Mr. Stone can reap a field in 5 days, and Mr. Gray can reap it in 6 days, in how many days can both together reap it?

43. Mr. French can do a piece of work in 20 days, Mr. Thompson can do it in 25 days, and Mr. Ellis can do it in 30 days. In how many days can all do it?

44. When $\frac{3}{4}$ of a barrel of crackers cost \$2.37, how much will $3\frac{2}{5}$ barrels cost?

45. When a man can earn \$10.50 in $\frac{4}{7}$ of a week, how much can he earn in 3 weeks?

46. If a ship sails 4.25 miles in .51 of an hour, how many miles will she sail in 1 day?

47. If .09 of a man's estate is worth \$854.82, how much is the whole of it worth?

48. If .75 of an acre of land costs \$98.55, how much will 3.4 acres cost?

49. If .005383 is the product and .07 is the multiplier, what is the multiplicand?

50. $\frac{7}{8} = \frac{3}{4}$ of what number?

51. If $\frac{2}{3}$ of a yard of cloth is worth \$$1\frac{5}{6}$, how much are $2\frac{3}{4}$ yd. worth?

52. When \$$4\frac{1}{6}$ will pay for $22\frac{6}{7}$ melons, how many melons will \$$4\frac{2}{3}$ pay for?

53. If $22\frac{6}{7}$ melons cost \$$4\frac{1}{6}$, how much will $25\frac{3}{5}$ melons cost?

54. If \$$4\frac{2}{3}$ will pay for $25\frac{3}{5}$ melons, how many melons will \$$4\frac{1}{6}$ pay for?

55. If a man can earn $\$2\frac{13}{18}$ in $1\frac{5}{9}$ days, how many dollars can he earn in $2\frac{8}{21}$ days?

56. If 18 horses can be kept $1\frac{13}{15}$ days on $12\frac{3}{5}$ bu. of corn, how many days can 14 horses be kept on $20\frac{1}{4}$ bu.?

Solution.—As the answer is to be in days, we first write the $1\frac{13}{15}$ days = $\frac{28}{15}$ of a day. If 18 horses can be kept so many days, 1 horse can be kept 18 times as many days, expressed by making 18 a factor of the numerator, thus, $\frac{28 \times 18}{15}$.

If $12\frac{3}{5}$ bushels, or $\frac{63}{5}$ of a bushel will keep a horse so many days, $\frac{1}{5}$ of a bushel will keep him $\frac{1}{63}$ of this number of days, expressed by making 63 a factor of the denominator, and $\frac{5}{5}$ of a bushel will keep him 5 times the last result, expressed by making 5 a factor of the numerator, thus $\frac{28 \times 18 \times 5}{15 \times 63}$.

If 1 horse can be kept so many days, 14 horses can be kept $\frac{1}{14}$ as many days, expressed by making 14 a factor of the denominator, thus, $\frac{28 \times 18 \times 5}{15 \times 63 \times 14}$. If 14 horses can be kept so many days on 1 bushel of grain, they can be kept $20\frac{1}{4}$ or $\frac{81}{4}$ times as many days on $20\frac{1}{4}$ bushels, expressed by making 81 a factor of the numerator, and 4 a factor of the denominator, thus $\frac{28 \times 18 \times 5 \times 81}{15 \times 63 \times 14 \times 4}$.

In practice, the written work would take the following form:

$$\text{Days, } \frac{\overset{2}{\cancel{28}} \times \overset{3}{\cancel{18}} \times \cancel{5} \times \overset{9}{\cancel{81}}}{\underset{3}{\cancel{15}} \times \underset{7}{\cancel{63}} \times \cancel{14} \times \underset{2}{\cancel{4}}} = \frac{3 \times 9}{7} \text{ days} = 3\tfrac{6}{7} \text{ days.}$$

Note.—In solving such problems, each condition may be considered by itself, without reference to the other conditions, and the problem considered as though made up of several simple problems. For instance, the above problem was solved as though composed of the following:

1. If 18 horses can be kept $1\frac{13}{15}$ days on a given quantity of grain, how many days can 1 horse be kept on the same quantity?

2. If 1 horse can be kept so many days (the number obtained as the answer to the preceding question) on $12\frac{3}{5}$ or $\frac{63}{5}$ bu., how many days can he be kept on $\frac{1}{5}$ of a bushel? How many days on a bushel?

3. If 1 horse can be kept so many days (the last answer) on a bushel of grain, how many days can 14 horses be kept on it?

4. If 14 horses can be kept so many days (the last answer), on 1 bu. of grain, how many days can they be kept on $20\frac{1}{4}$ or $\frac{81}{4}$ bushels?

. The student should observe that the question in each case has been the same as that of the original problem, *i. e.* "How many days," and that the denomination of each answer is *days*.

57. If $43\frac{3}{4}$ lb. of meat will last 25 men 7 days, how many days will $58\frac{1}{3}$ lb. last 13 men?

58. If $9\frac{1}{3}$ acres of land will produce hay enough to keep 11 cows $4\frac{4}{5}$ months, how many acres will produce hay enough to last 8 cows $2\frac{7}{13}$ months?

59. If 12 men can build a wall $31\frac{1}{2}$ rods long in $5\frac{1}{3}$ days, in how many days can 20 men build a wall $73\frac{4}{5}$ rods long?

60. How many days will it take a man to dig a trench $78\frac{6}{11}$ ft. long, $4\frac{3}{4}$ ft. wide, and $2\frac{3}{5}$ ft. deep, if it takes him $9\frac{3}{5}$ days to dig a trench $20\frac{4}{7}$ ft. long, $2\frac{2}{11}$ ft. wide, and $1\frac{1}{3}$ ft. deep?

61. If 10 men, by working 12 hours per day and $5\frac{1}{2}$ days in the week, earn $\$366\frac{3}{5}$ in 2 weeks, how many dollars will 15 men, working $9\frac{3}{7}$ hours per day and 6 days per week, earn in $3\frac{1}{5}$ weeks?

62. If 18 bu. of corn are worth 24 bu. of oats, and 15 bu. of oats are worth 27 lb. of butter, how many pounds of butter are 12 bu. of corn worth?

SOLUTION. — As the answer is to be in pounds of butter, we write 27 (the number of pounds of butter mentioned in the question), above a line as a numerator, and take care to express the values throughout in the same denomination. Since 15 bu. of oats were worth 27 lb. of butter, 1 bu. of oats is worth $\frac{1}{15}$ of 27 lb. of butter, expressed by making 15 a factor of the denominator, and 24 bu. of oats must be worth 24 times the last result, expressed by making 24 a factor of the numerator. But as this is also the value of 18 bu. of corn, 1 bu. of corn must be worth $\frac{1}{18}$ of this, expressed by making 18 a factor of the denominator, and 12 bu. of corn must be worth 12 times the last result, expressed by making 12 a factor of the numerator. This gives the following written work:

$$\text{lb. of butter, } \frac{\overset{3}{\cancel{27}} \times \overset{8}{\cancel{24}} \times \overset{6}{\cancel{12}}}{\underset{5}{\cancel{18}} \times \underset{2}{\cancel{18}}} = 24\tfrac{4}{5} \text{ lb. of butter. Ans.}$$

63. If 8 yd. of broadcloth are worth 15 yd. of silk, and 20 yd. of silk are worth 3 bbl. of flour, and 5 bbl. of flour are worth $38, how many dollars are 18 yd. of broadcloth worth?

64. If 7 peaches are equal in value to 4 pears, and 9 pears to 24 apples, and 16 apples to 35 plums, and 8 plums to 60 cherries, how many cherries are equal in value to 56 peaches?

65. If Edward can walk 6 miles while Arthur walks 4, and Arthur can walk 10 miles while Robert can drive 21, and Robert can drive 7 miles while Samuel can read 12 pages, and Samuel can read 18 pages while David can perform 4 problems, how many problems can David perform while Edward is walking 8 miles?

66. When 16 sheep are equal in value to 5 cows, and 6 cows to 4 oxen, and 15 oxen to 8 horses, how many horses are equal in value to 36 sheep?

67. Multiply .06 by .036, divide the result by .0012, and add .04½ of 1.37 to the quotient.

68. Divide 53.7 acres of land into house-lots each containing .375 of an acre.

69. A piece of work was to be completed in 40 days, but, after 50 men had been employed 30 days, only ⅗ of it was done. How many men must be employed to complete the remainder of the work in the required time?

70. I gave 25.38 yards of cloth, worth $3.75 per yd., in exchange for 7.2 tons of hay. I sold .25 of the hay at an advance of $1.50 per ton on its cost, and the rest at the rate of $12.75 per ton. Did I gain or lose, and how much?

71. I owned a pile of wood 37.5 ft. long, 4 ft. wide, and 4.6 ft. high. From it I sold 1 cord 3½ cord feet at $6.25 per cord, and the rest for enough to make up $35. For how much per cord did I sell the last lot?

102. Duodecimal Fractions.

(a.) DUODECIMAL FRACTIONS, or simply DUODECIMALS, are fractions whose denominator is 12, or some power of 12, and, instead of being written, is indicated by one or more marks or accents placed at the right of the numerator and a little above it.

Thus: $4' = \frac{4}{12}$; $7'' = \frac{7}{144}$; $11''' = \frac{11}{1728}$, etc.

(b.) In reading duodecimals, *twelfths* are usually called **PRIMES**; *one-hundred-and-forty-fourths* are called SECONDS, etc.

(c.) Duodecimals are employed in measuring lengths, surfaces, and solids; so that the unit of measure is a foot of either Long, Square, or Cubic Measure, according to the nature of the thing measured.

(d.) Since a linear inch equals $\frac{1}{12}$ of a linear foot, a square inch $\frac{1}{144}$ of a square foot, and a cubic inch $\frac{1}{1728}$ of a cubic foot, it follows that in Long Measure the inch is represented by the *prime*, in Square Measure by the *second*, and in Cubic Measure by the *third*.

(e.) Duodecimals may be added, subtracted, multiplied, and divided, as other fractions are. In performing these operations, it is necessary to notice that a unit of any denomination equals 12 units of the next lower denomination, and $\frac{1}{12}$ of a unit of the next higher denomination; also that $1' \times 1' = \frac{1}{12} \times \frac{1}{12} = \frac{1}{144}$ or $1''$; that $1'' \times 1' = \frac{1}{144} \times \frac{1}{12} = \frac{1}{1728}$ or $1'''$, etc.

1. What are the contents of a floor 16 ft. 5′ long and 11 ft. 8′ wide?

REASONING PROCESS.—A floor 16 ft. 5′ long and 1 ft. wide must contain 16 sq. ft. 5′. Hence a floor 16 ft. 5′ long and 11 ft. 8′ wide must contain $11\frac{8}{12}$ times 16 sq. ft. 5′. The work may be written as below.

$$\begin{array}{rrr} 16 & 5' & \\ 11 & 8' & \\ \hline 10 & 11' & 4'' \\ 180 & 7' & \\ \hline 191 & 6' & 4'' = 191 \text{ sq. ft. } 76 \text{ sq. in.} \end{array}$$

EXPLANATION.—Multiplying by 8′, we have 8′ times 5′ = 40″ = 3′ 4″ (*i. e.* $\frac{8}{12} \times \frac{5}{12} = \frac{40}{144} = \frac{3}{12} + \frac{4}{144}$). Writing the 4″, we have 8′ times 16 ft. = 128′ (*i. e.* $\frac{8}{12} \times 16$ ft. $= \frac{128}{12}$ ft.), and 3′ added equal 131′ = 10 ft. 11′, which we write.

Now, multiplying by 11, we have 11 times 16 ft. 5′ = 180 ft. 7′, which, added to the product by 8′, gives 191 ft. 6′ 4″, which, being in Square Measure, the 4″ = 4 sq. in., and the 6′ = 6 times 12 sq. in. The result then equals 191 sq. ft. 76 sq. in.

2. What are the contents of a board 17 ft. 5′ long and 2 ft. 1′ wide?

3. What are the contents of a platform 18 ft. 7′ long and 9 ft. 4′ wide?

4. How many square feet and inches are there in the outer surface of a box 6 ft. 3′ long, 5 ft. 5′ wide, and 2 ft. 7′ high?

5. How many square feet and inches are there in the inner surface of the box described in the last problem, allowing that the boards composing it are 1 inch thick?

6. How many feet of boards will it take to make the above-named box?

7. What are the solid contents of a block of granite 8 ft. 11′ long, 3 ft. 5′ wide, and 2 ft. 6′ thick?

8. How many cubic feet of earth would be removed in digging a ditch 83 ft. 8′ long, 4 ft. 4′ wide, and 2 ft. 10′ deep?

9. How much will a pile of wood 26 ft. 3′ long, 4 ft. wide, and 5 ft. 8′ high, cost at $.75 per cord foot?

10. How many square yards of carpeting will it take to cover a floor 17 ft. 3′ long and 14 ft. 6′ wide?

11. How many yards of carpeting $\frac{3}{4}$ of a yard wide will it take to cover a floor 16 ft. 5′ long and 14 ft. 9′ wide?

12. My room is 15 ft. 6′ long, 14 ft. 10′ wide, and 9 ft. 2′ high. It contains 3 windows each 5 ft. 4′ high and 2 ft. 11′ wide: two doors each 6 ft. 11′ high and 2 ft. 11′ wide; a closet-door 6 ft. 11′ high and 2 ft. 9′ wide; and a mopboard 6′ wide extending around the room. How many square yards of plastering are there in its top and walls?

13. A man who owned a garden spot 127 ft. 10′ long and 96 ft. 8′ wide, built a heavy stone-wall around it 8 ft. 5′ high and 2 ft. 4′ thick, leaving space for one gate 7 ft. 6′ wide and as high as the wall, and for two smaller gates each 2 ft. 7′ wide and 6 ft. 8′ high. How many cubic feet did the wall contain?

NOTE.—Observe what allowances are to be made for the thickness of the wall.

14. How many bricks each 8′ long, 4′ wide, and 2′ thick will be required to build the four walls of a house 23 ft. 2′ long, and 18 ft. 7′ wide, the walls to be 21 ft. 3′ high and 1 ft. thick, and allowance to be made for 1 door 7 ft. 10′ high and 3 ft. 8′ wide, for 1 door 7 ft. 10′ high and 3 ft. 2′ wide, and for 18 windows each 5 ft. 1′ high and 3 ft. 2′ wide?

SECTION XVI.

RATIO AND PROPORTION.

103. Ratios.

(a.) RATIO is the part which one number is of another, or the quotient of one number divided by another.

ILLUSTRATIONS.—The ratio of 3 to 4 is $\frac{4}{3}$, or $1\frac{1}{3}$, because 4 equals $\frac{4}{3}$ of 3, or $1\frac{1}{3}$ times 3; or because $4 \div 3 = \frac{4}{3} = 1\frac{1}{3}$.

The ratio of 6 to 18 is $\frac{18}{6}$, or 3, because 18 equals $\frac{18}{6}$ of 6, = 3 times 6; or because $18 \div 6 = \frac{18}{6} = 3$.

The questions in **90** were really questions in Ratio.

NOTE.—Some writers consider the ratio of 3 to 4 to be $\frac{3}{4}$, and that of 6 to 18 to be $\frac{6}{18}$, instead of being $\frac{4}{3}$ and $\frac{18}{6}$, as in the above illustrations. The difference is not practically of much consequence, for the term ratio is almost invariably used in some such connection as the following: "9 has the same ratio to 12 that 6 has to 8," which, by the first interpretation, means that $\frac{12}{9} = \frac{8}{6}$, or $\frac{4}{3} = \frac{4}{3}$; and, by the second, $\frac{9}{12} = \frac{6}{8}$, or $\frac{3}{4} = \frac{3}{4}$, both of which are manifestly true. The important thing is to apply the same interpretation to all ratios which are compared with each other.

(b.) A ratio can always be established between abstract numbers, but it can only exist between concrete numbers when they are of the same denomination.

ILLUSTRATION.—6 plums are no part of 3 peaches, and hence have no ratio to them.

(c.) Every ratio is a true fraction, and may be written and dealt with as such.

(d.) Ratios are usually expressed by writing one number after the other, and placing two dots between them.

ILLUSTRATION.—The ratio of 5 to 7 = 5 : 7, or $\frac{7}{5}$.

(e.) The numbers which form any ratio are called TERMS of the ratio. The first term is called the ANTECEDENT of the ratio, and the second term is called the CONSEQUENT of the ratio.

ILLUSTRATION.—In the ratio 6 : 9, 6 is the antecedent and 9 the consequent.

(f.) Ratios, like fractions, may be SIMPLE, COMPLEX, or COMPOUND.

(g.) A SIMPLE RATIO is the ratio of two entire numbers; as, 5 : 7, 4 : 5, or $\frac{7}{5}$, $\frac{5}{4}$.

(h.) A COMPLEX RATIO is the ratio of two fractional numbers, or of an entire and a fractional number; as, $\frac{4}{5}$: $4\frac{1}{2}$, $6\frac{7}{8}$: 3, or $\frac{4\frac{1}{2}}{\frac{4}{5}}$, $\frac{3}{6\frac{7}{8}}$.

(i.) A COMPOUND RATIO is the indicated product of two or more ratios; as, (3 : 8) × (6 : 4), or $\frac{8}{3}$ of $\frac{4}{6}$.

(j.) A compound ratio is usually expressed by writing the ratios which compose it under each other.

ILLUSTRATION.— $\left.\begin{array}{r}7 : 11\\ 5 : \ \ 3\end{array}\right\}$ expresses that the product of the two ratios 7 : 11 and 5 : 3 is to be obtained; or, which is the same thing, it means $\frac{11}{7}$ of $\frac{3}{5}$, or $\frac{11}{7} \times \frac{3}{5}$, or 7 × 5 : 11 × 3.

(k.) The principles involved in operations on ratios are similar in every respect to those involved in the corresponding operations on other fractions; hence—

(l.) Multiplying or dividing both terms of a ratio by the same number does not alter its value.

1. Reduce 9 : 12 to its lowest terms.

SOLUTION.—Dividing both terms by 3 gives 9 : 12 = 3 : 4, *i. e.* $\frac{12}{9} = \frac{4}{3}$, or $\frac{9}{12} = \frac{3}{4}$.

(m.) Reduce each of the following to its lowest terms:

2. 8 : 6.   4. 5 : 15.   6. 24 : 36.
3. 12 : 16.   5. 9 : 7.   7. 18 : 10

8. Reduce $5\frac{1}{4}$: $4\frac{2}{3}$ to a simple ratio.

SOLUTION.—$5\frac{1}{4} : 4\frac{2}{3} = \frac{4\frac{2}{3}}{5\frac{1}{4}} = \frac{8}{9} = 9 : 8$.

(n.) Reduce the following to simple ratios.

9. $4\frac{2}{5} : 5\frac{1}{2}$.
10. $6\frac{2}{3} : 2\frac{1}{12}$.
11. $\frac{1}{2} : \frac{2}{3}$.
12. 4.5 : .15.
13. .06 : .6.
14. 7.5 : .05.

15. Reduce $\left.\begin{matrix} 3 : 7 \\ 8 : 9 \\ 21 : 16 \end{matrix}\right\}$ to a simple ratio.

Solution. — The given ratio expresses $\frac{7}{3}$ of $\frac{9}{8}$ of $\frac{16}{21} = \frac{7 \times 9 \times 16}{3 \times 8 \times 21}$ $= 2$, or 1 : 2.

Note. — By the above solution, it appears that, in reducing a compound ratio to a simple one, all its antecedents become factors of the new antecedent, and all its consequents factors of the new consequent.

(o.) Reduce the following to simple ratios:

16. $\left\{\begin{matrix} 12 : 7 \\ 5 : 24 \end{matrix}\right.$.
17. $\left\{\begin{matrix} 8 : 15 \\ 25 : 32 \end{matrix}\right.$.
18. $\left\{\begin{matrix} .9 : .08 \\ 20 : 2.7 \end{matrix}\right.$.
19. $\left\{\begin{matrix} .05 : .012 \\ .09 : .025 \end{matrix}\right.$.

104. Proportions.

(a.) A Proportion is an equality of ratios.

(b.) Proportions may be either Simple, Complex, or Compound.

(c.) A Simple Proportion is the equality of two simple ratios.

(d.) A Complex Proportion is the equality of two complex ratios, or of a complex and a simple ratio.

(e.) A Compound Proportion is the equality of two ratios, one of which is compound.

(f) A proportion is expressed by writing two ratios one after the other, and placing four dots between them.

Illustrations. — 4 : 6 : : 12 : 18 is a proportion. It expresses that the ratio of 4 to 6 equals the ratio of 12 to 18, and would be read, "4 is to 6 as 12 is to 18." It means that 6 is the same part of 4 that 18 is of 12, or that 4 is the same part of 6 that 12 is of 18.

The compound proportion $\left.\begin{array}{r}4:7\\14:16\end{array}\right\}::1:2$ expresses that the ratio of 4 × 14 to 7 × 16 equals the ratio of 1 to 2. It would be read, "4 times 14 is to 7 times 16 as 1 is to 2," and means that 7 times 16 is the same part of 4 times 14, that 2 is of 1, or that 4 times 14 is the same part of 7 times 16 that 1 is of 2.

(g.) The sign of equality is sometimes used instead of the four dots.

Illustration.—Instead of $4:6::2:3$, we may have $4:6=2:3$.

(h.) Every proportion may be expressed as the equality of two fractions.

Illustration.—We may express the first of the proportions under f by "$\frac{6}{4}=\frac{18}{12}$," and the second by "$\frac{7}{4}$ of $\frac{16}{14}=2$."

The outer terms (*i. e.* the first and fourth) of a proportion are called the **Extremes**; and the inner terms (*i. e.* the second and third) are called the **Means** of the proportion.

Illustrations.—In the first proportion under f, 4 and 18 are the extremes, and 6 and 12 are the means; and, in the second, 4 × 14 and 2 are the extremes, and 7 × 16 and 1 are the means.

(i.) If any term of a proportion is omitted, it may easily be supplied; for, from the nature of a proportion, it follows that—

1st. The missing antecedent of any proportion must be the same part of its consequent that the given antecedent is of its consequent.

2d. The missing consequent of any proportion must be the same part of its antecedent that the given consequent is of its antecedent.

1. What is the missing term of $8:3::24:$ — ?

Solution.—The missing term is the same part of 24 that 3 is of 8, *i. e.* it is $\frac{3}{8}$ of 24. Hence, $8:3::24:\frac{24\times 3}{8}=9$.

2. What is the missing term of $7\frac{1}{5}:3\frac{9}{10}::$ — $:8\frac{2}{3}$?

Solution.—The missing term is the same part of $8\frac{2}{3}$ that $7\frac{1}{5}$ is of $3\frac{9}{10}$, *i. e.* it is $\frac{7\frac{1}{5}}{3\frac{9}{10}}$ of $8\frac{2}{3}$. Hence, $7\frac{1}{5}:3\frac{9}{10}::\frac{7\frac{1}{5}\times 8\frac{2}{3}}{3\frac{9}{10}}:8\frac{2}{3}$, or by reducing, $7\frac{1}{5}:3\frac{9}{10}::16:8\frac{2}{3}$.

3 What is the missing term of $\left.\begin{matrix}8:7\\14:9\end{matrix}\right\}::64:—$?

Solution.—The missing term is the same part of 64 that 7×9 is of 8×14, *i. e.* it equals $\frac{7\times9}{8\times14}$ of $64=36$. Hence, $\left.\begin{matrix}8:7\\14:9\end{matrix}\right\}::64:\frac{7\times9\times64}{8\times14}$ $=36$.

Find the missing term of each of the following:

4. $6:8::12:—$.
5. $9:25::—:15$.
6. $2:—::7:5$.
7. $4\frac{1}{2}:5\frac{2}{5}::6\frac{1}{4}:—$.
8. $.06:2.5::—:.15$.
9. $.012:.01\frac{5}{7}::.5\frac{3}{5}:—$.
10. $\left\{\begin{matrix}3:8\\7:5\end{matrix}\right\}::56:—$.
11. $\left\{\begin{matrix}2\frac{2}{3}:7\frac{1}{2}\\4\frac{1}{2}:6\frac{2}{5}\end{matrix}\right\}::12:—$.
12. $\left\{\begin{matrix}.03:1.2\\1.5:8\end{matrix}\right\}::—:.096$.
13. $\left\{\begin{matrix}80:.08\\300:.003\end{matrix}\right\}::100000:—$.

(j.) From the foregoing explanations and illustrations, we may infer that--

1st. Either extreme is equal to the quotient obtained by dividing the product of the means by the other extreme.

2d. Either mean is equal to the quotient obtained by dividing the product of the extremes by the other mean.

(k.) Hence, in a proportion, the product of the means is equal to the product of the extremes.

105. Problems in Proportion.

Note.—These problems may be solved by analysis instead of proportion, if the teachor prefers it.

(a.) The forming of a proportion from the conditions of a problem is called Stating it.

(b.) In stating a proportion, we write the number which is of the same denomination as the answer, for the third term, and arrange the first and second terms so that they may show the ratio of the the third term to the reouired answer.

1. If 7 cows cost $171.43, how much will 3 cows cost?

Solution.—As the answer is to be dollars, we write $171.43 for the third term, and as 7 cows cost this sum, 3 cows will cost the same part of this that 3 is of 7, expressed by making 3 the second term, and 7 the first.

Statement.—$7 : 3 :: \$171.43 : \$\frac{171.43 \times 3}{7} = \$73.47.$

Note.—The preceding process is the most logical, but, practically, it may be more convenient, after having found the third term, to consider that if the conditions of the problem are such as to require an answer greater than the third term, the second term must be greater than the first; but if they are such as to require an answer smaller than the third term, the second term will be smaller than the first. This is illustrated in the

2d Solution.—Since the answer is to be dollars, we write $171.43 for the third term, and as 3 cows will cost less than 7 cows, we make 3 (the smaller number) the second term, and 7 (the larger number) the first, which gives the same statement as before.

2. If a ton of hay will keep 8 cows $2\frac{1}{2}$ weeks, how many weeks will it keep 5 cows?

1st Solution.—Since the answer is to be weeks, we write $2\frac{1}{2}$ as the third term, and as the hay will last 8 cows so many weeks, it will last 5 cows the same part of this that 8 is of 5.*

Hence the statement, $5 : 8 :: 2\frac{1}{2} : \frac{2\frac{1}{2} \times 8}{5} = 4.$ Ans.

2d Solution.—We make $2\frac{1}{2}$ the third term, as before. Then, as the hay will keep 5 cows longer than it will keep 8, we make 8 (the larger number) the second term, and 5 (the smaller number) the first, which gives the same statement as before.

3. If 6 sleighs cost $150, how much will 8 cost?

4. If 9 horses eat 12 tons of hay in a winter, how many tons will 5 horses eat in the same time?

5. If it cost $3.50 to transport 17 cwt. 3 qr. 19 lb. a certain distance, how much will it cost to transport 8 cwt. 3 qr. 22 lb. the same distance?

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* For it will last 1 cow 8 times as long as it will last 8 cows, and will last 5 cows $\frac{1}{5}$ as long as it will last 1 cow.

6. How many pounds of sugar, at the rate of 8 pounds for a dollar, can be bought for $5.75?

7. If a man can earn $1 in $\frac{3}{4}$ of a day, how much can he earn in $\frac{4}{5}$ of a day?

8. If $2\frac{3}{8}$ acres of land cost $137.84, how much will $5\frac{5}{8}$ acres cost?

9. If $3\frac{3}{4}$ yards of cloth can be bought for $$4\frac{1}{2}$, how many yards can be bought for $$2\frac{1}{7}$?

10. How many bushels of grain at $\frac{5}{7}$ of a dollar per bushel can be bought for $$5\frac{5}{8}$?

11. I bought $8\frac{3}{5}$ barrels of apples for $$15\frac{1}{6}$, and afterwards $12\frac{4}{5}$ barrels at the same rate. What did I pay for the last lot?

12. How many men will it take to perform a piece of work in 8 days which 12 men can perform in 6 days?

13. How many days will it take 50 men to do a piece of work which 80 men can do in 40 days?

14. If 9 men can build a wall in $2\frac{4}{5}$ weeks, how many men must be employed to build it in $6\frac{3}{10}$ weeks?

15. If a ship's crew of 20 men have provisions enough for 18 months, how many months will the provisions last if 10 men be added to the crew?

16. How many men must be employed to do a piece of work in $\frac{7}{8}$ of a month, which 35 men can do in $1\frac{4}{5}$ months?

17. If sugar is worth 12 cents per pound when weighed by the correct weight, how much is it worth per pound Troy?

18. How many yards of silk $\frac{3}{4}$ of a yard wide will it take to line $2\frac{5}{8}$ yards of broadcloth $1\frac{3}{8}$ yards wide?

19. If George does as much work in 9 days as Joseph does in 8, in how many days will George do as much work as Joseph does in 9 days?

20. If Henry does as much work in 7 days as William does in 11, how many days will it take William to do as much work as Henry does in 7 days?

21. If 18 bags of coffee each weighing 96 pounds can be bought for $288, how many bags each weighing 75 pounds can be bought for the same sum?

22. I bought 2 pieces of cloth for the same sum per piece, but the first piece cost $1\frac{2}{3}$ times as much per yard as the second. If the second piece contained $25\frac{1}{2}$ yards, how many yards did the first piece contain?

23. I own two equal lots of land, but the length of the first lot is only $\frac{4}{7}$ that of the second. Now, allowing that the second lot is $35\frac{1}{3}$ rods wide, how wide is the first lot?

106. Problems in Compound Proportion.

(**a.**) In solving problems in Compound Proportion, the third term is determined as in Simple Proportion. Then, taking any two similar conditions, we determine what ratio an answer depending on them alone would have to the third term, and, after writing this ratio, consider what ratio an answer depending on any two other conditions which are similar to each other would bear to this, and so proceed till all the conditions are considered. Solving the resulting proportion will give the result sought.

1. If 9 men can mow a field containing $6\frac{2}{3}$ acres in 3 hours, how many hours will it take 12 men to mow a field containing $8\frac{3}{4}$ acres?

SOLUTION.—3 hours will be the third term, for the answer is to be in hours. As it takes 9 men so many hours, it will take 12 men the same part of this that 9 is of 12, expressed by making 9 the second term, and 12 the first. As it takes so many hours to mow $6\frac{2}{3}$ acres, it will take the same part of this time to mow $8\frac{3}{4}$ acres that $8\frac{3}{4}$ is of $6\frac{2}{3}$, which may be expressed by making $8\frac{3}{4}$ a factor of the second term and $6\frac{2}{3}$ a factor of the first. We thus have the proportion, $\left.\begin{matrix} 12 : 9 \\ 6\frac{2}{3} : 8\frac{3}{4} \end{matrix}\right\} : : 3 : \dfrac{3 \times 9 \times 8\frac{3}{4}}{12 \times 6\frac{2}{3}} = 2\frac{61}{64}$.

2D SOLUTION.—We write 3 hours for the third term. Then, as it will take 12 men a less number of hours to do a piece of work than it will take 9 men, we make 9 the second term and 12 the first. As it will take them longer to mow $8\frac{3}{4}$ acres than it will to mow $6\frac{2}{3}$ acres, we make $8\frac{3}{4}$ the second term and $6\frac{2}{3}$ the first. We thus have the same proportion and answer as before.

2. If 6 men can earn $73.28 in 8 days, how much can 11 men earn in 18 days?

3. If 6 men can earn $73.28 in 8 days, in how many days will 11 men earn $302.28?

4. How many men will it take to earn $302.28 in 18 days, if it takes 6 men 8 days to earn $73.28?

5. How many men will it take to earn $73.28 in 8 days, if it takes 11 men 18 days to earn $302.28?

6. If 11 men earn $302.28 in 18 days, how many dollars will 6 men earn in 8 days?

7. If 11 men earn $302.28 in 18 days, in how many days will 6 men earn $73.28?

8. If 7 men earn $75.60 in $5\frac{1}{4}$ days, how many dollars will 8 men earn in 49 days?

9. If 54 yards of cloth 5 quarters wide will make 12 suits of clothes, how many yards $5\frac{1}{2}$ quarters wide will it take to make 18 suits?

10. If a block of wood 6 ft. long, $1\frac{1}{2}$ ft. wide, and $1\frac{3}{7}$ ft. thick, weighs $771\frac{3}{7}$ pounds, how much will a block of wood 8 ft. long, $1\frac{5}{8}$ ft. wide, and $1\frac{3}{4}$ ft. thick, weigh?

11. If $7\frac{2}{3}$ acres of land are worth $\$191\frac{2}{3}$, how much are $5\frac{3}{4}$ acres worth, allowing that $2\frac{1}{2}$ acres of the first lot are equal in value to $\frac{4}{5}$ of an acre of the last?

12. If a trench 100 ft. long, 4 ft. wide, and 3 ft. deep, can be dug for $100, for how many dollars can a trench 150 ft. long, 3 ft. wide, and $2\frac{2}{3}$ ft. deep, be dug?

13. A ship's crew of 25 men have provisions enough to give each man 30 oz. per day for 48 days, how many days would it last if they should take on board 15 more men?

14. If 15 ounces of wool make 3 yards of cloth 1 yard wide, how many ounces will be required to make $10\frac{1}{2}$ yards 6 quarters wide?

15. If 200 tiles 9 inches long and 8 inches wide will pave my yard, how many tiles 1 ft. long and 10 inches wide will it take to pave it?

16. If 6 house-lots 84 ft. long and 72 ft. wide are given for 40 double-eagles, how many house-lots 96 ft. long and 78 ft. wide ought to be given for $462\frac{2}{3}$ quarter-eagles?

17. If 8 men, by working $10\frac{2}{3}$ hours per day, $5\frac{1}{2}$ days per week, 4 weeks per month, and $11\frac{1}{4}$ months per year, can earn $8800 in $3\frac{4}{5}$ years, how many dollars can 9 men working $9\frac{3}{5}$ hours per day, $5\frac{4}{9}$ days per week, $3\frac{2}{3}$ weeks per month, and 11 months per year, earn in $2\frac{1}{4}$ years?

18. If 7.5 pieces of cloth each containing 30 yards .9 of a yard wide can be bought for $112.50, how many pieces each containing 32.5 yards 1.62 yards wide can be bought for $425?

19. If 3 men can pick the apples in an orchard of 24 trees in $1\frac{7}{8}$ days of $10\frac{2}{7}$ hours each, how many days of $9\frac{3}{5}$ hours each would it take 10 boys to pick the apples in an orchard of 54 trees, allowing that the apples on 5 trees in the second orchard can be gathered as easily as those on 3 of the first, that 6 men do as much work as 9 boys, and that the boys are idle $\frac{1}{10}$ of the time?

SECTION XVII.

INTEREST.

107. Preliminary Definitions.

(a.) IF I use an article of value belonging to another, it is but right that I should pay him for the use of it.

ILLUSTRATION.—We pay rent for the use of a house or store.

(b.) Money paid for the use of money borrowed from others or due to them is called INTEREST.

(c.) The money used is called the Principal.

(d.) The principal and interest added together form the Amount.

Illustration.—If I should pay $9 for the use of $150 for 6 months, $9 would be the interest, $150 would be the principal, and $150 + $9, or $159, would be the amount.

(e.) Interest is usually reckoned at a certain number of hundredths of the principal *for each year it is used.* This number of hundredths is called the Rate Per Cent, or simply the Rate.

Illustration.—If a man pays a sum equal to .06 of the principal for each year that he uses it, the rate is 6 per cent.

Note.—The term Per Cent means simply one-hundredths, and applies to other quantities as well as to dollars and cents.

(f.) In computing interest, it is customary in this country to regard each calendar month as 30 days. In England, however, the months are reckoned according to the number of days which they contain.

(g.) Interest may be either Simple or Compound.

(h.) It is Simple when reckoned on the principal only.

(i.) It is Compound when, at regular intervals, the interest is added to the principal, so as to form a new principal on which interest is to be reckoned as on the original principal.

108. Legal Rate.

(a.) In most countries, laws called Usury Laws have been passed, regulating in some way or other the rates of interest.

(b.) Usury Laws commonly embrace the following particulars —

1st. They fix the rate which shall be paid when no special rate has been agreed upon by the parties. The rate thus fixed is called the Legal Rate.

2d. They forbid persons to receive interest at more than some given rate.

3d. They impose penalties for their violation.

(c.) Any excess over the rates allowed by these laws is called USURY.

(d.) In most States, the legal rate is also the highest rate allowed by law, even on special contracts.

(e.) The legal rate of interest in the several States of the Union, Jan. 1, 1858, was as follows:

5 per cent in Louisiana.

7 per cent in New York, South Carolina, Georgia, Michigan, and Minnesota.

8 per cent in Alabama and Texas.

10 per cent in California.

6 per cent in the rest of the Union, in the United States' Courts, and in the British Provinces.

(f.) By *special written contract*, as high as 8 per cent may be received in Florida and Louisiana; as high as 10 per cent in Mississippi, Ohio, Illinois, Iowa, Missouri, Michigan, and Arkansas; as high as 12 per cent in Texas; and any rate whatever in California. In Wisconsin, interest exceeding 12 per cent cannot be collected by law.

(g.) The legal rate of England and France is 5 per cent.

NOTE.—In this treatise, 6 per cent is to be understood in all cases where the rate is not specified.

109. To compute Interest at 6 per cent.

(a.) 6 per cent per year, interest for 2 months is $\frac{1}{6}$ of 6 per cent, or 1 per cent of the principal, which may be found by removing the decimal point two places towards the left, and is as many cents as there are dollars in the principal.

ILLUSTRATIONS.—The interest for 2 months of $64 is $.64; of $379 is $3.79; of $856.73 is $8.567; of $.90 is $.009, etc.

(b.) What is the interest of each of the following sums for 2 months?

1. $600?
2. $328?
3. $43?
4. $8?
5. $5.70?
6. $897.20?
7. $61?
8. $56.859?
9. $328.74?

(c.) Interest for 2 months being 1 per cent of the principal, interest for 1 month must be $\frac{1}{2}$ per cent of the principal; and, generally—

Interest for any number of months must be $\frac{1}{2}$ as many per cent of the principal as there are months.

Illustrations.—At 6 per cent, interest for 8 mo. equals $\frac{1}{2}$ of 8 per cent, or 4 per cent of the principal; for 13 mo. it equals $\frac{1}{2}$ of 13 per cent, or $6\frac{1}{2}$ per cent of the principal.

10. What is the interest of $824.38 for 9 mo. 15 da.?

Solution.—At 6 per cent per year, interest for 9 mo. 15 da., or $9\frac{15}{30}$, or $9\frac{1}{2}$ mo., must equal $\frac{1}{2}$ of $9\frac{1}{2}$ per cent, or $4\frac{3}{4}$ per cent. of the principal, Hence, the required interest is $4\frac{3}{4}$ per cent, or .04$\frac{3}{4}$ of $824.38, which is $39.158.

Note.—The denominations below mills need not be given in the answer; but if the number of tenths of a mill is more than 5, 1 should be added to the number of mills. Care should be taken, especially when multiplications are to be made, to carry out the work far enough to secure accuracy in the mills' place.

11. What is the interest of $849.64 for 4 mo.?

12. What is the interest of $927.38 for 9 mo.?

13. What is the interest of $25768 for 1 yr. 6 mo.?

14. What is the interest of $537.89 for 7 mo.?

15. What is the interest of $26.59 for 11 mo.?

16. What is the interest of $876.28 for 2 yr. 5 mo.?

17. What is the interest of $37.94 for 8 mo. 20 da.?

18. What is the interest of $869.73 for 4 mo. 16 da.?

19. What is the interest of $3.89 for 11 mo. 6 da.?

20. What is the interest of $694.83 for 15 mo. 11 da.?

21. What is the interest of $297.38 for 13 mo. 19 da.?

22. What is the interest of $528 73 for 11 mo. 7 da.?

(d.) 6 days being $\frac{1}{10}$ of 2 months, or of 60 days, interest for 6 days must equal $\frac{1}{10}$ of 1 per cent, which is .001 of the principal. Hence —

(e.) Interest for any number of days must be $\frac{1}{6}$ as many thousandths of the principal as there are days.

ILLUSTRATIONS. — Interest for 1 mo. 18 da. or for 48 days = $\frac{1}{6}$ of .048 = .008 of the principal. Interest for 1 mo. 23 da. or 53 da. = $\frac{1}{6}$ of .053 = .008$\frac{5}{6}$ of the principal.

(f.) Apply these principles to the solution of the following examples.

23. What is the interest of $824.38 for 9 mo. 15 da.?

SOLUTION. — 9 mo. 15 da. = 285 da., and interest for 285 da. = $\frac{1}{6}$ of .285 = .047$\frac{1}{2}$ of the principal. .047$\frac{1}{2}$ of $824.38 = $39.158 = Ans.

NOTE. — Many business men adopt this method exclusively.

24. What is the interest of $637.28 for 2 mo. 12 da.?
25. What is the interest of $759.42 for 1 mo. 24 da.?
26. What is the interest of $928.64 for 11 mo. 18 da.?
27. What is the interest of $379.48 for 3 mo. 19 da.?
28. What is the interest of $694.83 for 5 mo. 16 da.?
29. What is the interest of $287.29 for 10 mo. 10 da.?
30. What is the interest of $150 for 6 mo. 25 da.?
31. What is the interest of $369.50 for 1 yr. 1 mo. 1 da.?
32. What is the interest of $278.75 for 3 yr. 5 mo. 18 da.?
33. What is the interest of $908.62 for 2 yr. 11 mo. 17 da.?

(g.) Some combine the last two methods, thus: Add $\frac{1}{2}$ as many hundredths as there are months to $\frac{1}{6}$ as many thousandths as there are days, and multiply the principal by the result.

34. What is the interest of $543.28 for 1 yr. 7 mo. 17 da.?

SOLUTION. — 1 yr. 7 mo. 17 da. = 19 mo. 17 da. Interest for 19 mo. = $\frac{1}{2}$ of .19 = .09$\frac{1}{2}$, or .095 of the principal, and interest for 17 da. = $\frac{1}{6}$ of .017 = .002$\frac{5}{6}$ of the principal. Hence, interest for 19 mo. 17 da. = .095 + .002$\frac{5}{6}$ = .097$\frac{5}{6}$ of the principal. .097$\frac{5}{6}$ of $543.28 = $53.151, the interest required.

(**h.**) Perform the following examples by this method:

35. What is the interest of $329.48 for 8 mo. 24 da.?
36. What is the interest of $286.37 for 14 mo. 12 da.?
37. What is the interest of $579 for 7 mo. 18 da.?
38. What is the interest of $636.12 for 1 mo. 15 da.?
39. What is the interest of $256.69 for 3 mo. 21 da.?
40. What is the interest of $158.36 for 1 yr. 2 mo. 17 da.?
41. What is the interest of $807 for 2 yr. 5 mo. 29 da.?
42. What is the interest of $425.30 for 1 yr. 3 mo. 6 da.?
43. What is the interest of $900 for 3 yr. 2 mo. 11 da.?
44. What is the interest of $829.37 for 4 yr. 8 mo. 2 da.?

(**i.**) Perform the following examples by the method which you regard as most convenient.

45. What is the interest of $671.38 for 5 mo. 9 da.?
46. What is the interest of $328.16 for 14 mo. 15 da.?
47. What is the interest of $69.84 for 23 mo. 28 da.?
48. What is the interest of $523.19 for 1 mo. 10 da.?
49. What is the interest of $4798 for 1 yr. 3 mo. 12 da.?
50. What is the interest of $1384 for 21 mo. 12 da.?
51. What is the interest of $697.39 for 1 mo. 5 da.?
52. What is the interest of $57 for 9 mo. 18 da.?
53. What is the interest of $853.65 for 2 mo. 3 da.?
54. What is the interest of $1798.56 for 2 yr. 3 mo. 9 da.?

55. What is the amount of $52.96 for 16 mo. 24 da.?
56. What is the amount of $697 for 15 mo. 15 da.?
57. What is the amount of $257.36 for 7 mo. 7 da.?
58. What is the amount of $950.75 for 4 mo. 15 da.?
59. What is the amount of $2167 for 17 mo. 19 da.?
60 What is the amount of $625.50 for 9 mo. 12 da.?
61. What is the amount of $379.68 for 21 mo. 11 da.?
62. What is the amount of $5843 for 11 mo. 27 da.?
63. What is the amount of $327.96 for 18 mo. 1 da.?

64. What is the amount of $93.24 for 3 mo. 18 da.?
65. What is the amount of $2978.15 for 3 mo. 15 da.?
66. What is the amount of $137.28 for 16 mo. 21 da.?

110. Interest at 6 per cent for Multiples and Divisors of 200 Months.

(a.) Since at 6 per cent, interest for 2 months is 1 per cent of the principal, interest for 20 months must equal 10 per cent, or $\frac{1}{10}$, of the principal, and interest for 200 months must equal 100 per cent of the principal, or the principal itself. Hence —

(b.) Interest for $\frac{1}{2}$ of 200 months, or 100 months, equals $\frac{1}{2}$ of the principal; interest for $\frac{1}{3}$ of 200 months, or 66 months 20 days, equals $\frac{1}{3}$ of the principal, etc.; and interest for $\frac{1}{2}$ of 20 months, or 10 months, $=\frac{1}{2}$ of $\frac{1}{10}$ of the principal; interest for $\frac{1}{3}$ of 20 months, or 6 mo. 20 da., equals $\frac{1}{3}$ of $\frac{1}{10}$ of the principal, etc.

(c.) These considerations will suggest very simple methods of computing interest for any convenient multiples, or fractional parts of 2 mo., 20 mo., or 200 mo. They may be applied in the solution of the following examples.

1. What is the interest of $547.96 for 33 mo. 10 da.?

SOLUTION.—33 mo. 10 da. = 33$\frac{1}{3}$ mo. = $\frac{1}{6}$ of 200 mo. Hence, the required interest equals $\frac{1}{6}$ of the principal, or $\frac{1}{6}$ of $547.96, which is $91.326.

2. What is the interest of $84.28 for 200 mo.?
3. What is the interest of $60.72 for 100 mo.?
4. What is the interest of 36.60 for 66$\frac{2}{3}$ mo.?
5. What is the interest of $24.33 for 50 mo.?
6. What is the interest of $21.75 for 40 mo.?
7. What is the interest of $18.72 for 33$\frac{1}{3}$ mo.?
8. What is the interest of $29.75 for 25 mo.?
9. What is the interest of $54.80 for 20 mo.?
10. What is the interest of $13.74 for 16$\frac{2}{3}$ mo.?
11. What is the interest of $75.30 for 13$\frac{1}{3}$ mo.?

12. What is the interest of $64.16 for 12½ mo.?

13. What is the interest of $4.67 for 16 yr. 8 mo.?

14. What is the interest of $35.82 for 2 yr. 9 mo. 10 da.?

15 What is the interest of $5.46 for 1 yr. 4 mo. 20 da.?

16. What is the interest of $429 for 8 yr. 4 mo.?

17. What is the interest of $725 for 2 yr. 1 mo.?

18. What is the interest of $768 for 4 yr. 2 mo.?

19. What is the interest of $6.54 for 1 yr. 1⅓ mo.?

20. What is the interest of $87.32 for 1 yr. 8 mo.?

21. What is the interest of $25.30 for 3 yr. 4 mo.?

22. What is the interest of $58.35 for 5 yr. 6 mo. 20 da.?

23. What is the interest of $14.72 for 1 yr. 0 mo. 15 da.?

24. What is the interest of $25.47 for 6 mo. 20 da.?

Solution.—6 mo. 20 da. = 6⅔ mo. = ⅓ of 20 mo. Hence, the required interest must equal ⅓ of $\frac{1}{10}$ of the principal, or ⅓ of $2.547, which is $.849.

25. What is the interest of $57.84 for 20 mo.?

26. What is the interest of $684 for 6 mo. 20 da.?

27. What is the interest of $2872 for 5 mo.?

28. What is the interest of $69.80 for 4 mo.?

29. What is the interest of $3.24 for 3 mo. 10 da.?

30. What is the interest of $578 for 2 mo. 15 da.?

31. What is the interest of $36.24 for 1 mo. 20 da.?

32. What is the interest of $45.15 for 1 mo.?

33. What is the interest of $357 for 20 da.?

34. What is the interest of $41 for 15 da.?

35. What is the interest of $837 for 12 da.?

36. What is the interest of $582 for 10 da.?

37. What is the interest of $15 for 4 da.?

38. What is the interest of $896 for 3 da.?

39. What is the interest of $557.80 for 2 da.?

40. What is the interest of $479 for 1 da.?

41 What is the interest of $954 for 4 mo.?

42. What is the interest of $83.60 for 10 mo.?

43. What is the interest of $95.36 for 3 mo. 10 da.?

44. What is the interest of $57.24 for 2 mo. 15 da.?

(d.) Interest for any time whatever can usually be computed easily by dividing the time into some of the above convenient parts, as illustrated in the following solutions.

Note. — In these solutions, letters are used to mark the numbers and indicate to the eye how they have been obtained. Thus, in the first solution, "$.04$ of $a = b =$ \$17.895" means that $17.895 is .04 of $447.38.

45. What is the interest of $447.38 for 8 mo. 10 da.?

1st Solution. —

$$\begin{array}{rrl} a = & \$447.38 & = \text{Prin.} \\ \hline .04 \text{ of } a = b = & 17.895 & = \text{Int. 8 mo.} \\ \frac{1}{6} \text{ of } .01 \text{ of } a = c = & .746 & = \text{`` 10 da.} \\ \hline b + c = & \$18.641 & = \text{`` 8 mo. 10 da.} \end{array}$$

2nd Solution. —

$$\begin{array}{rrl} a = & \$447.38 & = \text{Prin.} \\ \hline \frac{1}{2} \text{ of } .1 \text{ of } a = b = & 22.369 & = \text{Int. 10 mo.} \\ \frac{1}{6} \text{ of } b = c = & 3.728 & = \text{`` 1 mo. 20 da.} \\ \hline b - c = d = & \$18.641 & = \text{`` 8 mo. 10 da.} \end{array}$$

46. What is the interest of $873.29 for 2 yr. 7 mo. 20 da.?

1st Solution. —

$$\begin{array}{rrl} a = & \$873.29 & = \text{Prin.} \\ \hline .1 \text{ of } a = b = & 87.329 & = \text{Int. 20 mo.} \\ \frac{1}{2} \text{ of } b = c = & 43.664 & = \text{`` 10 mo.} \\ \frac{1}{6} \text{ of } c = d = & 7.277 & = \text{`` 1 mo. 20 da.} \\ \hline b + c + d = e = & \$138.271 & = \text{`` 31 mo. 20 da.} \end{array}$$

2nd Solution. —

$$\begin{array}{rrl} a = & \$873.29 & = \text{Prin.} \\ \hline \frac{1}{6} \text{ of } a = b = & 145.548 & = \text{Int. 33 mo. 10 da.} \\ \frac{1}{12} \text{ of } .1 \text{ of } a = c = & 7.277 & = \text{`` 1 mo. 20 da.} \\ \hline b - c = d = & \$138.271 & = \text{`` 31 mo. 20 da.} \end{array}$$

3d Solution. —

$$\begin{array}{rrl} a = & \$873.29 & = \text{Prin.} \\ \hline \frac{1}{8} \text{ of } a = b = & 109.161 & = \text{Int. 25 mo.} \\ \frac{1}{3} \text{ of } .1 \text{ of } a = c = & 29.110 & = \text{`` 6 mo. 20 da.} \\ \hline b + c = d = & \$138.271 & = \text{`` 31 mo. 20 da.} \end{array}$$

47. What is the interest of $57.20 for 7 mo. 15 da.?
48. What is the interest of $42.32 for 2 yr. 3 mo.?
49. What is the interest of $958.42 for 7 mo. 10 da.?
50. What is the interest of $538.69 for 23 mo. 10 da.?
51. What is the interest of $847.58 for 27 mo. 15 da.?
52. What is the interest of $359.14 for 19 mo. 29 da.?
53. What is the interest of $413.60 for 16 mo. 14 da.?
54. What is the interest of $537.28 for 1 mo. 24 da.?
55. What is the interest of $279.44 for 3 mo. 16 da.?
56. What is the interest of $638.71 for 21 mo. 20 da.?
57. What is the interest of $247.94 for 9 mo. 27 da.?
58. What is the interest of $482.24 for 24 mo. 24 da.?

59. What is the amount of $697 for 6 mo. 26 da.?
60. What is the amount of $478 for 26 mo. 10 da.?
61. What is the amount of $983.96 for 199 mo. 29 da.?
62. What is the amount of $93.65 for 1 mo. 19 da.?
63. What is the amount of $487.36 for 7 mo. 26 da.?
64. What is the amount of $257.86 for 3 mo. 22 da.?
65. What is the amount of $586.90 for 3 mo. 7 da.?
66. What is the amount of $859.46 for 9 mo. 3 da.?
67. What is the amount of $29468 for 1 mo. 13 da.?
68. What is the amount of $5873 for 14 mo. 14 da.?
69. What is the amount of $294.36 for 17 mo. 17 da.?
70. What is the amount of $368.25 for 9 mo. 13 da.?

111. To compute the Time.

(a.) To compute the time during which money has been on interest, *i. e.* the time between the dates on which interest began and ended, first reckon the number of entire years, then the number of entire calendar months remaining, and then the remaining days.

Note.—This method, although different from that usually given in school arithmetics, is the one adopted by banks and business men, except when the interest is to be computed for some number of days, as 60 days, 90 days, etc., when they count the days from one date to the other.

1. What is the time from April 13, 1853, to Nov. 21, 1856?

Solution.—From April 13, 1853, to April 13, 1856, is 3 years; from April 13 to Nov. 13 is 7 months; and from Nov. 13 to Nov. 21 is 8 days. Hence, the time is 3 yr. 7 mo. 8 da.

2. What is the time from June 23, 1854, to April 8, 1857?

Solution.—From June 23, 1854, to June 23, 1856, is 2 yr.; from June 23, 1856, to March 23, 1857, is 9 months; from March 23 to March 31 (the last day of March) is 8 days, to which adding the 8 days in April gives 16 days. Hence, the time is 2 yr. 9 mo. 16 da.

3. What is the time from Oct. 30, 1855, to March 17, 1857?

Solution.—From Oct. 30, 1855, to Oct. 30, 1856, is 1 year; from Oct. 30, 1856, to Feb. 28, 1857 (the last day of February), is 4 months. There are no days left in February, but there are 17 in March. Hence, the time is 1 yr. 4 mo. 17 da.

4. What is the time from Aug. 18, 1853, to March 12, 1856?

Solution.—From Aug. 18, 1853, to Aug. 18, 1855, is 2 years; from Aug. 18, 1855, to Feb. 18, 1856, is 6 mo.; from Feb. 18, 1856, to Feb. 29, 1856 (February having 29 days in leap-year), is 11 days, to which adding the 12 days in March gives 23 days. Hence, the time is 2 yr. 6 mo. 23 da.

5. What is the time from April 5, 1847, to June 12, 1857?
6. What is the time from Aug. 3, 1850, to June 27, 1855?
7. What is the time from May 23, 1848, to Sept. 11, 1852?
8. What is the time from Dec. 21, 1853, to March 6, 1857?
9. What is the time from Jan. 20, 1855, to March 19, 1856?
10. What is the time from Jan. 20, 1855, to March 20, 1856?

(b.) What is the interest of—

11. $1000 from Feb. 13, 1852, to June 9, 1856?
12. $628.13 from Nov. 8, 1850, to Dec. 3, 1852?
13. $358.36 from June 20, 1856, to June 13, 1857?

14. $924.53 from Oct. 31, 1850, to March 13, 1853?

15. $6000 from March 9, 1855, to March 8, 1856?

16. $6000 from March 9, 1856, to March 8, 1857?

17. $6000 from March 9, 1856, to March 9, 1857?

18. $358.70 from April 13, 1857, to Aug. 1, 1857?

(c.) What is the amount of—

19. $928.16 from Aug. 23, 1847, to Nov. 30, 1849?

20. $235.96 from April 19, 1851, to June 11, 1854?

21. $86.78 from July 29, 1856, to March 20, 1857?

22. $62984 from Aug. 30, 1855, to July 7, 1857?

23. $1849.36 from Dec. 6, 1856, to April 3, 1857?

24. $94.86 from Aug. 2, 1853, to Sept. 19, 1853?

25. $2459.76 from May 4, 1855, to July 19, 1855?

26. $386.94 from Nov. 13, 1850, to June 4, 1857?

27. Aug. 3, 1847, I borrowed $450; and, Nov. 23, 1847, I borrowed $683.50 more. Jan. 1, 1848, I paid both debts with interest. What was their amount?

28. April 27, 1852, I bought 7 acres of land at $128.36 per acre; Nov. 18, 1852, I sold it at $150.73 per acre. If money was worth 6 per cent per year, how much did I gain by the transaction?

29. Oct. 5, 1857, I bought a lot of grain for $619.27; Nov. 3, I sold a part of it for $328.13; and, Nov. 7, I sold the remainder for $250. Money being worth 6 per cent interest, how much did I lose by the transaction?

112. Interest at various Rates

(a.) We may find the interest when the rate is other than 6 per cent, by first computing the interest at 6 per cent, and then getting such part of this as the given rate is of 6 per cent.

Illustrations.—Interest at 9 per cent. $= \frac{9}{6} = \frac{3}{2} = 1\frac{1}{2}$ times the interest at 6 per cent.; interest at $7\frac{1}{2}$ per cent. $= \frac{7\frac{1}{2}}{6} = \frac{5}{4} = 1\frac{1}{4}$ times the interest at 6 per cent, etc.

1. What is the amount of $628.37 for 1 yr. 5 mo. 5 da. at 7½ per cent?

Solution.— $a =$ **$628.37** = Prin.

$\frac{1}{12}$ of $a = b =$ 52.364 = Int. 16 mo. 20 da. at 6 per cent.
$\frac{1}{4}$ of .01 of $a = c =$ 1.571 = " 15 da. at 6 per cent.
$b + c = d =$ $53.935 = " 17 mo. 5 da. at 6 per cent.
$\frac{1}{4}$ of $d = e =$ 13.484 = " 17 mo. 5 da. at 1½ per cent.
$a + d + e =$ $695.789 = Amt. 17 mo. 5 da. at 7½ per cent.

(b.) What is the interest of—

2. $847.98 for 1 yr. 10 mo. 15 da. at 5 per cent?
3. $3865 for 2 yr. 9 mo. 10 da. at 7 per cent?
4. $594.32 for 9 mo. 19 da. at 4½ per cent?
5. $219.74 for 3 mo. 4 da. at 3 per cent?
6. $392.58 from June 9, 1846, to June 3, 1847, at 8 per cent?
7. $936.20 from Oct. 1, 1855, to Nov. 17, 1855, at 1½ per cent?

(c.) What is the amount of—

8. $540 for 8 mo. 19 da. at 7 per cent?
9. $3600 for 15 mo. 25 da. at 9 per cent?
10. $8293 for 29 mo. 29 da. at 5½ per cent?
11. $95.63 from Jan. 1, 1855, to Oct. 31, 1856, at 8½ per cent?
12. $289.46 from March 27, 1852, to June 4, 1857, at 5¾ per cent?

(d.) Although the preceding method may always be employed, it will often, if not usually, be better to—

Compute the interest at the given rate for any time for which it may be easily computed, and then take such multiples and parts of this as will give the interest for the required time.

13. What is the amount of $976.25 for 1 yr. 3 mo. 18 da. at 7 per cent?

Solution.— $a =$ $976.25 = Prin.

.07 of $a = b =$ 68.337 = Int. 1 yr.
$\frac{1}{4}$ of $b = c =$ 17.084 = " 3 mo.
$\frac{1}{5}$ of $c = d =$ 3.416 = " 18 da.
$a + b + c + d =$ $1056.087 = Amt. 1 yr. 8 mo. 31 da.

14. What is the interest of $879.63 for 3 yr. 8 mo. 15 da. at 4½ per cent.?

Solution. —

$$\begin{array}{rrl}
 a = & \$879.63 & = \text{Prin.} \\
.09 \text{ of } a = b = & 79.167 & = \text{Int. } 2 \text{ yr.} \\
\tfrac{1}{2} \text{ of } b = c = & 39.583 & = \text{`` } 1 \text{ yr.} \\
\tfrac{1}{3} \text{ of } b = d = & 26.389 & = \text{`` } 8 \text{ mo.} \\
\tfrac{1}{16} \text{ of } d = e = & 1.649 & = \text{`` } 15 \text{ da.} \\
\hline
b + c + d + e = & \$146.788 & = \text{`` } 3 \text{ yr. } 8 \text{ mo. } 15 \text{ da.}
\end{array}$$

(e.) What is the interest of—

15. $137.25 for 1 yr. 9 mo. at 8 per cent.?

16. $637.94 for 2 yr. 5 mo. 10 da. at 3½ per cent.?

17. $857.93 for 9 mo. 20 da. at 3 per cent.?

18. $1728 for 4 yr. 6 mo. at 3⅓ per cent.?

19. $98.43 for 3 yr. 9 mo. 9 da. at 5⅓ per cent.?

20. $144 for 19 mo. 25 da. at 4 per cent.?

21. $1575.50 for 4 yr. 8 mo. 24 da. at 2½ per cent.?

22. $694.83 for 1 yr. 6 mo. 18 da. at 8 per cent.?

23. $279.38 for 3 yr. 2 mo. 20 da. at 5 per cent.?

24. $948.37 for 2 yr. 9 mo. 18 da. at 8⅓ per cent.?

25. $1673.29 for 5 mo. 13 da. at 6½ per cent.?

26. $913.27 for 10 mo. 20 da. at 5 per cent.?

27. $628.43 for 6 yr. 6 mo. 6 da. at 7½ per cent.?

28. Jan. 1, 1857, I bought 1000 bbl. of flour at $8.75 per bbl., borrowing the money at 6 per cent to pay for it. Feb. 1, I sold ½ the flour at $9.25 per bbl., and immediately put the money on interest at 7 per cent. Feb. 15, I sold the remainder at $9.62 per bbl., and put the money received for it on interest at 7½ per cent. July 1, I collected the money due me and paid the amount of that which I had borrowed. What was my gain?

113. Compound Interest.

(a.) When interest is to be paid at regular intervals, or, if unpaid, is to be added to the principal to form a new principal on which interest is to be computed, it is called Compound Interest.

NOTE.—The difference between Simple and Compound Interest is that, by the latter, interest is reckoned on interest, while by the former it is not.

(b.) To compute compound interest, we find the amount of the principal at simple interest to the time when the first interest is due; then the amount of this amount to the time when the second interest is due; and so on to the time of settlement. The last amount found is the required amount, and the difference between it and the given principal is the compound interest.

1. To what sum will \$432 amount in 1 yr. 8 mo. 15 da. at 4 per cent, payable semi-annually, and what is its compound interest for that time?

SOLUTION.—$a = \$432.00 = \text{Principal.}$
$.02 \text{ of } a = b = 8.64 = \text{Int. 1st 6 mo.}$
$a + b = c = \$440.64 = \text{Amt. end of 1st 6 mo.}$
$.02 \text{ of } c = d = 8.813 = \text{Int. 2d 6 mo.}$
$c + d = e = \$449.453 = \text{Amt. end of 2d 6 mo.}$
$.02 \text{ of } e = f = 8.989 = \text{Int. 3d 6 mo.}$
$e + f = g = \$458.442 = \text{Amt. end of 3d 6 mo.}$
$.00\frac{5}{6} \text{ of } g = h = 3.820 = \text{Int. for 2 mo. 15 da.}$
$g + h = i = \$462.262 = \text{Amt. due end of 1 yr. 8 mo. 15 da.} = \textbf{1st Ans.}$
$a = \$432. = \text{Principal.}$
$i - a = 30.262 = \text{Comp. Int. req.}$

2. To what sum will \$950 amount in 2 yr. 6 mo. at 6 per cent, payable semi-annually?

3. What is the compound interest of \$175.50 for 4 yr. 8 mo. 12 da. at 4 per cent, payable annually?

4. What is the compound interest of \$450 for 3 yr. 9 mo. at 9 per cent, payable quarterly?

5 To what sum will \$875.50 amount in 5 yr. 8 mo. 10 da. at 3 per cent, payable annually?

6. To what sum will \$1500 amount in 2 yr. 2 mo. at 10 per cent, payable semi-annually?

(c.) The following table shows what part of any principal is equal to its amount at 3, 4, 5, 6, 7, and 8 per cent annual compound interest, for any number of years not exceeding 25.

Years.	3 per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.	8 per cent.
1.	1.03	1.04	1.05	1.06	1.07	1.08
2.	1.0609	1.0816	1.1025	1.1236	1.1449	1.1664
3.	1.092727	1.124864	1.157625	1.191016	1.225043	1.259712
4.	1.125509	1.169858	1.215506	1.262477	1.310796	1.360489
5.	1.159274	1.216653	1.276281	1.338225	1.402552	1.469328
6.	1.194052	1.265319	1.340096	1.418519	1.500730	1.586874
7.	1.229874	1.315932	1.407100	1.503630	1.605781	1.713824
8.	1.266770	1.368569	1.477455	1.593848	1.718186	1.850930
9.	1.304773	1.423312	1.551328	1.689479	1.838459	1.999005
10.	1.343916	1.480244	1.628894	1.790848	1.967151	2.158925
11.	1.384234	1.539454	1.710339	1.898298	2.104852	2.331639
12.	1.425761	1.601032	1.795856	2.012196	2.252191	2.518170
13.	1.468534	1.665073	1.885649	2.132828	2.409845	2.719624
14.	1.512590	1.731676	1.979931	2.260903	2.578534	2.937194
15.	1.557967	1.800943	2.078928	2.396558	2.759031	3.172169
16.	1.604706	1.872981	2.182874	2.540351	2.952164	3.425943
17.	1.652848	1.947900	2.292018	2.692772	3.158815	3.700018
18.	1.702433	2.025816	2.406619	2.854339	3.379932	3.996019
19.	1.753506	2.106849	2.526950	3.025599	3.616527	4.315701
20.	1.806111	2.191123	2.653298	3.207135	3.869684	4.660957
21.	1.860294	2.278768	2.785962	3.399563	4.140562	5.033834
22.	1.916103	2.369918	2.925260	3.603537	4.430402	5.436540
23.	1.973586	2.464715	3.071524	3.819749	4.740530	5.871464
24.	2.032794	2.563304	3.225100	4.048934	5.072367	6.341181
25.	2.093778	2.665836	3.386355	4.291870	5.427433	6.848475

(d.) The amount of any sum of money, at annual compound interest, for any time and rate mentioned in the table, may be found by multiplying the principal by the appropriate number selected from the table.

7. What is the amount of $562.48 for 8 yr. 4 mo. at 7 per cent?

SOLUTION. — By the table, it appears that the amount of a sum of money for 8 years, at 7 per cent compound interest, is 1.718186 times the principal. Multiplying $562.48 by 1.718186, we have $966.44526128, or, omitting the denominations below mills,

$$\begin{aligned} a &= \$966.445 = \text{Amount for 8 years.} \\ 2\tfrac{1}{3} \text{ per cent. of } a = b &= \underline{22.55} = \text{Interest for 4 mo.} \\ a + b = c &= \$988.995 = \text{Amount for 8 yr. 4 mo.} \end{aligned}$$

What is the amount at compound annual interest of—

8. $253.45 for 7 yr. 6 mo. at 4 per cent?
9. $945 for 14 yr. 8 mo. 12 da. at 6 per cent?
10. $1000 for 11 yr. 3 mo. at 7 per cent?

What is the compound interest of—

11. $1525.75 for 18 yr. 6 mo. at 3 per cent?
12. $1525.75 for 18 yr. 6 mo. at 6 per cent?
13. $872.50 for 4 yr. 3 mo. 10 da. at 5 per cent?

SECTION XVIII.

PRACTICAL APPLICATIONS OF INTEREST AND PERCENTAGE.

114. Promissory Notes.

(a.) A Promissory Note, a Note of Hand, or, as it is more commonly called, a Note, is a written promise to pay a specified sum of money.

(b.) The person who signs a note is called the Promisor, or maker of the note, and the one to whom it is made payable is called the Promisee. The one who has legal possession of it is called the Holder of the note. The Face of a Note is the sum for which it is given.

Illustrations.—In Form 1, Elbridge Clapp is the Promisor or Maker of the note, and Hiram A. Pratt is the Promisee. Mr. Pratt will be the Holder of the note until it is paid, or till he transfers it to some other person. The Face of the Note is $500.

(c.) The words "value received" should always be placed in a note, as an acknowledgment that the maker gave it in consideration of something of equivalent value.

(d.) Notes are either made "ON TIME" or "ON DEMAND." They are on time when payable at the expiration of a certain time, as 60 days, 3 months, etc. (see Forms 2 and 3), and on demand when payable whenever payment may be demanded by the Promisee or holder of the note (see Form 1).

(e.) Notes on time are not legally due till three days after the time specified in the note. Thus, a note on 30 days is due in 33 days; a note on 6 months is due in 6 months and 3 days. The days thus added are called DAYS OF GRACE, OR GRACE.

(f.) Notes are NEGOTIABLE when so written that they may be transferred or sold by one person to another.

(g.) Negotiable notes are of two kinds, viz. those payable to a person OR ORDER (see Forms 1 and 2), and those payable to a person or BEARER, or to the BEARER (see Form 3).

(h.) Notes of the second class are negotiable merely by delivery (bank bills are of this class); but notes of the first class require a written order of the Promisee, or legal holder of the note, to authorise any one else to collect the money due on them. This order is usually written on the back of the note, and is called an INDORSEMENT. The most common form of indorsement is for the indorser to simply write his name on the back of the note. Such an indorsement is called an INDORSEMENT IN BLANK, and is equivalent to the order, "Pay the within to the bearer."

(i.) A note is DISHONORED if the signer neglects or refuses to pay it on the demand of the holder, the demand being made the day the note is due.

(j.) Every indorser of a note renders himself liable to pay it if it is dishonored, provided the holder gives him immediate notice that payment has been demanded and refused.

(k.) A formal notice, made by an officer called a NOTARY PUBLIC, that a note has been dishonored, is called a PROTEST.

(l.) A note for a sum "WITH INTEREST," is on interest from date. If the phrase "with interest" is omitted, the note is not on interest. If, however, a note on demand is not paid when the demand is made, or a note on time is not paid when due, interest may afterwards be demanded, even though no mention is made of it in the note.

(m.) The body of a note may be written by any one, but the SIGNATURE should always be written by the promisor, or by some one specially authorised to sign it for him. When the note is paid, the signature should be erased or torn off.

(n.) If a part only of the money due on a note is paid, the sum paid is indorsed, *i. e.* receipted on the back of the note.

NOTE.—A more full discussion of this subject may be found in "Arithmetic and its Applications."

FORM 1.

$500. Boston, March 11, 1858.

For value received, I promise to pay Hiram A. Pratt, or order, five hundred dollars, on demand, with interest.

Elbridge Clapp.

FORM 2.

964\frac{79}{100}$ New York, July 1, 1858.

Sixty days after date, I promise to pay to the order of George A. Walton, nine hundred and sixty-four $\frac{79}{100}$ dollars. Value received.

N. T. Allen.

FORM 3.

$1000. Philadelphia, Jan. 17, 1858.

Three months after date, I promise to pay D. B. Hagar, or bearer, one thousand dollars. Value received.

Levi T. Dodge.

115. Partial Payments.

(a.) The principle adopted by the Supreme Court of the United States, and by that of Massachusetts and most of the other States, as the one to be applied in determining the sum due on a promissory note or bond on which payments have been made, may be thus stated —

(b.) As much of the payment as is necessary to pay the interest due at the time the payment is made, should be appropriated for that purpose, and the surplus to the payment of the principal. The balance then due will form a new principal on interest, as was the original principal. If, however, any payment is less than the interest at the time due, the principal remains unaltered, and on interest until some payment is made, which, with the preceding neglected payments, is more than sufficient to pay the interest, when we proceed as if a single payment, equal to the sum of the last payment and the preceding neglected ones, had been made.

NOTE.—The method adopted by the court of Connecticut differs from the above in this respect, that if a payment greater than the interest at the time due be made before the principal has been on interest one year, the person making it is allowed interest on it to the end of the year, *i. e.* the amount of the payment from the time it was made to the end of the year, is deducted from the amount of the principal to the same time. If settlement be made before the principal has been on interest one year, interest is allowed on the payments from the time they were made to the time of settlement.

1.

$1200. Providence, June 17, 1851.

For value received, I promise to pay Arthur Stanhope, or order, twelve hundred dollars, on demand, with interest.

Joseph Andrews.

On this note are the following indorsements:

Feb. 15, 1852, received one hundred and twenty-five dollars.
Nov. 30, 1852, received seventy-five dollars.
Sept. 19, 1853, received twenty dollars.
Jan. 1, 1854, received forty-five dollars.
July 9, 1854, received five hundred dollars.

This note was settled Jan. 1, 1855. What was then due?

WRITTEN WORK.

$1200.00 = Face of note, June 17, 1851.
47.80 = Int. 7 mo. 29 da. to Feb. 15, 1852.
$1247.80 = Amt. due " "
125.00 = 1st payment " "
$1122.80 = Balance due " "
53.333 = Int. 9 mo. 15 da. to Nov. 30, 1852.
$1176.133 = Amt. due " "
75.000 = 2d Payment " "
$1101.133 = Balance due " "
106.259 = Int. 19 mo. 9 da. to July 9, 1854.
$1207.392 = Amt. due " "
565.000 = 3d, 4th, and 5th pay'ts " "
$642.392 = Balance due " "
18.522 = Int. 5 mo. 23 da. to Jan. 1, 1855.
$660.914 = Amt. due " "

Explanation. — It is readily seen that the first payment, $125, is more than the interest at the time due; hence, we compute the amount of the note to Feb. 15, 1852, and deduct from it the payment. The remainder is a new principal. The second payment of $75 is evidently more than the interest of this new principal to the time when the payment was made; hence, we compute the amount to that time, Nov. 30, 1852, and subtract from it the payment. The remainder is a new principal. The third payment, $20, made Sept. 19, 1853, is evidently less than the interest then due. Again, $65, the sum of the third and fourth payments, is less than the interest to Jan. 1, 1854, the time when the fourth payment was made. But as $565, the sum of the third, fourth, and fifth payments, is greater than the interest due July 9, 1854 (the day when the fifth payment was made), we get the amount to that date and subtract from it $565, the sum of the three payments. The remainder is a new principal; and its amount to June 1, 1855, is the sum due at the time of settlement.

The following series of questions may illustrate more clearly the principle involved in the above solution:

What is the face of this note? When did interest commence? Is the interest of the face of the note to Feb. 15, 1852, greater or less than $125, the first payment? What, then, is the amount of $1200 from June 17, 1851, to Feb. 15, 1852? What is due after deducting the payment? Is the interest of this to Nov.

30, 1852, greater or less than $75, the second payment? What, then, is the amount? What is due after deducting the payment? Is the interest of this to Sept. 19, 1853, greater or less than $20, the third payment? Is its interest to June 1, 1854, greater or less than $65, the sum of the third and fourth payments? Is its interest to July 9, 1854, greater or less than $565, the sum of the third, fourth, and fifth payments? What, then, is its amount to that time? What is due after deducting the sum of the payments? What is the amount of this to Jan. 1, 1855? What, then, was due Jan. 1, 1855?

2.

$900. Boston, Jan. 1, 1853.

For value received, I promise to pay George A. Walton, or order, nine hundred dollars, on demand, with interest?

O. F. Bryant.

On this note were the following indorsements: Oct. 9, 1853, received one hundred and fifty-six dollars and twenty-eight cents.

June 3, 1854, received ten dollars.

Jan. 1, 1855, received three hundred and fifty dollars.

How much was due July 15, 1855?

3.

$1559\frac{47}{100}$ Springfield, Nov. 3, 1855.

For value received, I promise to pay S. D. Bowen, or order, fifteen hundred and fifty-nine dollars and forty-seven cents, on demand, with interest.

Freeman Fisher.

Indorsements: Dec. 1, 1855, received thirty dollars.

July 15, 1856, received twenty-five dollars.

Aug. 7, 1857, received fifty dollars.

Dec. 1, 1857, received one thousand dollars.

What was due Jan. 1, 1858?

4.

$1000. Salem, July 9, 1850.

On demand, I promise to pay Richard Edwards, or bearer, one thousand dollars, with interest, for value received.

J. F. Brown.

Indorsements: Sept 30, 1850, received one hundred dollars.
June 19, 1851, received one hundred dollars.
March 21, 1852, received one hundred dollars.
Dec. 3, 1855, received one hundred dollars.
May 15, 1856, received one hundred dollars.
June 7, 1856, received one hundred dollars.
Feb. 28, 1857, received one hundred dollars.
July 1, 1857, received one hundred dollars.
Dec. 5, 1857, received one hundred dollars.

What was due Jan. 1, 1858?

5.

$783. Providence, April 30, 1852.

On demand, after three months, I promise to pay to the order of John Gorham seven hundred and eighty-three dollars, with interest, value received.

Lewis Dexter.

Indorsements: Jan. 24, 1853, received one hundred dollars.
March 20, 1854, received one hundred and forty-five dollars and seventy-five cents.
Oct. 12, 1855, received two hundred dollars.
March 3, 1856, received one hundred and fifty dollars.
Jan. 1, 1857, received ninety-eight dollars and fifty-six cents.

What was due April 9, 1857?

116. Merchants' Method when Debts are paid within a Year.

(a.) When notes or debts of any kind, on which partial payments have been made, are paid in full within one year from the time interest commences, merchants often determine the sum to be paid on settlement, as they would if nothing is due on a note till it is paid in full.

(b.) In such cases, they find the amount of the note to the time of settlement, and the amount of each payment from the time it was made to the time of settlement; then the excess of the amount of the note over the sum of the amounts of the several payments will be the sum due on settlement.

1.

$450.
Providence, Aug. 1, 1856.

For value received, I promise to pay Wm. H. Ayer, or order, four hundred and fifty dollars, on demand, with interest.

F. C. Rider.

Indorsements: Nov. 13, 1856, received fifty dollars.
Jan. 7, 1857, received thirty-three dollars and fifty cents.
March 24, 1857, received one hundred and fifty dollars.
May 1, 1857, received ninety dollars.

This note was settled July 3, 1857. What was the amount due on it?

Solution.

$450.00 = Face of note, Aug. 1, 1856.
24.90 = Int. 11 mo. 2 da. to July 3, 1857.
$474.90 = Amount of note " "
$50.00 = 1st payment, Nov. 13, 1856.
1.917 = Int. 7 mo. 20 da. to July 3, 1857.
33.500 = 2d payment, Jan. 7, 1857.
.983 = Int. 5 mo. 26 da. to July 3, 1857.
150.000 = 3d payment, March 24, 1857.
2.475 = Int. 3 mo. 9 da. to July 3, 1857.
90.000 = 4th payment, May 1, 1857.
.930 = Int. 2 mo. 2 da. to July 3, 1857.
$329.805 = Amount of payments, July 3, 1857.
$145.095 = Balance due " "

2.

$\$725\frac{50}{100}$
Hartford, Feb. 19, 1857.

For value received, I promise to pay James B. Atwood, or order, seven hundred and twenty-five dollars and fifty cents, on demand, with interest.

Henry E. Harris.

Indorsements: April 6, 1857, received seventy-five dollars.
July 9, 1857, received one hundred and ninety-five dollars.

Sept. 23, 1857, received three hundred and fifteen dollars.

Nov. 15, 1857, received fifty dollars.

This note was settled Dec. 1, 1857. What was the amount due?

3.

$1275. Providence, Sept. 30, 1856.

For value received, I promise to pay H. C. Bradford, or order, twelve hundred and seventy-five dollars, on demand, with interest.

J. L. Burroughs.

Indorsements: Dec. 5, 1856, received fifty-five dollars.

Jan. 9, 1857, received seven hundred and sixty dollars.

June 3, 1857, received four hundred dollars.

Aug. 5, 1857, received the balance due on the above note. How much did I receive?

4.

$547\frac{75}{100}$ Worcester, May 7, 1856.

For value received, I promise to pay E. S. Winsor, or order, five hundred and forty-seven dollars and seventy-five cents, on demand, with interest.

S. S. Rider.

Indorsements: Aug. 19, 1856, received one hundred dollars.

Nov. 23, 1856, received forty-five dollars and thirty-seven cents.

Jan. 29, 1857, received ninety-four dollars.

March 3, 1857, received seventy-six dollars and fifty cents.

April 15, 1857, received the balance due on the above note. How much did I receive?

117. Banks and Banking.

(a.) A BANK is an institution established for the purpose of trafficking in money.

(b.) Banks usually receive money on deposit, loan money on interest, and issue bank notes, or bank bills, *i. e.* notes payable in specie to the bearer on demand at the bank, and intended to circulate as money.

(c.) When money is loaned by a bank, it is commonly made payable at the end of a given time, and the interest for that time and the three days of grace *is deducted at the time it is loaned.*

Illustration. — On a note of $800, payable in 60 days, I shall receive at a bank $800 minus the interest of $800 for 63 days, *i. e.* $800 — $8.40 = $791.60.

(d.) By this arrangement the banks receive interest on a larger sum of money than they lend.

Illustration.—In the above example, the bank receives interest on $800, while the sum actually lent is only $791.60.

(e.) Bank interest is called Discount, because it is thus deducted from the face of the note.

(f.) The note on which the money is received is said to be DISCOUNTED.

1. How much would be received at a bank on a note for $1000 due in 60 days?

Solution.—The interest of $1000 for 63 days is $10.50, which, subtracted from $1000, leaves $989.50, the sum received.

(g.) How much would be received at a bank on a note for—

2. $800 payable in 30 days?
3. $147.28 payable in 90 days?
4. $396.84 payable in 40 days?
5. $1137.65 payable in 4 months?
6. $250 payable in 6 months?
7. $825.37 payable in 5 months?

8. A note for $1327.58, dated July 1, 1857, and payable in 3 months, was discounted July 27, 1857. When was it due and how much was received on it?

Suggestion.—The note was due in 3 mo. 3 da. after July 1, 1857, *i. e.* on Oct. 4, 1857. Hence, it had 2 mo. 7 da. to run from the day of discount, and interest for that time would be deducted. Ans. $1312.756.

9. A note for $1327.58, dated July 1, 1857, and payable in 90 days, was discounted at a bank July 27, 1857. When was it due and what was received on it?

Suggestion.—The note was due in 93 days after July 1, 1857, *i. e.* (by actually counting the days in the months included) on Oct. 2d. As 26 days had elapsed from the date of the note to the date of the discount, it had 67 days to run, and interest for that time would be deducted. Ans. $1312.756.

Note.—The last two examples illustrate the inequalities which may arise in reckoning time by the methods adopted in this country, one note becoming due 2 days before the other, yet giving the same interest. The only strictly accurate method is to compute the interest by days, reckoning 365 days to the year, as is done in England.

10. A note for $978.43, dated Aug. 7, 1856, and payable in 5 months, was discounted at a bank Sept. 17, 1856. When was it due and how much was received on it?

11. A note for $978.43, dated Sept. 1, 1857, and payable in 6 months, was discounted at a bank Jan. 1, 1858. When was it due and how much was received on it?

12. A note for $197.64, dated Jan. 19, 1856, and payable in 60 days, was discounted at a bank Jan. 21, 1856. When was it due and how much was received on it?

13. My note for $1200, payable in 3 months, was discounted at a bank, and I immediately put the proceeds at interest. When the note at the bank became due, I renewed it for the same time as before. When the second note became due, I collected the amount of the money I had lent, and paid my note at the bank. How much did I lose by the transaction?

Suggestion.—To understand the process of renewal, it is only necessary to consider that, when the first note became due, I owed the bank $1200. To renew it, I got a new note for $1200 discounted at the bank, which would pay $1181.40 of the debt, leaving the balance, which is the bank discount, to be paid in cash. When the new note became due, I paid $1200 in cash. The total expense to me, then, was the amount of the bank discount for 3 mo. 3 da. plus the face of the note, $1200, and my total receipts were the amount of the sum received at the bank for 6 mo. 6 da. My loss is the difference between the total expenses and total receipts.

14. My note on 4 months, for $2000 was discounted at a bank, and I immediately put the proceeds on interest. When the note became due, I renewed for the same term as before; and, when this note became due, I again renewed it for the same time. When the last note became due, I collected the amount of the money I had lent, and paid the note at the bank. How much did I lose by the transaction?

118. To find the Time, when the Principal, Interest, and Rate are known.

(a.) The time which it will take any principal at a given rate to gain a given interest must be as many years, months or days, as the interest of the given principal for 1 year, 1 month, or 1 day is contained times in the given interest. Hence if the given interest be divided by the interest of the given principal for 1 year, 1 month, or 1 day, the quotient will equal the required time in years in months, or in days.

Note.—Usually it will be best to compute the time in days, in which case it will not ordinarily be necessary in computing the division to carry out the interest of the principal for one day, farther than to mills.

1. How long will it take $749.76 to gain $21.868 at 6 per cent?

Solution. — The interest of $749.76 for 1 day at 6 per cent is $.125. Hence it will take as many days to gain $21.868 as $.125 is contained times in $21.868, which is $174\frac{118}{125}$ times. Therefore the required time is $174\frac{118}{125}$ days, or, practically, 175 days = 5 mo. 25 da.

Note.—Any fractional part of a day equal to or greater than ½ should be called 1 day. Smaller fractions may be disregarded.

2. How long will it take $856 to gain $17.976 at 7 per cent?

Solution —The interest of $856 for 1 day at 7 per cent is $.166. Hence it will take as many days to gain $17.976 as $.166 is contained times in $17.976, which gives for an answer 108 days = 3 mo. 18 da.

(b.) In how long time at interest will —

3. $975.73 gain 97.573 at 6 per cent?

4. $153.453 gain $10 at 6 per cent?

5. $87.25 gain $19.762 at 4 per cent?
6. $2250 gain $89.93 at 7 per cent?
7. $750.87 gain $100 at 9 per cent?
8. $425.62 gain $50 at 5½ per cent?
9. $900 gain $225 at 6 per cent?
10. $794.60 gain $39.73 at 3⅓ per cent?

119. Equation of Payments.

(a.) When one man owes another sums of money payable at different times, it may be desirable to determine when the whole can be paid without gain or loss to either party. The process of doing this is called EQUATION OF PAYMENTS, and the time sought is called the EQUATED TIME.

(b.) It is obvious that if a debt be not paid till after it has become due, the debtor gains the use or interest of it from the time it became due to the time of payment; while, if it be paid before it becomes due, the debtor loses the use or interest of it from the time of payment to the time when it would justly have been due. The thing sought, then, is to ascertain when any sum must be paid, in order that the interest lost by paying one portion of it before it is due, shall be equal to the interest gained by keeping another portion after it has become due.

(c.) In solving problems in equation of payments, we first find to what interest a person is entitled on account of the various debts considered, and then find how long the sum of the debts, or the balance due, must be kept to gain this interest.

1. A owes B $150, due in 4 mo., $200 due in 5 mo., and $450 due in 7 mo. When can the whole be paid without gain or loss to either party?

1ST SOLUTION. — By the conditions of the question, A is entitled to the use, or —

Interest of $150 for 4 mo. = $3.00
" $200 for 5 mo. = 5.00
" $450 for 7 mo. = 15.75

or to the use of $800 till its int. = $23.75, which, determined as in Art. 118, is 178⅛ days = 5 mo. 28⅛ da., or, as it would be reckoned practically, mo. 28 da.

2D SOLUTION.—A is entitled to the use of—

$150 for 4 mo. or of $1 for 150 times 4 mo. = 600 mo.
$200 for 5 mo. or of $1 for 200 times 5 mo. = 1000 mo.
$450 for 7 mo. or of $1 for 450 times 7 mo. = 3150 mo.

or of $800 till its int. equals the int. of $1 for 4750 mo., which will be $\frac{1}{800}$ of 4750 mo. = $5\frac{15}{16}$ mo. = 5 mo. $28\frac{1}{8}$ da. = practically, 5 mo. 28 da.

2. I owe $400 due in 3 mo., $100 due in 4 mo., $200 due in 5 mo., and $500 due in 6 mo. What is the equated time of payment?

3. When may $50 due in 1 mo., $75 due in 4 mo., $250 due in 5 mo., and $125 due in 6 mo., be paid without gain or loss?

4. What is the equated time for paying $120 due in 6 mo , $420 due in 8 mo., $360 due in 7 mo., $180 due in 5 mo., and $60 due in 9 mo.?

5. I owe $250 due in 10 days, $300 due in 15 days, $450 due in 30 days, and $125 due in 36 days. What is the equated time of payment?

6. What is the equated time for paying $100 due March 7, $75 due March 15, $350 due April 6, and $500 due April 29?

NOTE.—Find in how many *days* from any assumed time, as, for instance, March 7, each debt is due, and then proceed as before explained. The time is found to be $36\frac{28}{41}$ days, or, practically, 37 days after March 7, which is April 13.

7. What is the equated time for paying $326.35 due July 8, $529.47 due Aug. 1, $823.14 due Aug. 19, and $500 due Aug. 27?

8. What is the equated time for paying $296.36 due March 4, $896.82 due April 9, $794.15 due May 16, and $823.10 due July 10?

9. A owes B $1000, payable in 6 mo.; but, to accommodate B, he pays half of it in 3 mo. When ought he to pay the remainder?

10. I owe $500 payable in 8 mo., $600 payable in 9 mo., and $900 payable in 10 mo. To accommodate my creditor, I pay $500 in 1 mo., $300 in 2 mo., and $200 in 3 mo. When ought I to pay the remainder?

11. A owes B $600 due in 5 mo., and B owes A $900 due in 8 mo. At what time can both debts be paid without gain or loss to either party?

12. A owes B $500 due in 2 mo., and $400 due in 4 mo., while B owes A $200 due in 3 mo., and $600 due in 5 mo. A agrees to pay the whole of his debt in 1 mo., if B will pay his enough sooner to compensate for the loss. When ought B to pay his?

13. I owe $225 due in 30 days, $325 due in 45 days, $130 due in 50 days, and $300 due in 60 days. If I should not pay anything till the end of 60 days, and then should pay the whole, how much ought I justly to pay, money being worth 6 per cent per year?

14. I owe $800 due in 2 mo. 15 da., $900 due in 3 mo. 10 da., $1000 due in 4 mo., $800 due in 5 mo., and $600 due in 6 mo. 20 da.; but, to accommodate my creditor, I make a complete settlement in 4 mo. How much ought I to pay, money being worth 6 per cent?

15. I owe $450 due Sept. 3, $500 due Sept. 29, $375 due Oct. 10, and $750 due Nov. 1. If I should pay nothing till Oct. 20, and should then make a complete settlement, how much ought I justly to pay, money being worth 6 per cent? What is the equated time for paying the above debts?

120. Equation of Accounts.

1. The account-books of A and B show that—

A owes B	and that B owes A
$425.37 due Feb. 3, 1857.	$219.86 due Jan. 27, 1857.
$428.13 due March 7, 1857.	$1000. due March 30, 1857.
$285.93 due April 23, 1857.	$900. due May 1, 1857.
$428.25 due May 5, 1857.	$2119.86 = sum due A.
$1567.68 = sum due B.	

$2119.86 — $1567.68 = $552.18 = balance due A.

When can this balance be paid without gain or loss to either party?

Solution.—The first step towards solving this problem is to find the interest which will be gained or lost by paying the balance at any assumed time, say April 1, 1857. It is obvious that A should allow interest on all sums which he keeps after they have become due, and on all which he receives before they become due; and that interest should be allowed him on all sums which he pays before they are due, and on all which he fails to receive till after they become due. Hence, the interest account would be as follows:

Interest of—	Due B.	Due A.
$425.37 Feb. 3 to April 1, 57 da. =	$4.041	
$428.13 March 7 to April 1, 25 da. =	1.784	
$285.93 April 1 to April 23, 22 da. =		$1.048
$428.25 April 1 to May 5, 34 da. =		2.427
$219.86 Jan. 27 to April 1, 64 da. =		2.345
$1000 March 30 to April 1, 2 da. =		.333
$900 April 1 to May 1, 30 da. =	4.500	
Total interest, =	$10.325	$6.153

$10.325 — $6.153 = $4.172 = Balance of interest due B, *i. e.* gained by A and lost by B.

The question now resolves itself into this: If, by B's paying A the balance, $552.18, April 1, he loses $4.17 interest, when can he pay it without any gain or loss of interest?

The answer evidently is as many days after April 1 as it will take for $552.18 to gain $4.17 interest, which, found as explained in **118**, is *about* 45 days, and 45 days after April 1 is May 16, which is, therefore, the equated time.

Note.—Had the above account been settled April 1, 1857, B should have paid A $552.18 — $4.17 = $548.01. If the balance of interest had been due to A instead of B, B should have paid $552.18 + $4.17 = 556.35.

2. The account-books of Henry Clark and George Barton show that—

Clark owes Barton	And that Barton owes Clark
$328.14 due Oct. 1, 1857.	$148.16 due Sept. 28, 1857.
$425.96 due Nov. 3, 1857.	$452.19 due Oct. 17, 1857.
$604.50 due Nov. 25, 1857.	$83.75 due Dec. 1, 1857.

What is the equated time for payment?

3 The account-books of L. F. Wilson and S. P. Allen sh. that—

Wilson owes Allen	And that Allen owes Wilson
$640 due Jan. 4, 1858.	$200 due Jan. 16, 1858.
$900 due Feb. 6, 1858.	$1200 due Jan. 31, 1858.
$500 due March 1, 1858.	$1000 due Feb. 10, 1858.
$600 due March 20, 1858.	$1000 due Feb. 28, 1858.

What is the equated time for payment?

4. When was the balance of the following account due?

Dr. John H. Arnold in account with W. C. Burlingame. Cr.

1858.					1858.				
Jan.	9	To Mdse. 6 mo.*	$253	25	Feb.	11	By Mdse. 5 mo.	$500	00
Feb.	7	To Mdse. 4 mo.*	573	87	Feb.	20	By Mdse. 4 mo.	326	27
Feb.	25	To Mdse. 4 mo.	421	19	March	9	By Cash	349	20
March	1	To Mdse. 3 mo.	147	00	March	23	By Mdse. 2 mo.	472	18

5. What was due on the following account July 1, 1858, interest being reckoned at 6 per cent?

Dr. Lewis Dexter in account with John Gorham. Cr.

1858.					1858.				
Feb.	3	To Mdse. 6 mo.	$2000	00	Feb.	10	By Mdse. 4 mo.	$800	00
March	1	To Mdse. 5 mo.	1500	00	Feb.	25	By Mdse. 6 mo.	1000	00
April	5	To Mdse. 2 mo.	1800	00	April	11	By Mdse. 4 mo.	2500	00
April	27	To Mdse. 4 mo.	1000	00	May	26	By Mdse. 2 mo.	1900	00
May	12	To Mdse. 3 mo.	1200	00	June	20	By Mdse. 1 mo.	1700	00
June	7	To Mdse. 2 mo.	3000	00	June	30	By Mdse. 2 mo.	1000	00

121. To find the Principal or the Interest when the Amount, Rate, and Time are given.

1. What sum of money will amount to $367.65 in 1 year, at $7\frac{1}{2}$ per cent?

Solution.—Since, at the given rate, the interest for 1 year must equal $\frac{7\frac{1}{2}}{100} = \frac{15}{200} = \frac{3}{40}$ of the principal, the amount must equal $1\frac{3}{40}$, or $\frac{43}{40}$

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* *i. e.* on 6 mo. credit, 4 mo. credit, etc.

times the principal. Hence, the principal must equal $\frac{40}{43}$ of the amount, or $\frac{40}{43}$ of $367.65 = $342.

2. What principal will amount to $252.791 in 1 yr. 3 mo. 18 da. at 6 per cent, and what is its interest for that time?

Solution.—Since, at 6 per cent per year, interest for 1 yr. 3 mo. 18 da., or 468 days, is $\frac{468}{6000} = \frac{78}{1000}$ of the principal, the amount must equal $1\frac{78}{1000}$ or $\frac{1078}{1000}$ times the principal. Hence, the principal must equal $\frac{1000}{1078}$, and the interest must equal $\frac{78}{1078}$ of the amount. $\frac{1000}{1078}$ of $252.791 = $234.50 = principal; $\frac{78}{1078}$ of $252.791 = $18.291 = interest.

3. The amount of a certain sum for 1 yr. 5 mo. 15 da., at 5 per cent, is $1855.545. What is the principal and what the interest?

Solution.—Since, at 5 per cent per year, interest of any sum for 1 yr. 5 mo. 15 da., or 525 days, or $\frac{525}{360}$ of a year, is $\frac{525}{360}$ of $\frac{5}{100} = \frac{7}{96}$ of the principal, the amount must equal $1\frac{7}{96}$ or $\frac{103}{96}$ times of the principal. Hence, the principal must equal $\frac{96}{103}$, and the interest must equal $\frac{7}{103}$ of the amount. $\frac{96}{103}$ of $1855.545 = $1729.44 = principal; $\frac{7}{103}$ of $1855.44 = $126.105 = interest.

(a.) From these solutions, it appears that, to find the principal or the interest when the amount, rate, and time are given, we find the fraction which expresses what part interest for the given time at the given rate is of the principal, and divide the given amount by 1 plus this fraction.

Note.—The above-named fraction will always be the same part of the given annual rate that the given time is of 1 year, or of 360 days; or, if the rate is 6 per cent, it will equal the fraction expressing the part which the given time is of 200 months, or of 6000 days.

(b.) To find the interest, we subtract the principal, found as before, from the amount; or, we multiply the given amount by the fraction which shows what part interest for the given time and rate is of the amount.

What principal will amount to—

4. $342.40 in 1 yr. 2 mo. at 6 per cent?
5. $486.404 in 8 mo. 16 da. at 6 per cent?
6. $500.55 in 2 yr. 5 mo. 5 da. at 7½ per cent?
7. $1834.24 in 11 mo. 15 da. at 4 per cent?

8. $305.207 in 3 yr. 3 mo. 3 da. at 9 per cent?
9. $490.912 in 8 mo. 8 da. at $3\frac{3}{10}$ per cent?
10. $363.48 in 1 yr. 4 mo. 20 da. at 6 per cent?

122. Discount and Present Worth.

(a.) Discount is an allowance made for the payment of money before it is due.

(b.) The **Present Worth** of a debt due at a future time is the sum which it is supposed to be worth now.

(c.) According to the principles involved in simple interest, the present worth of a debt due at a future time is that sum of money which, put on interest now, will amount to the given debt at the time it becomes due. Hence, the given debt corresponds to the amount, the present worth to the principal, and the discount to the interest; and the problems in this article are merely additional applications of the principles involved in the last.

1. What is the present worth of $470.12, due in 1 yr. 4 mo. hence, at 9 per cent?

Solution.—The present worth required is that sum of money which, put on interest at 9 per cent, will amount to $470.12 in 1 yr. 4 mo. Since, at 9 per cent per year, interest for 1 yr. 4 mo. or 16 mo. equals $\frac{16}{12}$ of 9 per cent, equals 12 per cent of the principal, the amount must equal 112 per cent of the principal, and the principal, which is the present worth required, must equal $\frac{100}{112}$ of the amount. But $\frac{100}{112}$ of $470.12 = $419.75 = present worth.

Proof.—The amount of $419.75 at 9 per cent. for 16 mo. = $470.12 = given debt.

What is the present worth of—

2. $1453.20 due in 2 yr. 9 mo. 20 da. at 6 per cent?
3. $465.26 due in 1 yr. 4 mo. 12 da. at 6 per cent?
4. $634.827 due in 2 mo. 15 da. at 8 per cent?
5. $600 due in 60 da. at 6 per cent?
6. $1000 due in 4 mo. at 9 per cent?

7. $206 due in 6 mo. at 6 per cent?
8. $145.36 due in 4 mo. at 7 per cent?
9. $628.35 due in 1 yr. at 4½ per cent?
10. $1438 due in 9 mo. at 6 per cent?

11. What is the discount of $997.542 due in 2 mo. 12 da. hence, at 7½ per cent?

Solution.—The discount required is the interest of that sum of money which, at 7½ per cent, will amount to $997.542 in 2 mo. 12 da. Interest for 2 mo. 12 da., or $\frac{1}{5}$ of a year, at 7½ per cent, equals $\frac{1}{5}$ of $\frac{7\frac{1}{2}}{100} = \frac{3}{200}$ of the principal; hence, the amount must equal $\frac{203}{200}$ of the principal, and the discount must equal $\frac{3}{203}$ of the amount, equal to $14.742.

Note.—The discount might have been found by finding the present worth and subtracting it from the given debt.

What is the discount of—

12. $507.50 due in 3 mo. at 6 per cent?
13. $469.464 due in 6 mo. 20 da. at 6 per cent?
14. $594.615 due in 1 yr. 1 mo. 10 da. at 6 per cent?
15. $253.46 due in 1 yr. at 6 per cent?
16. $825.14 due in 7 mo. 19 da. at 6 per cent?
17. $824.36 due in 5 mo. 17 da. at 6 per cent?
18. $927.41 due in 2 yr. at 4½ per cent?
19. $683.76 due in 3 yr. at 3½ per cent?
20. $437.26 due in 1 mo. 23 da. at 9 per cent?

123. Business Method of Discount.

(a.) Merchants and business men are usually willing to allow a larger rate of discount than that obtained by the method of the preceding article. Sometimes they deduct the interest of the entire debt for the time it has to run, as is always done on notes discounted at bank (see **117**), and sometimes they deduct a still larger sum.

1. What will be received on a debt of $1000 due in 2 mo., discount being allowed at the rate of 6 per cent per year?

Answer.—$1000 minus the interest of $1000 for 2 mo. = $990.

2. How much money will be required to pay a note of $800 due in 3 mo., discount at 6 per cent per year being allowed? *

3. Bought goods to the amount of $1200, for which I can pay either by giving my note payable in 6 mo., or by cash at a discount of 6 per cent per annum. How much shall I gain by borrowing enough money to pay for them now, and then paying the borrowed money with interest at the time when the note would have become due?

4. What is the difference between the present worth of a debt of $2000 due in 6 mo. and the sum which would be received on it, if discount be reckoned as in this article, money being worth 6 per cent per year?

5. I bought goods to the amount of $1500, payable in 4 mo.; but the seller offering to deduct 5 per cent of the debt for cash, I hired money enough to pay for them. At the end of the 4 mo., I paid the borrowed money with interest at 6 per cent. What did I gain by hiring the money?

Solution.—5 per cent of $1500 = $75 = discount. $1500 — $75 = $1425 = sum borrowed. Amount of $1425 for 4 mo. = $1453.50, and $1500 — $1453.50 = $46.50.

6. For how much can a debt of $695.34, due in 5 mo., be paid if a discount of 8 per cent of the debt be allowed for cash?

7. I bought a note for $2500 due in 4 mo., at a discount of 5 per cent. Allowing that money was worth to me 6 per cent per year, and that the note was paid when due, how much did I gain by the transaction?

8. I sold my note for $1000 due in 2 mo., at a discount of 1½ per cent per month. How much did I receive for it?

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* Observe that grace is always to be allowed on *notes* on time.

9. For how much must a note payable in 2 mo. be given to realize $1000 at bank, money being worth 6 per cent?

Solution.—$1000, the money received, must be the difference between the face of the note and its interest for 2 mo. 3 da., *i. e.* it must equal the face of the note minus $\frac{63}{6000}$ of it = $\frac{5937}{6000}$ or $\frac{1979}{2000}$ of the face of the note Hence, the note must be given for $\frac{2000}{1979}$ of $1000, which is $1010.611.

10. For how much must a note payable in 90 da. be given, in order that $2000 may be received on it at bank?

11. For how much must a note payable in 4 mo. be given, in order that $800 may be received on it at a New York bank, interest being 7 per cent?

12. For how much must a note payable in 3 mo. be given, so that when *shaved* at 2 per cent per month I may receive $1200 for it?

13. I owe $1000 payable in 3 mo. My creditor offering to deduct 4 per cent for cash, I pay him $500. How much is still due?

Solution.—Since 4 per cent of the debt was to be deducted for cash, the cash payment would be 96 per cent of the part of the debt cancelled, and the part of the debt cancelled would be $\frac{100}{96}$ of the cash payment. $\frac{100}{96}$ of $500 = $520.83⅓ = part of debt cancelled. $1000 = $520.83⅓ = $479.16⅔ still due.

14. I owed $1260 due in 4 mo. My creditor offering to deduct 6 per cent for cash, I paid $900 down. How much did I still owe?

15. I bought goods to the amount of $1500, and can pay for them by a note on 6 mo., or by cash at 6 per cent discount. Allowing that I pay $900 in cash, for how much must my note be given?

16. I bought a lot of goods for $728.96, payable in 3 mo.; but the seller offering to deduct 4 per cent for cash, I got a note payable in 3 mo. discounted at bank for such a sum that the money received on it was just sufficient to pay for the goods. For how much was the note given? How much less money was required to pay the note when due, than would have been required to pay for the goods at that time?

124. To find the Rate when the Principal, Interest, and Time are given.

Note.—Problems like the following occur but rarely in business life.

(a.) To solve them, we first find what part of the principal the given interest is, and then find from this what part of the given principal the interest for one year is, reducing the result to hundredths.

1. At what rate per cent must \$324 be on interest to gain \$19.44 in 1 yr. 2 mo. 12 da.?

Solution.—\$19.44 = $\frac{1944}{32400}$ of \$324; or the given interest is $\frac{1944}{32400}$ of the principal. If the interest for 1 yr. 2 mo. 12 da., or 432 days, is so much, the interest for 1 yr. or 360 da. must equal $\frac{360}{432}$ of $\frac{1944}{32400}$ of the principal = $\frac{1}{20}$ or 5 per cent of the principal.

Required the rate of interest when—

2. \$475.50 gains \$38.04 in 1 yr. 4 mo.
3. \$933.48 gains \$31.116 in 6 mo. 20 da.
4. \$255 gains \$40.80 in 1 yr. 9 mo. 10 da.
5. \$444 gains \$14.43 in 8 mo. 20 da.
6. \$356.40 gains \$116.127 in 4 yr. 4 mo. 4 da.

125. To find the Principal when the Interest, Rate, and Time are given.

Note.—These problems, like those of the last article, are of rare occurrence in business life.

(a.) To solve them, we first find what part interest for the given time and rate is of the principal. The given interest being this fractional part of the principal, the principal must be the part of the interest expressed by this fraction inverted.

1. What principal on interest at 7 per cent will gain \$88.536 in 1 yr. 4 mo.?

Solution.—Since 1 yr. 4 mo. = 16 mo. = $\frac{16}{12}$, or $\frac{4}{3}$ of a year, the interest must equal $\frac{4}{3}$ of $\frac{7}{100}$ = $\frac{7}{75}$ of the principal. If \$88.536 is $\frac{7}{75}$ of the principal, the principal must equal $\frac{75}{7}$ of \$88.536, which is \$948.60.

What principal on interest —

2. At 6 per cent will gain $5.038 in 1 yr. 5 mo. 15 da.?
3. At 6 per cent will gain $8.76 in 6 mo. 20 da.?
4. At 6 per cent will gain $8.841 in 1 mo. 26 da.?
5. At 2½ per cent will gain $46.55 in 2 yr. 2 mo. 4 da.?
6. At 10 per cent will gain $6.246 in 11 mo. 17 da.?
7. At 7 per cent will gain $19.551 at 4 mo. 27 da.?
8. At 5½ per cent will gain $81.873 in 2 yr. 3 mo. 17 da.?

126. Commission.

(a.) Money received for services in buying and selling goods for others is called COMMISSION, and is usually reckoned at a certain per cent of the cost of the goods bought, or of the price of those sold.

(b.) A merchant who makes it his business to buy and sell on commission is called a COMMISSION MERCHANT.

1. A commission merchant sold goods for me to the amount of $775.50, for which he charged a commission of 1½ per cent. What was his commission, and how much ought he to remit to me?

SOLUTION. — His commission = .01½ of $775.50 = $11.632, and he ought to remit me $775.50 — $11.632 = $763.868.

2. A commission merchant bought goods for me to the amount of $573.25, for which he charged a commission of 2 per cent. What was his commission, and what sum must I remit to pay for the goods and commission?

SOLUTION. — His commission = .02 of $573.25 = $11.465, and I must remit $573.25 + $11.465 = $584.715.

3. I sold goods for R. & S. Ives to the amount of $3275.28, for which I charged a commission of 7¼ per cent. What was the amount of my commission, and how much money must I remit to them?

4. I bought goods for John Munroe to the amount of $1500, charging him a commission of 3⅝ per cent. What was my com-

mission, and how much money was required to pay for the goods and my commission?

5. I sold for a manufacturer 9342 yards of cotton cloth at 9 cents per yard, 17250 yards at 10 cents per yard, and 5000 yards at 9½ cents per yard. What was my commission at 2 per cent on the sales? How much money ought I to pay him?

6. I bought for a merchant a bill of goods amounting to $5172.75. What was my commission at 2½ per cent? How much money ought he to send me to pay for the goods and my commission?

7. John Williamson has sent me $3550 with which to buy flour, after deducting my commission of 2½ per cent on the amount expended. What will be my commission, and how much ought I to expend for flour?

Solution.—Since $3550 includes the sum to be invested and the commission of 2½ per cent, it must equal 102½ per cent, or $\frac{41}{40}$ of the sum to be invested. Hence, I ought to invest $\frac{40}{41}$ of $3550, which is $3463.415, and my commission ought to be $\frac{1}{41}$ of $3550, which is $86.585.

Proof.—2½ per cent of $3463.415 equals $86.585.

8. Thomas Simmons has bought a lot of goods for me, on which he charges a commission of 2½ per cent. If $8520 is just sufficient to pay for the goods and his commission, what was the cost of the goods and what the amount of his commission?

9. I have bought a lot of furs at a commission of 10 per cent. The cost of the furs and the amount of my commission are together equal to $1376.65. What was the cost of the furs and what the amount of the commission?

10. I sent $904.96 to my agent, directing him to expend it in corn, after deducting his commission of 1 per cent on the purchase. He purchased the corn at 50 cents per bushel. How many bushels did he buy?

11. I sent 15625 yards of sheeting to Brown & Rogers, Commission Merchants in New Orleans, directing them to sell it, and, after deducting all expenses, to invest the balance in cotton. They sold the sheeting at 12 cents per yard, for which they charged a com-

mission of 4 per cent. They then invested the balance, after deducting a commission of 2 per cent on the purchase, in cotton, at 15 cents per pound. How many pounds of cotton did they buy?

127. Insurance.

(a.) Insurance is an obligation assumed by one individual or company to pay to another a certain sum of money on the occurrence of some contingent event.

ILLUSTRATIONS. — My house is insured against loss by fire, when a company or an individual have agreed to pay me a certain sum in case it is burned within a specified period. My life is insured when a company agrees to pay to my heirs a certain sum in the event of my death.

(b.) FIRE INSURANCE is insurance against loss by fire.

(c.) MARINE INSURANCE is insurance against losses at sea.

(d.) HEALTH INSURANCE is an obligation to pay an individual a certain sum for each week that he may be sick.

(e.) LIFE INSURANCE is an obligation to pay to some person a certain sum of money in the event of the death of the person insured.

(f.) The written certificate of insurance is called a POLICY.

(g.) The sum paid for insurance is called the PREMIUM.

(h.) If *property* is insured, the premium is a certain per cent of the sum covered, *i. e.* of the sum for which it is insured, and is usually paid at the time of effecting the insurance.

(i.) When *health* or *life* is insured, the premium is usually a given sum paid annually during the time for which the insurance continues. As its amount is calculated from tables prepared by the different insurance companies, it will be unnecessary to give examples in life or health insurance.

1. I had my factory insured for $10000, at a premium of 7 per cent, paying $1 for the policy. How much did the insurance cost me?

ANSWER. — $1 + 7 per cent of $10000 = $701.

2. What would be the expense of getting a building insured for $1500, at a premium of 4½ per cent, the policy costing $1?

3. A ship was insured for a certain voyage to the amount of $27500. The premium was 5½ per cent, and the policy cost $1.50. What was the cost of insurance?

4. I had my house insured for $3000, for which I paid a premium of 5 per cent, and $.75 for the policy. 11 months afterwards, the house was burned; and, 1 month after the fire, the insurance company paid me the insurance. How much did I really save by the insurance?

5. Aug. 4, 1856, I bought a lot of cloth for $5000, and shipped it to Buenos Ayres. Aug. 7, I got it insured for its cost, paying 4¼ per cent for the insurance, and $1.50 for the policy. Sept. 15, I received intelligence that the vessel was lost at sea. Oct. 15, the insurance company paid me the insurance. What was my real loss, money being worth 8 per cent per year?

6. I had my house insured for $2500 for 7 years, paying a premium of 6 per cent, and $1 for the policy. The house was not burned. Allowing that money was worth 6 per cent a year compound interest, how much had the insurance cost me when the policy expired? How much should I have gained by the insurance had the house been burned, and the insurance paid me at the end of the 7 years?

128. Orders, Drafts, and Bills of Exchange.

(a.) A Draft or an Order is a business paper, requesting one person to pay to another a specified sum of money on account of a third person, who is always the signer of the order.

ILLUSTRATION.

$300.   Providence, Jan. 1, 1858.

Please pay to the order of Dexter S. Stone, three hundred dollars, and charge the same to me

Charles B. Hawkins.

To E. Wheelock, Jr.
140 Westminster St.

(b.) The preceding is an ORDER or DRAFT drawn by Charles B. Hawkins on E. Wheelock, Jr., IN FAVOR OF Dexter S. Stone.

(c.) The signer of the order is the DRAWER. The one to whom it is addressed is the DRAWEE; and the one in whose favor it is drawn, or the one who becomes its legal owner, is the HOLDER.

(d.) The draft may be made payable on time, as, "Thirty days after date," or, "At so many days sight," *i. e.* in so many days after the holder presents it to the drawee for ACCEPTANCE.

(e.) A draft is ACCEPTED when the drawee has written the word "ACCEPTED" and his name across the face of it, to indicate that he will pay it when due. An accepted draft is, to all intents and purposes, a promissory note against the party accepting it.

(f.) If the drawee refuses either to accept a draft, or to pay it when due, it may be protested in the same manner as a promissory note, and the holder may demand payment of the drawer or of any indorser.

(g.) Drafts payable in a different place from that in which they are dated, are called BILLS OF EXCHANGE. If payable in the same country, they are called INLAND or DOMESTIC BILLS; if payable in a foreign country, they are called FOREIGN BILLS.

(h.) To guard against loss in the transportation of foreign bills of exchange, it is customary to prepare two or three (usually three) copies, and to send them by different vessels. These bills constitute a SET OF EXCHANGE, and are called FIRST, SECOND, and THIRD of exchange. They are so worded that the payment of one renders the others void.

(i.) A bill of exchange gives to the holder funds in the country on which it is drawn, and thus saves the expense and risk of transporting specie for the making of purchases or the payment of debts.

ILLUSTRATION.—The bill under k gives to Williams & Grovei £2000 in London.

(j.) Such bills are usually sold by the drawer to the party in whose favor they are drawn, and may bring more or less than their nominal or par value. If they bring more, they are sold at a PREMIUM; if less, they are sold at a DISCOUNT.

(k.) FORM OF A BILL OF EXCHANGE.

New York, Jan. 1, 1858.

Exchange for £2000.

Thirty days after sight of this first of exchange (second and third unpaid), pay to the order of Williams & Grover two thousand pounds, and charge the same to our account.

G. F. Austin & Co.

To Geo. Peabody & Co.
London.

(l.) A pound sterling in exchange is reckoned at $\frac{40}{9}$ of a dollar, or $\$4.44\frac{4}{9}$, though the real value of the sovereign, which represents it, varies from \$4.83 to \$4.86 (see **17, b**). A dollar in exchange, therefore, is really worth about \$1.09; and exchange on England is *really* at par when \$1.00 sells for about \$1.09. It is at a premium when it sells for more, and at a discount when it sells for less, than \$1.09.

ILLUSTRATION.—If a Bill of Exchange on England for £1000 be purchased at 108, it would cost \$4800, as follows:

$$
\begin{array}{rl}
1000 \times \$4\tfrac{4}{9} = \$4444.444 & = \text{nominal value.} \\
355.556 & = 8 \text{ per cent nominal premium.} \\
\hline
\$4800.00 & = \text{total cost.}
\end{array}
$$

If the value of the sovereign was \$4.85, the above bill would bring in England 1000 × \$4.85 = \$4850; from which it appears that this sale, though made nominally at a premium, was really at a discount.

1. What would a bill of exchange on England for £1000 cost at 108$\frac{1}{2}$?

SOLUTION.

$$
\begin{array}{rl}
1000 \times \$4\tfrac{4}{9} = \$4444.444 & = \text{par value.} \\
377.778 & = 8\tfrac{1}{2} \text{ per cent premium.} \\
\hline
\$4822.222 & = \text{cost.}
\end{array}
$$

2. What will a bill of exchange on England for £1500 cost at 109¼?

3. What will a bill of exchange on London for £1800 cost at 108¾?

4. The sovereign being worth $4.83, were the last three bills really sold at a premium or a discount?

5. What is the exact cost of a pound of exchange bought at 108¾?

6. What is the exact cost of a pound of exchange bought at 109?

7. What will a bill of exchange on France for 6400 francs cost at 1 per cent discount, the franc being reckoned at $.186?

8. What will a bill of exchange on Havana for $5475 cost at 2¼ per cent premium?

129. Stocks.

(a.) Money invested in any property designed to yield an income is called STOCK.

ILLUSTRATIONS.—Money invested in business is called STOCK IN TRADE; money invested in government securities, bonds, etc. is called GOVERNMENT STOCK.

(b.) The CAPITAL STOCK of an incorporated company is the money paid in by its members for the general purposes for which the company was formed. It is divided into equal parts called SHARES. Any person owning one or more of these shares is a STOCKHOLDER, OR MEMBER OF THE CORPORATION. Stocks is a general term applied to the shares themselves, which may be bought and sold like any other property.

(c.) The first or original value of the shares of any corporation *i. e.* the value at which they are rated in estimating its capital stock, is called their NOMINAL VALUE, or their PAR VALUE, and is always the same.

(d.) The price which they will bring, if exposed for sale, is their TRUE or REAL VALUE, and is different at different times.

(e.) If the real value equals the par value, the stocks are AT PAR; if it be greater, they are ABOVE PAR, and sell at a PREMIUM or ADVANCE; if it be less, they are BELOW PAR, and sell at a DISCOUNT.

(f.) The profits accruing to the corporation, if any, are at intervals distributed among the members, in proportion to the number of shares each holds, and are then called DIVIDENDS. The dividends are usually reckoned at a certain per cent of the *par value* of the shares.

1. What will 8 shares of Suffolk Bank stock cost at an advance of 7 per cent, the par value being $100 per share?

SOLUTION. —
$800 = par value of 8 shares.
56 = 7 per cent premium.
$856 = real value, or required cost.

2. How much will 19 shares of railroad stock cost at a discount of 4 per cent, the par value being $50 per share?

SOLUTION. —
$950 = par value of 19 shares.
38 = 4 per cent discount.
$912 = real value, or required cost.

3. How much will 14 shares City Bank stock cost at a premium of 11 per cent, the par value being $100 per share?

4. What will 23 shares of railroad stock cost at a discount of 19 per cent, the par value being $100 per share?

5. A broker bought 11 shares of stock at a premium of $3\frac{3}{4}$ per cent. How much did they cost him, the par value being $50 per share?

6. I bought 20 shares in a mining company at 42 per cent below par, and sold them at 37 per cent below par. How much did I gain, the par value being $100 per share?

7. How much shall I lose by buying 15 shares bank stock at a premium of 7 per cent, and selling them 4 months afterwards at a discount of 3 per cent, the par value being $100 per share, and money being worth 6 per cent per year?

8. I bought 25 shares of Fitchburg railroad stock at 25 per cent discount, and a broker sold them for me at 16 per cent discount, charging 25 cents per share for his services. How much did I gain by the transaction, the par value of the stock being $100 per share?

9. I paid $440 for stock at an advance of 10 per cent. How many shares did I buy, the par value being $50 per share?

10. I paid $879.75 for Weybosset Bank stock at a premium of 3½ per cent. How many shares did I buy, the par value being $50 per share?

11. March 1, 1856, I bought 4 shares of stock at an advance of 1¼ per cent on the par value of $500 per share. July 1, 1856, I received a dividend of 5½ per cent. Sept. 1, 1856, I sold the stock at a premium of 12 per cent. What did I gain by the transaction, money being worth 7 per cent per year?

NOTE.—In examples of this kind, interest should be reckoned on the dividends as well as on the original cost of the stock.

12. A man bought 16 shares of stock at a discount of 2½ per cent. Six months afterwards he received a dividend of 3 per cent, and in 6 months more he received a dividend of 4 per cent. Two months after receiving the second dividend, he sold the stock at a premium of 5 per cent. Money being worth 6 per cent simple interest, what was his gain by the transaction, the par value of the stock being $100 per share?

13. By buying stock at 76 per cent of its par value, and selling it at par, I gained $144. What was the par value of the stock?

14. By buying stock at 20 per cent below par, and selling it for 90 per cent of its par value, I gained $337.50. What was the par value of the stock bought? How much did I give for it? For how much did I sell it?

130. Profit and Loss.

(a.) PROFIT AND LOSS treats of the percentage of profit and loss made by merchants in their business transactions.

(b.) Most problems in profit and loss come under one of three classes —

1st. Those in which the cost and per cent of profit or loss being given, the selling price or the number of dollars gained or lost is required.

2d. Those in which, the cost and selling price being given, the percentage of profit or loss is required.

3d. Those in which, the selling price and percentage of profit or loss being given, the cost or some per cent of the cost is required.

(c.) For how much must I sell articles —

1. Costing $4 to gain 16⅔ per cent?

ANSWER. — $4 + 16⅔ per cent of $4 = $4.66⅔.

2. Costing $7.50 to gain 15 per cent?
3. Costing $12 to lose 12½ per cent?
4. Costing $3.56 to lose 25 per cent?

(d.) How much shall I gain or lose by selling articles —

5. Costing $9 at an advance of 12 per cent?

ANSWER. — $1.08 gain.

6. Costing $27.50 at a loss of 16 per cent?
7. Costing $12.50 at an advance of 12½ per cent?
8. Costing $25 at a loss of 25 per cent?
9. Costing $8.70 for 112 per cent of cost?

ANSWER. — $1.044 gain.

10. Costing $125 for 125 per cent of cost?
11. Costing $96 for 96 per cent of cost?
12. Costing $8 for 87½ per cent of cost?

13. How many pounds of sugar, costing 8 cents per pound, must be sold for a dollar, in order to make a profit of 12½ per cent?

SOLUTION. — To gain 12½ per cent, I must sell it for 112½ per cent, or $\frac{9}{8}$ of the cost. Therefore I must sell it for $\frac{9}{8}$ of 8 cents, or for 9 cents per pound, which will give 11⅑ lb. for a dollar.

14. How many pounds of coffee, costing 15 cents per pound, must be sold for a dollar, in order to gain 11⅑ per cent?

15. Bought a cask of rice containing 213 pounds, at 3⅕ cents per pound. How many pounds must I sell for a dollar, in order to gain 25 per cent?

16. I bought 112 barrels of apples at $1.75 per bbl., and it cost $12.50 for having them brought to my store. For how much per bbl. must I sell them, in order to make a clear profit of 33⅓ per cent?

17. A house carpenter purchased a house for $2500. He worked with 2 journeymen and 1 apprentice 24 days in repairing it. He valued his own services at $2.25 per day, the services of each journeyman at $1.75 per day, and those of the apprentice at $1 per day. The stock used in repairing cost $225, and he paid $100 for having the house painted. He then sold the house so as to gain 25 per cent on the whole amount which it had cost him. For how much did he sell it and what was his gain?

18. An importer sold to a wholesale dealer a lot of sugar which had cost him 5¼ cents per pound, at an advance of 33⅓ per cent. The wholesale dealer sold it to retail dealers at an advance of 25 per cent. The retail dealers sold it at an advance of 20 per cent on what it had cost them. For how many cents per pound did the retail dealers sell it?

19. What per cent shall I gain by selling articles costing $1.54 for $1.68?

Solution.—I shall gain $.14, which equals $\frac{14}{154} = .09\frac{1}{11}$, or $9\frac{1}{11}$ per cent of the cost.

(e.) What per cent shall I gain or lose by selling articles costing—

20. $7 for $6?
21. $7.50 for 5?
22. $.62½ for $.50?
23. $.50 for $.62½?
24. $137.80 for $172.25?
25. $172.25 for $137.80?

(f.) What per cent of the cost shall I *receive* by selling articles costing—

26. $4 for $5?
27. $5 for $4?
28. $15 for $15.75?
29. $15.75 for $15?

(g.) What per cent of the *selling price* shall I gain or lose by selling articles costing —

30. \$.75 for \$1?
31. \$1 for \$.75?
32. \$11 for \$12?
33. \$12 for \$11?
34. \$1 for \$2?
35. \$2 for \$1?

36. What per cent of the selling price shall I gain, if I gain 18 per cent of the cost?

Solution. — If I gain 18 per cent of the cost, I shall sell for 118 per cent of the cost. Hence, the gain, 18 per cent, is $\frac{18}{118} = .15\frac{15}{59}$ of the selling price, or $15\frac{15}{59}$ per cent of the selling price.

(h.) What per cent of the selling price shall 1 gain or lose if I —

37. Lose 10 per cent of cost?
38. Gain 10 per cent of cost?
39. Lose 20 per cent of cost?
40. Gain 20 per cent of cost?
41. Gain 50 per cent of cost?
42. Gain 100 per cent of cost?

43. What per cent of the cost shall I lose if my loss equals 10 per cent of the selling price?

Solution. — The loss added to the selling price must equal the cost. Hence, if I lose 10 per cent of the selling price, the cost must be 110 per cent of the selling price, and the loss must be $\frac{10}{110} = \frac{1}{11} = 9\frac{1}{11}$ per cent of the cost.

(i.) What per cent of the cost shall I gain or lose —

44. If my loss equals 25 per cent of the selling price?
45. If my loss equals 50 per cent of the selling price?
46. If my loss equals 20 per cent of the selling price?
47. If my gain equals 15 per cent of the selling price?
48. If my gain equals $16\frac{2}{3}$ per cent of the selling price?
49. If my gain equals $6\frac{1}{4}$ per cent of the selling price?

50. What is the cost of goods on which I gain $9\frac{1}{11}$ per cent by gaining 14 cents?

Solution. — Since 14 cents is $9\frac{1}{11}$ per cent of the cost, 100 per cent of the cost must equal $\frac{100}{9\frac{1}{11}}$ of 14 cents = \$1.54.

(j.) What is the cost of goods on which I—

51. Gain \$2 by gaining 10 per cent?
52. Gain \$.04 by gaining 50 per cent?
53. Lose \$.45 by losing 15 per cent?
54. Lose \$5.20 by losing $14\frac{2}{7}$ per cent?

(k.) What is the selling price of goods on which I—

55. Gain \$6 by gaining 6 per cent?
56. Lose \$6 by losing 6 per cent?
57. Gain \$.09 by gaining 3 per cent?
58. Lose \$.09 by losing 3 per cent?
59. Gain \$15.50 by gaining $16\frac{2}{3}$ per cent?
60. Lose \$15.50 by losing $16\frac{2}{3}$ per cent?

61. What is the cost of goods on which I gain 12 per cent by selling for \$3.08?

SOLUTION.—Since I gain 12 per cent, my selling price must equal 112 per cent of the cost. Hence, 100 per cent of the cost, or the cost itself, must equal $\frac{100}{112}$ of the selling price. $\frac{100}{112}$ of \$3.08 = 2.75 = cost.

62. What is the cost of goods on which I lose 12 per cent by selling for \$3.08?

SOLUTION.—Since I lose 12 per cent, my selling price must equal 88 per cent of the cost. Hence, 100 per cent of the cost, or the cost itself, must equal $\frac{100}{88}$ of the selling price. $\frac{100}{88}$ of \$3.08 = \$3.50 = cost.

What is the cost of goods on which I—

63. Gain 20 per cent by selling for \$1728?
64. Lose 20 per cent by selling for \$1728?
65. Gain $33\frac{1}{3}$ per cent by selling for \$15.50?
66. Lose 40 per cent by selling for \$40?

67. I bought a carriage for \$125, and, after paying \$12.50 for having it painted and varnished, I sold it for \$150. What was my gain per cent?

68. I bought a sail-boat for $200, and paid $5 for painting and repairs. I let it enough to come to $25, and then sold it for $250 What per cent of my total expenses did I gain?

69. I sold a rifle for $16.60, thereby losing 17 per cent. How much did it cost me?

70. I purchased a lot of merchandise for a certain sum, and, after incurring expenses upon it to the amount of $25, I sold it for $290, thereby gaining 16 per cent on its entire cost. How many dollars did I pay for it?

71. I sold a horse for $212.50, thereby losing 15 per cent. For how much ought I to have sold him to gain 20 per cent?

Solution. — As I lost 15 per cent, I sold him for 85 per cent of the cost. But to have gained 20 per cent, I should have sold him for 120 per cent of the cost. As 120 per cent is $\frac{120}{85}$ or $\frac{24}{17}$ of 85 per cent, I ought tc have sold him for $\frac{24}{17}$ of $212.50, which is $300.

72. I bought 2 lots of land at the same price per acre. I sold the first lot for $125.32 per acre, thereby gaining 16⅔ per cent; but was obliged to sell the second lot at a loss of 12½ per cent. For how much per acre did I sell the second lot?

73. I bought 2 cargoes of coal at the same price per ton. I sold the first at an advance of 12 per cent, and the second at a loss of 12 per cent. The first cargo contained 123½ tons, and I sold it for $622.44. The second cargo contained 128 tons. For how much per ton did I sell the second cargo?

74. By selling a lot of goods for $1001, I lost 9 per cent. For how much ought I to have sold them to gain 3½ per cent?

75. By selling a lot of goods for $545.49, I gain 14 per cent. For how much must I sell them to lose 6 per cent?

76. I bought a lot of goods, and, by selling ½ of them at an advance of 6 per cent, and the other half at an advance of 12 per cent, I gained $47.25. How much did they cost me?

77. I bought a cask of oil. One-fourth of it leaked out, and I sold the remainder at an advance of 25 per cent of its cost. What per cent did I lose on the investment?

78. I bought a cargo of sugar, and sold it at an advance of $33\frac{1}{3}$ per cent on its cost; but $12\frac{1}{2}$ per cent of my sales proved to be bad debts. What per cent did I gain on the investment?

79. I bought some cloth at $3 per yard. How much must I ask for it, that I may fall 10 per cent from my asking price and still gain 20 per cent on the cost?

80. I bought the wood standing on 7 acres of land for $37.50 per acre. I paid $.70 per cord for having it cut, and $1.50 per cord for having it carried to market, and incurred other expenses on account of it to the amount of $8.75. The wood averaged 18 cords per acre. I sold it on 4 months' credit, at an advance of 75 per cent on its total cost; but the purchaser offering to pay me cash if I would deduct 5 per cent from the amount of the bill, I accepted the proposition. How much did I receive per cord for the wood?

131. Partnership.

(a.) Two or more persons, uniting for the purpose of carrying on business together, form what is called a PARTNERSHIP, FIRM, or COMPANY. The capital invested by them is called their STOCK IN TRADE.

(b.) When the stocks of the several partners are invested for the same length of time, each partner's gain or loss will be the same part of the entire gain or loss that his stock is of the entire stock.

1. Jones, Brown & Smith trade in company. Jones puts in $500; Brown puts in $300; and Smith puts in $450. They gain $937.50. What is each partner's share?

PARTIAL SOLUTION. — They all put in $1250, of which Jones put in $\frac{500}{1250} = \frac{2}{5}$, Brown put in $\frac{300}{1250} = \frac{6}{25}$, Smith put in $\frac{450}{1250} = \frac{9}{25}$. Hence, Jones ought to have $\frac{2}{5}$, Brown $\frac{6}{25}$, and Smith $\frac{9}{25}$ of the gain.

2. A, B & C traded in company. A put in $1000, B $1200, C $1500. They gained $1000. What was each man's share?

3. Rogers, Goodwin & Clark formed a partnership, with a capital of $10000; of which Rogers put in $3500, Goodwin $3700, and Clark the remainder. When their partnership expired, they found that they had lost $2500. What would be each man's share of the remaining capital?

4. John Smith, failing in business, finds that he owes $2500, and that his property is worth only $625. If he divides the latter justly among his creditors, how much will he pay on a dollar? How much to Mr. Bradford, to whom he owes $600? How much to Mr. Warren, to whom he owes $98.50?

5. A, B, and C do a piece of work together for $34.15. A is employed upon it 7½ days, B 6⅔ days, and C 8⅗ days. How much money ought each to receive?

6. A and B traded in company. A put in twice as much as B They gained $300. What was each man's share?

7. A and B traded in company. A put in ⅔ as much as B. They gained $750. What was each man's share?

8. A, B, and C traded in company. A put in ½ as much as B, and C put in as much as A and B together. They lost $1275. What was each man's loss?

9. A, B, C, & D formed a partnership. A put in 4 times as much as B, while B put in ½ as much as C, and D put in as much as A and B together. They gained $1943.76. What was each man's share of the gain?

10. A and B formed a partnership for one year. A put in $8000 and B put in $7000. It was agreed that B should manage the business, for which he should receive a salary of $1000; and that, after allowing for this, the profits or losses should be shared in proportion to the stock. At the end of a year, they closed up their business, and divided the property of the firm. Allowing that they had gained $4000, how much ought each man to receive?

11. How much would each partner in the last example have received if they had lost $4000, instead of gaining it?

12. Edwin Adams, Henry B. Luther, and Horace K. Hull, formed a partnership, under the title of Adams, Luther & Co. Adams put in $7000, Luther $5000, and Hull $8000. Luther being better acquainted with the business than either of the other partners, it was agreed that his salary should be reckoned at $1200 per year, and the others at $600 each; and that, after allowing for these, the gains or losses should be shared in proportion to the stock. At the end of 2 years, they sold their stock in trade for twice their original capital. How ought the money to be divided?

13. How would the proceeds in the last example have been divided if the stock had sold for a sum equal to ½ the original capital?

132. Partnership on Time.

(a.) When the stocks of the several partners are invested for unequal times, it is evident that the profit or loss should be shared in proportion to what the use or interest of each man's stock for the time it is invested is worth.

1. A, B & C traded in company. A put in $500 for 12 months, B put in $450 for 10 months, C put in $600 for 9 months. They gained $1200. What was each man's share?

FIRST SOLUTION.

$30.00 =	interest of	A's stock	for	12	months.
$22.50 =	"	B's	"	10	"
$27.00 =	"	C's	"	9	"
$79.50 =	"	whole.			

Hence, A should have $\frac{3000}{7950}$ or $\frac{20}{53}$ of the gain = $452.830.
B should have $\frac{2250}{7950}$ or $\frac{15}{53}$ of the gain = $339,623.
C should have $\frac{2700}{7950}$ or $\frac{18}{53}$ of the gain = $407.547

SECOND SOLUTION.

The use of	$500 for 12 mo.	is worth the use of		$6000	for 1 mo.
"	$450 for 10 mo.	"	"	$4500	"
"	$600 for 9 mo.	"	"	$5400	"
"	whole stock	"	"	$15900	"

Therefore, A will have $\frac{6000}{15900}$ or $\frac{20}{53}$ of gain, B would have $\frac{4500}{15900}$ or $\frac{15}{53}$ of gain, and C would have $\frac{5400}{15900}$ or $\frac{18}{53}$ of gain, which gives the same results as before.

2. X, Y, and Z traded in company. X put in $1500 for 20 months, Y put in $1250 for 22 months, Z put in $2000 for 15 months. They gained $6000. How much ought each man to receive?

3. James Smith, George Brown, and Charles Swain formed a partnership, under the firm of Smith, Brown & Co. Smith put in $1200 for 1 year, Brown put in $800 for 18 months, and Swain put in $900 for 16 months. They gained $3000. What was each man's share?

4. Alfred White and Seth Jackson formed a partnership for 2 years. White put in $6000 for the whole time. Jackson was to manage the business, for which he was to receive $1000 per year. He did not put in any money till the end of 6 months, when he put in $5000. They gained $8000. What was each partner's just share?

5. Arthur Robbins, William Chase, Joseph Daggett, and James Freeman did a piece of work in company for $50. Robbins worked 8 hours per day for 6 days, Chase worked 10 hours per day for 3 days and 9 hours per day for 2 days, Daggett worked 7 hours per day for 4 days and 10 hours per day for 2 days, and Freeman worked 10½ hours per day for 2 days and 9¼ hours a day for 4 days. How much money ought each to receive?

6. William Mason, James Loring, and Thomas Davis formed a partnership for 2 years, commencing with a capital of $4000 each. At the end of the first year, Mason took out $1000. Loring put in $1000 and Davis put in $500. When the partnership expired, they found that they had gained $8000. How many dollars should each receive? How many dollars would each receive if the capital and gain were both divided?

7. A, B, and C traded in company. A put in $1000 for 6 months, and at the end of that time put in $1000 more. B put in $800 for 6 months, and at the end of that time put in $700 more. C put in $2000 for 4 months, and at the end of that time withdrew $800. At the end of a year they settled up their accounts, and the partnership was dissolved. Allowing that their loss in trade was $1600, how much capital would each man carry out of the firm?

8. Daniel Gould and Albert Austin formed a partnership for 2 years, under the firm of Gould & Austin. Gould at first put in $3000, and at the end of 1 year put in $2000 more. Austin put in at first $2500, and at the end of 6 months put in $4000 more. Gould managed the business, for which he was to receive a salary of $800 per year. When the partnership expired, they closed up their affairs, and found that the property of the firm was worth $5000 more than the capital which had been contributed. How should it be divided?

9. What would have been the answer to the last question had the property of the firm been worth $5000 less than the capital which had been contributed?

133. Assessment of Taxes.

(a.) Taxes are assessments laid on persons or property, usually for some public purpose.

(b.) A tax on persons is called a POLL TAX, and a tax on property is called a PROPERTY TAX.

(c.) The poll tax is only assessed on males between certain ages, as between twenty-one and seventy; and in some States it is not assessed.

(d.) In assessing taxes it is necessary—

1st. To estimate the value of all the property to be taxed, and make a complete inventory of it.

2d. To find the number of polls, *i. e.* the number of persons liable to pay a poll tax.

3d. To determine what portion of the tax is to be raised upon the polls, and to divide it equally among them.

4th. To find how much must be paid on each dollar of the taxable property, in order to raise the remainder of the tax.

(e.) This last may be done by dividing the amount to be raised by the estimated value of the property on which it is to be raised. It will then be easy to find the tax of any individual.

1. A tax of $2500 is to be raised in a certain town. The taxable property is valued at $850000, and there are 300 polls, each taxed

$1.25. What will be the tax on each dollar, and what will be the tax of each of the following persons?

A, who pays a tax on $4480, and 2 polls.
B, who pays a tax on $1700, and 1 poll.
C, who pays a tax on $360, and 1 poll.
D, who pays a tax on $11500, and 1 poll.

SOLUTION.—The tax on 300 polls at $1.25 each is $375, which, subtracted from $2500, leaves $2125 to be assessed on the property.

Since $850000 is to be taxed $2125, one dollar will be taxed $\frac{1}{850000}$ of $2125, which is 2½ mills.

A's tax on 2 polls would be twice $1.25, or $2.50, and on $4480 property would be 4480 times 2½ mills, which is $11.20. $2.50 + $11.20 gives $13.70 as the amount of A's tax.

Find the tax of the others in the same way.

2. A tax of $6108.72 is to be raised by a certain town. The taxable property is valued at $1,824,600, and there are 240 polls, each taxed $1.12½. How much is the tax on $1? How much is the tax on each of the following persons?

W, who pays a tax on $3620, and 1 poll.
X, who pays a tax on $17400, and 3 polls.
Y, who pays a tax on $8350, and 1 poll.
Z, who pays a tax on $400, and 1 poll.

SECTION XIX.

POWERS AND ROOTS.

134. Definitions.

(a.) THE product of a number taken any number of times as a factor is called a POWER of the number. (See **68**, j, k, and l.)

(b.) A ROOT of a number is such a number as taken some number of times as a factor will produce the given number.

(c.) If the root must be taken twice as a factor to produce the number, it is the SQUARE ROOT, or the SECOND ROOT; if three times, it is the CUBE ROOT, or the THIRD ROOT; if four times, it is the FOURTH ROOT, etc.

ILLUSTRATION.—3 is the square root of 9, the third root of 27, and the fourth root of 81; because $3^2 = 9$, $3^3 = 27$, $3^4 = 81$.

(d.) The character, $\surd$, called the RADICAL SIGN, is used to indicate that the root of the number before which it is placed is to be extracted.

(e.) The DEGREE of the root is indicated by a small figure called an INDEX, which is placed a little above and at the left of the radical sign. When no index is written, the square root is required.

ILLUSTRATIONS.— $\sqrt{49}$ or $\sqrt[2]{49}$ means the square root of 49.
$\sqrt[4]{16}$ means the fourth root of 16.
$\sqrt[7]{4^6}$ means the seventh root of the sixth power of 4.

(f.) We may also indicate that a root is to be extracted by using a FRACTIONAL EXPONENT.

ILLUSTRATIONS.—$4^{\frac{1}{2}} = \sqrt{4}$; $81^{\frac{1}{4}} = \sqrt[4]{81}$; $8^{\frac{3}{4}} = \sqrt[4]{8^3}$, etc.

(g.) The process of finding the powers of numbers is called INVOLUTION; and the process of finding their roots is called EVOLUTION, or the EXTRACTION OF ROOTS.

135. Relation of Square to its Root.

(a.) TABLE OF SQUARES.

$1^2 = 1$
$2^2 = 4$
$3^2 = 9$
$4^2 = 16$
$5^2 = 25$
$6^2 = 36$
$7^2 = 49$
$8^2 = 64$
$9^2 = 81$
$10^2 = 100$
$100^2 = 10000$
$1000^2 = 1000000$
$10000^2 = 100000000$
$100000^2 = 10000000000$
$1000000^2 = 1000000000000$

(**b.**) The preceding table shows, first, that there are below 100 but 9 entire numbers which are perfect squares—

(**c.**) Second, that the entire part of the square root of any number below 100 will be less than 10, and therefore contain but one figure; of any number between 100 and 10000 will lie between 10 and 100, and therefore contain 2 figures; of any number between 10000 and 1000000 will lie between 100 and 1000, and therefore contain 3 figures, etc.; or, in other words—

(**d.**) If any number contains either 1 or 2 figures, the entire part of its root must contain 1 figure; if it contains 3 or 4 figures, the entire part of its root must contain 2 figures, etc.

(**e.**) Hence, if we should begin at the right of any number, and separate it into periods of two figures each, the number of periods would be the same as the number of figures in its square root. The square of the highest denomination of the root would be found in the left-hand period; the square of the two highest denominations would be found in the two left-hand periods, etc.

(**f.**) In order to understand the method of extracting the square root, it is necessary to consider how the square of a number consisting of two parts is formed from those parts. To do this, let a represent any number whatever, and b represent any other number. Then will $a + b$ represent the sum, and $(a + b)^2$, or $(a + b) \times (a + b)$, the square of the sum of any two numbers whatever.

(**g.**) Performing the multiplication, we have a times $a = a^2$; a times $b = a \times b$, or, as it may be written, $a\,b$; b times $a = a$ times $b = a \times b$, or $a\,b$; b times $b = b^2$.

(**h.**) Writing the work as below, and adding the partial products, we have—

$$\begin{array}{rlll}
 & a + b & & \\
 & a + b & & \\
\hline
a \times (a + b) = & a^2 + a \text{ times } b & = a^2 + a\,b & \\
b \times (a + b) = & \quad + a \text{ times } b + b^2 & = + \quad a\,b + b^2 & \\
\hline
(a + b) \times (a + b) = & a^2 + 2a \text{ times } b + b^2 & = a^2 + 2\,a\,b + b^2. &
\end{array}$$

(i.) Hence $(a + b)^2 = a^2 + 2\,a\,b + b^2$, or, since a^2 equals the square of the first number, and $2\,a\,b$ equals twice the product of the first number by the second, and b^2 equals the square of the second, it follows that —

(j.) The square of the sum of any two numbers equals the square of the first number plus twice the product of the first number by the second, plus the square of the second.

ILLUSTRATIONS.

$(7 + 3)^2$, or $10^2 = 7^2$ + twice 7 times $3 + 3^2 = 49 + 42 + 9 = 100$.

$(30 + 8)^2$, or $38^2 = 30^2$ + twice 30 times $8 + 8^2 = 900 + 480 + 64 = 1444$.

(k.) The square $a^2 + 2\,a\,b + b^2$ may take another form; for $2\,a\,b + b^2 = 2\,a$ times $b + b$ times $b = (2\,a + b) \times b$, or, by omitting the sign of multiplication, which may be done without ambiguity, $= (2\,a + b)\,b$.

(l.) Hence $(a + b)^2 = a^2 + 2\,a\,b + b^2 = a^2 + (2\,a + b)\,b$, or since a^2 equals the square of the first number, and $(2\,a + b)\,b$ equals the product obtained by multiplying the sum of twice the first plus the second, by the second, it follows that —

(m.) The square of the sum of any two numbers is equal to the square of the first number plus the product of two factors, one of which is the sum of twice the first number plus the second, and the other is the second number.

ILLUSTRATIONS.

$(7 + 3)^2$, or $10^2 = 7^2 + (14 + 3)\,3 = 49 + 51 = 100$.

$(30 + 8)^2$, or $38^2 = 30^2 + (60 + 8)\,8 = 900 + 544 = 1444$.

136. Method of extracting the Square Root.

1. What is the square root of 877969?

SOLUTION.—This number contains three periods of two figures each, which shows that there are three figures in its root. Moreover, the greatest square below 87 is the square of the highest denomination of the root, and the greatest square below 8779 is the square of the two highest denominations of the root. Hence, we may find the first two figures of the required root by finding the square root of 8779 *as though it were units.*

The greatest square below 87, the left-hand period, is 81, the root of which, 9, is the first figure of the required root. Subtracting 81 from 87, and annexing the next period to the remainder gives 679, which, for convenience, we call a dividend. This dividend contains the remaining part of the square of the two highest denominations, and must therefore contain the product of two factors (**135, m**) one of which is the sum of twice the part of the root already found plus the next figure of the root, and the other is the next figure of the root.

If we knew the first of these factors, which we may call the TRUE DIVISOR, we could readily find the other by dividing 679 by it. But as we only know the value of the highest denomination, we have to make twice that value, which is the largest part of the true divisor, a TRIAL DIVISOR.

Twice 9 = 18, but as the 9 is tens with reference to the next figure of the root, the 18 must be tens, and in dividing by it we drop the right-hand figure of the dividend. Dividing 67 by 18 gives 3 for a quotient, which must either be equal to or greater than the next figure of the root.*

We now complete the true divisor by adding 3 (the last quotient figure) to 18 tens (the trial divisor). The sum is 183, which multiplied by 3 gives 549. This being less than the dividend, shows that 3 is not too large. Hence, 93 represents the two highest denominations of the root, and, as the root contains three figures, the 93 must be tens.

WRITTEN WORK.

```
        .  .  .
        877969(937
        81
        ----
 183)   679
        549
        -----
1867)   13069
        13069
        -----
```

Subtracting the 549 from 679, and annexing the next period to the remainder, gives 13069 for a new dividend, which contains the remaining part of the square of the tens and units, and therefore must contain the product of two factors, such as described in (**135, m**). We therefore proceed to form a trial divisor by doubling the 93.

Twice 93 is 186, which, as the 93 is tens, must be tens. Dropping the right-hand figure of the dividend, as before, and dividing the remaining part, 1306, by 186, gives 7 for a quotient, which is either equal to or larger than the units figure of the root.

We now complete the true divisor by adding 7 (the last figure found) to 186 tens (the trial divisor). The result is 1867, which multiplied by 7 gives 13069. This being equal to the dividend, shows that 7 is the true units figure of the root. Hence, the required root is 937.

(**a.**) These principles are of universal application, and furnish the following rules—

1st. Divide the given number into periods of two figures each, beginning with the units. The left-hand period may contain one or two figures.

---

* Since the trial divisor is less than the true divisor, this figure can never be too small.

2d. Find the greatest square in the left-hand period, and place its root as the first figure of the required root.

3d. Subtract the square thus found from the left-hand period, and to the remainder bring down the next period, calling the result a dividend.

4th. Double the part of the root already found for a trial divisor.

5th. See how many times this trial divisor is contained in all of the dividend, excepting the right-hand figure, and write the quotient as the next figure of the root, and also place it at the right of the trial divisor, to form a true divisor.

6th. Multiply this true divisor by the root figure last found, and subtract the product from the dividend.

7th. Bring down the next period to the right of the remainder, to form the next dividend.

8th. Double the part of the root already found, for a trial divisor, and proceed as with the last trial divisor.

(b.) Find the square root of each of the following numbers—

2. 529
3. 2601
4. 6724
5. 6241
6. 9801
7. 784
8. 105625
9. 47524
10. 401956
11. 863041
12. 31684
13. 654481
14. 10323369
15. 4456321
16. 3407716
17. 33930625
18. 74080449
19. 81144064

137. Square Root of Fractions.

(a.) To square a fraction, we multiply it by itself, *i. e.* we square its numerator for a new numerator and its denominator for a new denominator.

(b.) Hence, to extract the square root of a fraction, we must extract the square root of its numerator for a new numerator, and the square root of its denominator for a new denominator.

ILLUSTRATIONS.—$\sqrt{\frac{4}{9}} = \frac{2}{3}$ $\sqrt{.25} = .5$ $\sqrt{.0081} = .09$

(c.) If both terms are not perfect squares, only an approximate value of the square root can be obtained.

(d.) If the *denominator* is not a perfect square, both terms should be multiplied by such a number as will make it so. This number may be either the denominator or the product of all the prime numbers which are found as factors in the denominator, 1, 3, 5, or any other odd number of times.

ILLUSTRATIONS. — $\sqrt{\frac{25}{84}} = \sqrt{\frac{25 \times 84}{84^2}} = \frac{\sqrt{2100}}{84} = \frac{46}{84}$ nearly.

(e.) What is the square root of —

1. $\frac{144}{169}$.
2. $\frac{9604}{9801}$.
3. $\frac{371}{532}$.
4. $\frac{20736}{157609}$.
5. $\frac{2}{3}$.
6. $\frac{196}{225}$.
7. $\frac{17}{27}$.
8. $\frac{1125}{1327}$.
9. $\frac{256}{495}$.

(e.) In order that the denominator of a DECIMAL FRACTION may be a perfect square, its numerator must contain an even number of decimal places.

ILLUSTRATION. — The denominators of .04, 3.27, and .1951 are perfect squares, but the denominators of .4, .327, and .00625 are not.

(f.) Hence, to extract the square root of a decimal fraction, annex a zero, if necessary to make its number of decimal places even. Then extract the root as in whole numbers, observing that there will be one decimal place in the root for every two places in the given fraction.

(g.) The root may be found to any number of decimal places by annexing two zeroes for every additional figure desired in the root.

ILLUSTRATION. — $\sqrt{.9} = \sqrt{.90} = \sqrt{.9000} = \sqrt{.900000}$, etc.

(h.) What is the square root to 5 decimal places of —

10. .5625?
11. .0025?
12. .1429?
13. 88?
14. 2?
15. 15?
16. 86.53?
17. 62.55?
18. .8579467?

138. Relation of Cube to Root.

$1^3 = 1$

$2^3 = 8$

$3^3 = 27$

$4^3 = 64$

$5^3 = 125$

$6^3 = 216$

$7^3 = 343$

$8^3 = 512$

$9^3 = 729$

$10^3 = 1{,}000.$

$100^3 = 1{,}000{,}000.$

$1000^3 = 1{,}000{,}000{,}000.$

$10000^3 = 1{,}000{,}000{,}000{,}000.$

(a.) The above table shows, first, that there are but nine entire numbers below 1000 which are perfect cubes.

(b.) Second, that the entire part of the cube root of any number below 1000 will be less than 10, and will therefore contain but one figure; of any number between 1000 and 1000000 will lie between 10 and 100, and will therefore contain two figures, or, in other words —

(c.) The entire part of the cube root of a number containing 1, 2, or 3 figures will contain one figure; the entire part of the cube root of a number containing 4, 5, or 6 figures will contain two figures, and so on with the larger numbers.

(d.) Hence, if we should begin at the right of any number and separate it into periods of three figures each, the number of periods will be the same as the number of figures in the entire part of the cube root. The cube of the highest denomination of the root will be found in the left-hand period. The cube of the two highest denominations will be found in the two left-hand periods, etc.

(e.) We have next to consider how the cube of a number consisting of two parts is formed from those parts. To do this, let a represent any number whatever, and b any other number. Then $a + b$ will represent the sum, and $(a + b)^3$ the cube of the sum of any two numbers whatever.

(f.) But $(a + b)^2 = a^2 + 2\,a\,b + b^2$. Multiplying this by a gives $a^3 + 2\,a^2\,b + a\,b^2$, and multiplying it by b gives $a^2\,b + 2\,a\,b^2 + b^3$. Adding these results together gives $a^3 + 3\,a^2\,b + 3\,a\,b^2 + b^3$, as in the following written work.

$$\begin{array}{l}
a^2 + 2ab + b^2 = (a+b)^2 \\
\qquad\quad a + b \\
\hline
a^3 + 2a^2b + ab^2 \qquad\quad = a \times a^2 + 2ab + b^2 \\
\qquad\quad a^2b + 2ab^2 + b^3 = b \times a^2 + 2ab + b^2 \\
\hline
a^3 + 3a^2b + 3ab^2 + b^3 = (a+b) \times (a^2 + 2ab + b^2) = (a+b)^3
\end{array}$$

(**g.**) But the last three terms $3a^2b + 3ab^2 + b^3 = 3a^2$ times $b + 3ab$ times $b + b^2$ times $b = (3a^2 + 3ab + b^2) \times b$. But the last two terms within the parenthesis $3ab + b^2 = 3a$ times b $+ b$ times $b = (3a + b) \times b$. Hence $(3a^2 + 3ab + b^2) \times b$ $= [3a^2 + (3a + b)b] \times b$ and $(a+b)^3 = a^3 + 3a^2b + 3ab^2$ $+ b^3 = a^3 + [3a^2 + (3a + b)b] \times b$.

(**h.**) Subtracting a^3 from this leaves $[3a^2 + (3a + b)b] \times b$, which shows that —

(**i.**) If from the cube of the sum of two numbers we subtract the cube of the first number, the remainder is the product of two factors, one of which, b, is the second number, and the other, $3a^2$ $+ (3a + b) \times b$, is the sum obtained by adding to 3 times the square of the first number, the product of 3 times the first number plus the second multiplied by the second.

139. Method of extracting the Cube Root.

1. What is the cube root of 279726264?

Solution.—This number contains three periods of three figures each, which shows that there are three figures in its root. Moreover, the greatest cube in 279 is the cube of the highest denomination of the root, and the greatest cube in 279726 is the cube of the two highest denominations of the root. Hence, we may find the first two figures of the required root by finding the cube root of 279726 *as though it were units.*

The greatest cube below 279 is 216, the root of which, 6, is the first figure of the required root. Subtracting 216 from 279, and annexing the next period to the remainder, gives 63726, which for convenience we call a dividend.

This dividend contains the remaining part of the cube of the two highest denominations of the root, and must therefore (see **138**, i) contain the product of two factors, one of which is the sum obtained by adding three times the square of the first figure of the root to the product of three times the first figure of the root plus the second multiplied by the second, and the other is the second figure of the root.

If we knew the first of these factors, which we will call the TRUE DIVISOR, we could find the other by dividing 63726 by it. But as we only know the value of the highest denomination, we have to make three times its square (which will be the largest part of the true divisor) a TRIAL DIVISOR.

3 times $6^2 = 108$, but as 6 is tens with reference to the next denomination of the root, the 108 (being 3 times the square of 6 tens) must be hundreds, and in dividing by it we must drop the two right-hand figures of the dividend. Dividing the 637 by 108 gives 5 for a quotient, which is either equal to or larger than the next figure of the root.*

To determine it, we complete the true divisor by adding to the trial divisor 3 times the first figure of the root plus the second, multiplied by the second (see **138**, i). Three times 6 = 18, which we regard as tens. 18 tens + 5 = 185, and this multiplied by 5 is 925, which, added to the trial divisor, 108 hundreds, gives 11725 for the true divisor.

Multiplying the true divisor by 5 gives 58625, which, being less than the dividend, shows that 5 is really the second figure of the root. Hence, 65 represents the two highest denominations of the root, and, as the root contains three figures, the 65 must be tens.

Subtracting 58625 from 63726, and annexing the next period to the remainder, gives 5101264 for a new dividend, which contains the remaining part of the cube of the tens and units, and must therefore contain the product of two factors, such as described in **138**, i. We therefore proceed to form a trial divisor.

We may do this the most easily by adding together the square of the last root figure found, the last true divisor, and the number which was added to the last trial divisor, which gives $25 + 11725 + 925 = 12675 = 3$ times 65^2, for the next trial divisor. This trial divisor, since the 65 is tens, must be hundreds.

Dropping the two right-hand figures of the dividend, as before, and dividing the remaining part, 51012, by the trial divisor, gives 4 for a quotient, which is either equal to or larger than the units figure of the root.

To complete the true divisor, we multiply 65, the part of the root before found by 3, annex the last root figure, 4, to the product, and multiply the result by 4. This gives 7816, which added to the trial divisor, 12675 hundreds, gives 1275316 for the true divisor.

Multiplying this true divisor by 4 gives 5101264 for a product, which, being equal to the dividend, shows that 4 is really the last figure of the root, or that the required root is 654.

---

* For the trial divisor is always smaller than the true divisor.

The work may be written thus:

		279726264 (654 = Cube Root.
		216
1st trial divisor	108)	63726 = First dividend.
185 × 5 =	925	
1st true divisor	11725	58625 = First true divisor multiplied by 5.
5 × 5 =	25	
2d trial divisor	12675)	5101264 = Second dividend.
1954 × 4 =	7816	
2d true divisor	1275316	5101264 = Second true divisor multiplied by 4.

(a.) These principles are of universal application, and furnish the following rules —

1st. Divide the given number into periods of three figures each, beginning with the units. The left-hand period may contain one, two, or three figures.

2d. Find the greatest cube in the left-hand period, and place its root as the first figure of the required root.

3d. Subtract this cube from the left-hand period, and to the remainder bring down the next period, calling the result a dividend.

4th. Find three times the square of the part of the root already found, and make it a TRIAL DIVISOR.

5th. See how many times the trial divisor is contained in the dividend, excepting the two right-hand figures, and write the quotient as the next figure of the root.

6th. To three times the part of the root previously found, annex the last root figure, multiply the result by the last figure, and, placing the product under the trial divisor, but two places further to the right, add it to the trial divisor. This will give the true divisor.

7th. Multiply the true divisor by the last root figure, placing the product under the dividend.

8th. Subtract the product from the dividend, and to the remainder annex the next period for a new dividend.

9th. Add together the square of the last quotient figure, the last true divisor, and the number standing over the last true divisor. The sum will equal three times the square of the root already found, and will be the second trial divisor.

10th. Now proceed as with the first trial divisor.

(b.) What is the cube root of —

2. 592704?
3. 912673?
4. 59319?
5. 17576?
6. 219256227?
7. 340068392?
8. 6434856?
9. 145531576?
10. 10460353203?
11. 125300240064?
12. 106429035419?
13. 1371330631?

140. Cube Root of Fractions.

(a.) To extract the cube root of a fraction, we extract the cube root of its numerator for a new numerator, and the cube root of its denominator for a new denominator.

(b.) If its numerator and denominator are not both perfect cubes, we can only get the approximate value of its cube root.

(c.) If the *denominator* is not a perfect cube, both terms should be multiplied by the square of the denominator, or by such other number as will make it so.

ILLUSTRATION. — $\sqrt[3]{\frac{8}{45}} = \sqrt[3]{\frac{8 \times 45 \times 45}{45 \times 45 \times 45}} = \frac{\sqrt[3]{16200}}{45} = \frac{25}{45} = \frac{5}{9}$ nearly.

(d.) Find the exact or approximate cube roots of the following fractions —

1. $\frac{343}{729}$.
2. $\frac{125}{512}$.
3. $\frac{27}{32}$.
4. $\frac{5}{9}$.
5. $\frac{18}{625}$.
6. $\frac{43}{47}$.

(e.) In order that a *decimal fraction* may have a perfect cube for its denominator, its numerator must contain three decimal places, or some multiple of three.

ILLUSTRATIONS.—The denominators of .008, 2.017, and .571632 are perfect cubes, but the denominators of .08, .2017, and 57.1632 are not.

(f.) Hence, to extract the cube root of a decimal fraction, annex zeroes if necessary to make its number of decimal places some multiple of 3. Then extract its root as in whole numbers, observing that there will be one decimal place in the root for every three in the given fraction. The root may be found to any number of decimal places by annexing three zeroes to the fraction for every additional figure desired in its root.

What is the cube root to 5 places of—

7. .6?
8. 8.7?
9. 5978?
10. 2?
11. 4.7863?
12. 60?
13. .00007?
14. 3.21?
15. 543.768?

SECTION XX.

MENSURATION

141. Plane Figures.

(a.) A Triangle is a figure having three sides and three angles

(b.) A Right-Angled Triangle is a triangle having a right angle.

(c.) The Angle, Rectangle, and Square are defined on pages 25 and 26.

(d.) Lines are Parallel when they lie in the same direction, as, for instance, the lines forming the sign of equality.

(e.) A Parallelogram is a four-sided figure, having its opposite sides parallel.

(f.) A Trapezoid is a four-sided figure, having two of its sides parallel.

(g.) A Polygon is a figure bounded on all sides by straight lines.

FIG. 1.

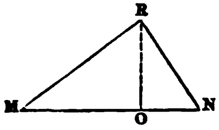


FIG. 2.

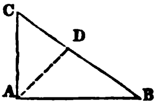


FIG. 3.

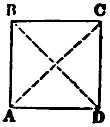


FIG. 4.

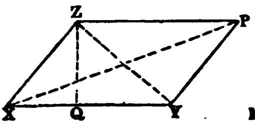


FIG. 5.

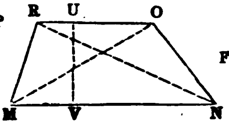
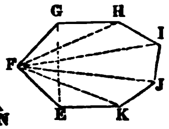


FIG. 6.



(h.) Similar figures are those which have the same shape, *i. e.* which have the angles of the one equal to the corresponding angles of the other, and the sides about the equal angles proportional.

(i.) In similar figures, then, the first side of the one is to the first side of the other as the second side of the one is to the second side of the other.

(j.) The BASE of a figure is the side on which it is supposed to stand. Any side of a figure may be regarded as its base. The side opposite the base of a parallelogram is often called its upper base.

(k.) The ALTITUDE of a triangle is the perpendicular distance from the side assumed as its base to the vertex of the opposite angle.

ILLUSTRATION. — In figure 2, when A B is the base, A C is the altitude; when B C is the base, A D is its altitude; and when A C is the base, A B is the altitude.

(l.) The ALTITUDE of a rectangle, a parallelogram, or a trapezoid, is the perpendicular distance between its parallel bases.

ILLUSTRATIONS. — A B is the altitude of figure 3, Z Q is the altitude of figure 4, and U V of figure 5.

(m.) A DIAGONAL of a polygon is a line connecting the vertices of two angles not adjacent.

ILLUSTRATIONS.—B D and A C are diagonals of figure 3; Z Y and X P are diagonals of figure 4; F K, F J, F I, and F E are diagonals of figure 6.

(n.) The AREA of a square or of a rectangle equals its length multiplied by its breadth.

(o.) The area of a triangle equals half the product of its base by its altitude.

ILLUSTRATION.—In figure 1, if the base, M N, is 8 ft., and the altitude, R O, is 5 ft., the area will be ½ of 8 times 5 sq. ft. = 20 sq. ft.

(p.) The area of a triangle is also equal to the square root of the product obtained by multiplying half the sum of its three sides by the three remainders found by subtracting each side separately from the half sum of the sides.

ILLUSTRATION.—To find the area of figure 1, when M N = 8 ft., R N = 5 ft., and M R = 7 ft., we proceed thus: ½ of the sum of the three sides = ½ of 8 + 5 + 7 = ½ of 20 = 10. This half sum minus the first side = 10 — 8 = 2. This half sum minus the second side = 10 — 5 = 5. The half sum minus the third side = 10 — 7 = 3. The product of the half sum and the three remainders = 10 × 2 × 5 × 3 = 300, and the square root of 300 = 17.3205 +. Hence, the area of the triangle is 17.3205 sq. ft.

(q.) The area of a parallelogram equals the product of its base by its altitude.

ILLUSTRATION.—If, in figure 4, the base, X Y, equals 12 ft., and the altitude, Z Q, equals 8 ft., the area will equal 12 times 8 sq. ft. = 96 sq. ft.

(r.) The area of a trapezoid equals half the product of its altitude by the sum of its parallel bases.

ILLUSTRATION.—If the lower base, M N, of figure 5 equals 12 ft., and the upper base, R O, equals 9 ft., and the altitude, U V, equals 8 ft., the area will equal ½ of the product of 8 multiplied by the sum of 12 and 9 = ½ of 8 times 21 = 84 sq. ft.

(s.) The area of an irregular polygon can be found by dividing it into triangles.

ILLUSTRATION.—The area of figure 6 equals the sum of the triangles F H G, F H I, F J I, F J K, and F K E, into which it is divided.

(t.) The areas of different triangles, squares, and parallelograms are to each other as the product of their bases by their altitudes.

Illustration.—The area of the triangle M R N, figure 1, is to the area of A C B, figure 2, as M N times R O is to A B times A C.

(u.) The areas of similar polygons are to each other as the squares of their like dimensions.

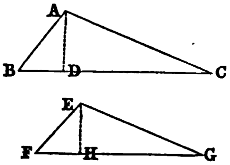


Illustration.—If the two triangles A B C and E F G are similar, the area of A B C will be to the area of E F G as A B^2 is to E F^2, or as B C^2 is to F G^2, or as A C^2 is to E G^2, or as A D^2 is to E H^2. If the side A B equals 5 ft., and the corresponding side, E F, equals 4 ft., the area of the first triangle will be to that of the second as 25 is to 16, or the area of A B C will be $\frac{25}{16}$ of the area of E F G.

(v.) The circumference of a circle equals very nearly 3.1416* times its diameter.

Illustration.—The circumference of a circle 9 ft. in diameter is 3.1416 times 9 ft. The diameter of a circle 20 ft. in circumference is $\frac{1}{3.1416}$ of 20 ft.

(w.) The circumferences of circles are to each other as their diameters or radii.

Illustration.—The circumference of a circle 4 ft. in diameter is $\frac{4}{5}$ of the circumference of a circle 5 ft. in diameter.

(x.) The area of a circle equals half the product of its circumference by its radius, or quarter the product of its circumference by its diameter.

Illustration.—If the circumference of a circle is 25.1328 ft., its diameter will be 8 ft. and its radius 4 ft. Its area then will equal $\frac{1}{4}$ of 8 times 25.1328 sq. ft., or $\frac{1}{2}$ of 4 times 25.1328 sq. ft. = 50.2656 sq. ft.

---

* More accurately, 3.141592653589; but the above is sufficiently exact for most purposes. Indeed, $3\frac{1}{7}$ is sometimes used where no great degree of accuracy is required.

(y.) The area of a circle also equals the square of its radius multiplied by 3.1416.

ILLUSTRATION.—If the radius of a circle is 4 ft., its area will equal $4^2 \times 3.1416$ sq. ft. $= 16 \times 3.1416$ sq. ft. $= 50.2656$ sq. ft.

(z.) The areas of circles are to each other as the squares of their diameters or of their radii.

ILLUSTRATION.—The area of a circle 6 ft. in diameter is 4 times the area of a circle 3 ft. in diameter, because the square of 6 is 4 times the square of 3. The area of a circle of 5 ft. radius is $\frac{25}{4}$ of the area of a circle of 2 ft. radius, because the square of 5 is $\frac{25}{4}$ of the square of 2.

NOTE.—The student should draw figures to correspond with the problems. When only approximate values can be obtained, the work may be carried out to 4 decimal places.

(aa.) Find the area of—

1. A triangle whose base is 8 ft. and altitude is 6 ft.
2. A triangle whose base is 17 ft. and altitude is 11 ft.
3. A triangle of which the sides are 12 ft. 8 ft. and 7 ft.
4. A triangle of which the sides are 8 ft. 3 ft. and 10 ft.
5. An equilateral triangle 8 ft. on a side?
6. An equilateral triangle 13 ft. on a side.
7. A parallelogram whose base is 18 ft. and altitude is 9 ft.
8. A parallelogram whose base is $16\frac{3}{4}$ ft. and altitude is $5\frac{1}{3}$ ft.
9. A trapezoid whose bases are 8 ft. and 7 ft. and altitude is 6 ft.
10. A trapezoid whose bases are 27 ft. and 26 ft., and altitude is 13 ft.
11. A rectangle 64 ft. long and 39 ft. wide.
12. A square 67 ft. on a side.
13. A circle 12 ft. in diameter.
14. A circle 100 ft. in diameter.
15. A circle of 8 ft. radius.
16. A circle of 100 ft. radius.
17. What is the side of a square containing 8649 sq. ft.?
18. What is the side of a square containing 379 sq. ft.?

19. What is the base of a triangle 100 ft. in area and 20 ft. in altitude?

20. A rectangle containing 15138 sq. ft. is twice as long as it is wide. What is its length?

SUGGESTION.—Divide it into squares.

21. A rectangle containing 41067 ft. is three times as long as it is wide. What is its length?

22. A rectangle containing 384 sq. ft. is ⅔ as wide as it is long. How long is it?

23. Find the side of a square equal in area to a triangle having a base of 50 ft. and an altitude of 72 ft.?

24. What is the circumference of a circle 50 ft. in diameter?

25. What is the diameter of a circle 34.5576 ft. in circumference?

26. What is the diameter of a circle 50 ft. in circumference?

27. What is the area of a circle 31.416 ft. in circumference?

SUGGESTION.—First find the diameter or the radius.

28. What is the area of a circle 4084.08 ft. in circumference?

29. What is the area of a circle 50 ft. in circumference?

30. What is the area of a circle 72 ft. in circumference?

31. What is the diameter of a circle containing 78.54 sq. ft.?

32. What is the diameter of a circle containing 109956 sq. ft.?

33. A parallelogram 20 ft. in altitude contains as much surface as a circle 20 ft. in diameter. What is its base?

34. The base of a triangle is 60 ft., and it contains as much surface as a circle 75 ft. in circumference. What is its altitude?

35. What is the side of a square containing 4 times as many sq. ft. as a circle 36 ft. in diameter?

36. What is the radius of a circle containing 9 times as much surface as a circle 10 ft. in diameter?

37. How will the area of a circle 6 inches in diameter compare with the area of a circle 2 inches in diameter?

38. If an iron rod 1 inch in diameter and 10 ft. long weighs 25.5664 lb., how much will an iron rod 2 inches in diameter and 5 ft. long weigh?

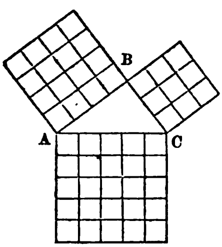
39. If a circular garden 100 ft. in diameter costs \$100, what will a circular garden 300 ft. in diameter cost at the same rate?

40. If a pipe 1 inch in diameter will empty a cask in 1 minute, how long will it take a pipe $\frac{1}{2}$ inch in diameter to empty it?

41. I bought 5 lots of land, each containing 1 acre. The first was in form a circle; the second was a square; the third was a rectangle twice as long as wide; the fourth was a triangle whose base was equal to its altitude; and the fifth was a trapezoid whose altitude was equal to $\frac{1}{2}$ the sum of its parallel bases. It is required to find in feet the diameter of the circle, the side of the square, the sides of the rectangle, the base and altitude of the triangle, and the altitude of the trapezoid.

142. Properties of the Right-angled Triangle, with Problems.

(a.) The side opposite the right angle of a right-angled triangle is called the hypothenuse. The square of the hypothenuse equals the sum of the squares of the other two sides.



NOTE.—This is illustrated by the annexed figure, and can be rigidly demonstrated by geometry.

A B C represents a right-angled triangle, right angled at B. Then will $AB^2 + BC^2 = AC^2$. If A B equals 4 ft. and B C equals 3 ft., AC^2 will equal $4^2 + 3^2 = 16 + 9 = 25$, and AC^2 will equal $\sqrt{16 + 9} = \sqrt{25} = 5$ ft.

(b.) What is the missing side of a right-angled triangle whose—

1. Base is 8 ft. and altitude 6 ft.?

ANSWER.—$\sqrt{8^2 + 6^2} = 10$.

2. Base is 15 ft. and altitude 20 ft.?
3. Base is 20 ft. and altitude 25 ft.?
4. Base is 8 ft. and hypothenuse 10 ft.?

ANSWER.—$\sqrt{100 - 64} = 6$.

5. Base is 20 ft. and hypothenuse 25 ft.?

6. Altitude is 32 ft. and hypothenuse 40 ft.?

7. Altitude is 30 ft. and hypothenuse 60 ft.?

8. Base is 90 ft. and hypothenuse 100 ft.?

9. Base and altitude are each 1 ft.?

10. If I travel east 12 hours at the rate of 4 miles per hour, and then travel north 12 hours at the rate of $5\frac{1}{3}$ miles per hour, how far shall I be from the starting-point?

11. What is the diagonal of a square measuring 50 ft. on a side?

12. What is the side of a square the diagonal of which is 50 ft.?

13. A rectangle is $\frac{3}{4}$ as wide as it is long, and contains 768 sq. ft. What is the length of its diagonal?

14. What is the side of the greatest square that can be inscribed in a circle 20 ft. in diameter?

NOTE.—The diameter of the circle will equal the diagonal of the inscribed square.

15. What is the side of the greatest square that can be inscribed in a circle 50 ft. in diameter?

16. What is the distance from the lower corner to the farthest upper corner of a hall 40 ft. long, 30 ft. wide, and 18 ft. high?

17. A church-spire, 180 ft. in height, stands on a plain. North of the spire, and at a distance of 300 ft. from its summit, is a stake. Measuring east from this stake a distance of 400 ft., I find that I am just 80 feet from the foot of a column which is directly between me and the spire. Allowing that the column is 15 ft. high, how many feet is it from the top of the column to the top of the spire?

143. Solids.

(a.) A SPHERE is a solid bounded by a curved surface, every part of which is equally distant from a point within, called the centre. (Fig. 1, p. 240.)

(b.) A line drawn from the centre of a sphere to the surface is called a RADIUS, and a line drawn from any point in the surface through the centre to the opposite point is called a DIAMETER (c.) A Prism is a solid having its lateral faces parallelograms and its bases equal and parallel polygons. (See Fig. 2.)

Note.—A Cube (defined in 25, e) is a kind of prism.

(d.) A Cylinder is such a solid as would be formed by revolving a rectangle about one of its sides. It has also been defined to be "a round body with circular ends." (See Fig. 3.)

(e.) A Pyramid is a solid bounded laterally by triangles, of which the vertices meet at a common point, and the bases form the sides of a polygon, which is called the base of the pyramid. (See Fig. 4.)

(f.) A Cone is a solid which has a circular base, and tapers regularly to a point called the vertex. (See Fig. 5.)

(g.) A Frustum of a cone or of a pyramid is a part cut off by a plane parallel to its base. (See Fig. 6.)

(h.) Similar Solids have the same shape, *i. e.* the angles of one of them equal the corresponding angles of the other, and the sides about the equal angles are proportional.

(i.) All spheres are similar. Two cones or two cylinders are similar when their altitudes are to each other as the radii or diameters of their bases.

Fig. 1.

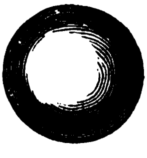


Fig. 2.

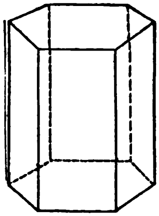


Fig. 3.

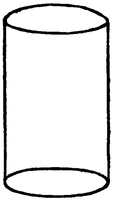


Fig. 4.

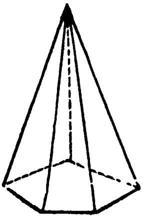


Fig. 5.

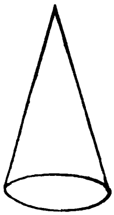
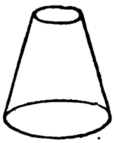


Fig. 6.



(j.) The SURFACE of a sphere equals the square of its diameter multiplied by 3.1416.*

ILLUSTRATION.—The surface of a sphere 4 inches in diameter = 3.1416 times 16 sq. ft. = 50.2656 sq. ft.

(k.) The SURFACES OF SPHERES are to each other as the squares of their radii or diameters.

ILLUSTRATION.—The surface of a sphere 2 inches in diameter is $\frac{4}{25}$ of the surface of a sphere 5 inches in diameter.

(l.) The SOLIDITY, or SOLID CONTENTS, of a sphere equals the product of the surface multiplied by $\frac{1}{3}$ of the radius, or by $\frac{1}{6}$ of the diameter, or it equals $\frac{1}{6}$ of the cube of the diameter multiplied by 3.1416.*

ILLUSTRATION.—If the diameter of a sphere is 12 feet, its surface will equal 3.1416 times 144 sq. ft., and its solid contents will equal $\frac{1}{6}$ of 12 times as many cubic feet; or $2 \times 3.1416 \times 144$ cu. ft. Or the solid contents will equal $\frac{1}{6}$ of 12^3 times $3.1416 = \frac{1}{6}$ of 1728 times $3.1416 = 288$ times 3.1416 cu. ft. = 904.7808 cu. ft.

(m.) The solidities of spheres are to each other as the cubes of their radii or diameters.

ILLUSTRATION.—A sphere of 3 ft. radius contains $\frac{27}{8}$ as many cubic feet as a sphere of 2 ft. radius.

(n.) The solidities of *similar* solids are to each other as the cubes of their like dimensions.

ILLUSTRATION.—A cube 2 ft. on a side is to a cube 5 ft. on a side as 2^3 is to 5^3, or as 8 is to 125.

(o.) The solidity of a prism or of a cylinder equals the product obtained by multiplying the area of its base by its altitude.

ILLUSTRATION.—If the base of the prism in Fig. 2, or of the cylinder in Fig. 3, contains 8 sq. ft. and the altitude is 5 ft., the prism and the cylinder each contain 40 cu. ft.

(p.) The convex surface of a cylinder is equal to the product obtained by multiplying the circumference of its base by its altitude.

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* See foot-note, page 235.

ILLUSTRATION.—If the circumference of the base of the cylinder in Fig. 3 equals 9 ft., and the altitude of the cylinder equals 4 ft., the convex surface must equal 4 times 9 sq. ft. = 36 sq. ft.

(q.) The solidity of a cone or of a pyramid equals the area of its base multiplied by $\frac{1}{3}$ of its altitude.

(r.) The solidity of a frustum of a cone, or of a pyramid, equals the product of $\frac{1}{3}$ of its altitude multiplied by the sum obtained by adding the areas of its upper and lower bases to the mean proportional between those areas.

NOTE.—The mean proportional of two numbers is the square root of their product. Thus, the mean proportional of 4 and 9 = $\sqrt{4 \times 9} = 6$.

ILLUSTRATION.—If the altitude of the frustum of a cone or of a pyramid is 9 ft., and its upper base contains 18 sq. ft. and its lower base 32 sq. ft., we should find its solid contents by finding the product of $18 + 32 + \sqrt{18 \times 32}$ multiplied by $\frac{1}{3}$ of $9 = (32 + 18 + 24) \times 3 = 74 \times 3 = 222$ cu. ft.

(s.) What is the solidity of—

1. A sphere 8 ft. in diameter?
2. A sphere of 5 ft. radius?
3. A sphere 20 ft. in circumference?
4. A sphere 10 ft. in circumference?
5. A cone 6 ft. high, with a base 3 ft. in diameter?
6. A cone 9 ft. high, with a base 5 ft. in diameter?
7. A cylinder 8 ft. high, with a base 4 ft. in diameter?
8. A cylinder 12 ft. high, with a base 1 ft. in diameter?
9. A prism 10 ft. high, whose base contains 17 sq. ft.?
10. A prism 15 ft. high, whose base contains 20 sq. ft.?
11. A prism 18 ft. high, whose base contains 18 sq. ft.?

12. The frustum of a cone 6 ft. high, 3 ft. in diameter at the upper base, and 4 ft. at the lower?

13. The frustum of a pyramid 10 ft. high, containing 20 sq. ft. in the upper base, and 30 sq. ft. in the lower?

14. What is the diameter of a sphere containing 100 cu. ft.?

15. What is the surface of a sphere containing 50 cu. ft.?

16. What is the surface of a sphere of 3 ft. radius?

(t.) What is the convex surface of —

17. A cylinder 12 ft. high, whose base is 3 ft. in diameter?

18. A cone 7 ft. high, whose base is 4 ft. in diameter?

19. A cone 12 ft. high, and 6 ft. in diameter at base?

20. A sphere of wood 3 inches in diameter weighs 2 lb. What will a sphere of wood 9 inches in diameter weigh?

21. How many spheres 1 inch in diameter will be required to balance a sphere 10 inches in diameter?

22. How many cubic inches of iron are there in a hollow sphere 1 ft. in diameter, if the iron is 1 inch thick?

SUGGESTION.—There will be as many cubic inches of iron as there are in the difference between a sphere 12 inches in diameter and one 10 inches in diameter.

23. How many cubic inches of iron are there in a spherical shell 14 inches in diameter and 2 inches thick?

SECTION XXI.

PROGRESSIONS.

144. Arithmetical Progression.

(a.) A SERIES OF NUMBERS IN ARITHMETICAL PROGRESSION, or an ARITHMETICAL SERIES, is a series of numbers, each of which differs from the preceding by the same number.

(b.) An arithmetical series is obtained by continually adding the same number to, or subtracting it from, any given number.

ILLUSTRATIONS.—We should have —

By adding 3's to 1..........................	1, 4, 7, 10, 13, 16, etc.
By subtracting 2's from 18.................	18, 16, 14, 12, 10, 8.

(c.) If the series is formed by addition, it is called an INCREASING SERIES; if by subtraction, it is called a DECREASING SERIES.

(d.) The numbers composing a series are called the TERMS of the series.

(e.) The difference between the consecutive terms of any series is called the COMMON DIFFERENCE, and is always the number by the addition or subtraction of which the series is formed.

(f.) Since the terms of a series are formed by continual additions or subtractions of the same number, it follows that the second term of any series equals the first, plus or minus the common difference; that the third term equals the first, plus or minus twice the common difference; that the fourth term equals the first, plus or minus three times the common difference, etc.

(g.) Hence, any term of an arithmetical series is equal to the first term, plus or minus the common difference taken one less times than there are terms in the series ending with the required term.

(h.) Moreover, if the first term of an increasing arithmetical series be subtracted from the last, or if the last term of a decreasing series be subtracted from the first, the remainder will be the product of the common difference multiplied by one less than the number of terms.

1. What is the 12th term of the increasing series of which 1 is the first term and 8 is the common difference?. (See g.)

2. What is the 6th term of the decreasing series of which 40 is the first term and 4 is the common difference?

3. What is the common difference of the series of which 1 is the first term and 25 the 8th? (See h.)

4. What is the common difference of the series of which 50 is the first term and 24 is the 14th?

5 How many terms are there in the series having 3 for the first term, 31 for the last, and 2 for the common difference?

6. How many terms are there in the series having 56 for the first term, 32 for the last, and 3 for the common difference?

7. Form a series of 4 terms, of which 3 is the first term and 9 is the last.

8. Form a series of 9 terms, of which 10 is the first and 6 is the last.

145. To find the Sum of an Arithmetical Series.

(a.) If we should invert any series, and add the first term of the result to the first term of the original series, the second term of the result to the second term of the original series, etc. each successive sum would equal the sum of the first and last term of the series. But all these sums taken together would equal the sum of both series, or (since the series are equal) twice the sum of either series.

ILLUSTRATION. — By inverting the series 5, 8, 11, etc. to 20, we have —

5 8 11 14 17 20 = given series.
20 17 14 11 8 5 = same series inverted.

25 25 25 25 25 25 = sums of the successive terms.

Hence, the sum of both series equals 6 × 25, and the sum of one of them equals ½ of 6 × 25, which is 3 × 25 = 75.

(b.) Hence, the sum of a series in arithmetical progression equals half the product obtained by multiplying the sum of the first and last terms by the number of terms.

1. What is the sum of a series of 12 terms, of which 2 is the first term and 35 the last?

2. What is the sum of a series of 10 terms, of which 81 is the first term and 36 is the last?

3. What is the last term and the sum of a series of 15 terms, of which 1 is the first term and 3 the common difference?

4. What is the last term and what the sum of a series of 16 terms, of which 90 is the first term and 4 is the common difference?

5. What is the sum of the numbers from 1 to 100?

6. What is the number of terms and what the common difference of the series of which 42 is the sum, 2 is the first term, and 12 is the last?

7. What is the number of terms and what the common difference of a series of which the sum is 121, the first term is 16, and the last term 6?

8. What is the last term and what the common difference of a series of 12 terms, of which the sum is 138, and the first term is 6?

9. What is the first term and what the common difference of a series of 10 terms, of which the last term is 10, and the sum is 85?

146. Geometrical Progression.

(a.) A series of numbers in geometrical progression, or a geometrical series, is a series of numbers, each of which bears the same ratio to the one which follows it.

(b.) Such a series would be obtained by continually multiplying by the same number.

ILLUSTRATIONS.—Beginning with 3 and multiplying by 2, we should have the series 3, 6, 12, 24, 48, 96, etc.

Beginning with 1728 and multiplying by $\frac{1}{4}$, we have the series 1728, 432, 108, 27, etc.

(c.) The numbers comprising the series are called the TERMS OF THE SERIES.

(d.) The ratio of each term to that which follows it is called the COMMON RATIO, and is always the number by which we should multiply to produce the series.

(e.) In an increasing series, the common ratio equals a whole number or an improper fraction; but in a decreasing series, the common ratio equals a proper fraction.

(f.) From the method of forming such a series, it is obvious that the second term must equal the first multiplied by the common ratio; that the third term must equal the first multiplied by the second power of the common ratio, etc.

(g.) Hence, any term of a geometrical series must equal the product of the first term multiplied by the common ratio raised to a power one degree less than the number of that term.

(h.) Moreover, if the last term of a geometrical series be divided by the first, the quotient will be the common ratio raised to a power one degree less than the number of the term.

1. What is the 9th term of the series of which 1 is the first term and 2 the common ratio?

2. What is the 8th term of the series of which $\frac{1}{81}$ is the first term and 3 the common ratio?

3. What is the 10th term of the series of which 81 is the first term and $\frac{2}{3}$ the common ratio?

4. What is the common ratio of a series of which 2 is the first term and 512 the 5th?

5. What is the common ratio of a series of which 15625 is the first term and 4096 the 7th?

6. Construct the series of which 16 is the first term and 10000 the 5th.

147. To find the Sum of a Geometrical Series

(a.) If a geometrical series should be multiplied throughout by the common ratio, all the terms except the last of the series thus derived would correspond to all the terms except the first of the given series. Hence, the difference between the original and the derived series would equal the difference between the first term of the original series and the last term of the derived series. But, from the manner in which the derived series is found, this difference must also equal the product of the original series multiplied by the difference between 1 and the common ratio. Hence, dividing the difference between the *first term* and the *common ratio times the last term*, by the difference between the common ratio and 1, must give the sum of the series.

ILLUSTRATIONS.—Multiplying the series 3, 12, etc. to 768 by 4, we have—

$a = 3, 12, 48, 192, 768 =$ given series.
$4a = \quad 12, 48, 192, 768, 3072 =$ derived series.
$4a - a = 3a = 3072 - 3 = 3069 = 3$ times given series. Hence,
$3069 + 3 = 1023 =$ sum of the series.

Multiplying the series 18, 6, etc. to $\frac{2}{27}$ by $\frac{1}{3}$, we have—

$$a = 18,\ 6,\ 2,\ \tfrac{2}{3},\ \tfrac{2}{9},\ \tfrac{2}{27} = \text{given series.}$$
$$\tfrac{1}{3}\,a = \quad 6,\ 2,\ \tfrac{2}{3},\ \tfrac{2}{9},\ \tfrac{2}{27},\ \tfrac{2}{81} = \text{derived series.}$$
$$a - \tfrac{1}{3}\,a = \tfrac{2}{3}\,a = 18 - \tfrac{2}{81} = \tfrac{2}{3} \text{ of the given series. Hence,}$$
$$17\tfrac{79}{81} \div \tfrac{2}{3} = 26\tfrac{26}{27} = \text{sum of the series.}$$

(b.) It thus appears that, to find the sum of a geometrical series, we may multiply the last term by the common ratio, and divide the difference between the product and the first term by the difference between 1 and the common ratio.

1. What is the sum of a series of which 1 is the first term, 256 is the last, and 2 is the common ratio?

2. What is the sum of a series of which 486 is the first term, $\frac{2}{3}$ is the last, and $\frac{1}{3}$ is the common ratio?

3. What is the common ratio and what the sum of a series of 4 terms, of which 9 is the first term and 64827 is the last?

4. What is the common ratio and what the sum of a series of 4 terms, of which 3267 is the first term and 121 the last?

5. If the last term is 64, the number of terms is 8, and the common ratio is $\frac{1}{2}$, what is the sum of the series?

6. If the first term of a geometrical series is 2, the number of terms is 11, and the common ratio is 2, what is the last term, and what the sum of the series?

148. Infinite Decreasing Series.

(a.) Since each term of a decreasing series is smaller than the preceding, it follows that if the series be carried far enough, the terms will become so small that they may be disregarded without affecting sensibly the sum of the series.

(b.) An infinite decreasing series will always be of this character, and hence its sum will equal the quotient obtained by dividing the first term by the difference between 1 and the common ratio.

1. What is the sum of the infinite decreasing series of which 6 is the first term and $\frac{3}{7}$ the common ratio?

ANSWER.—$6 \div (1 - \frac{3}{7}) = 6 \div \frac{4}{7} = 10\frac{1}{2}$.

What is the sum of the infinite decreasing series of which—

2. 2 is the first term and $\frac{1}{4}$ is the common ratio?
3. 1 is the first term and $\frac{1}{2}$ is the common ratio?
4. 5 is the first term and $\frac{2}{5}$ is the common ratio?
5. $\frac{3}{4}$ is the first term and $\frac{3}{4}$ is the common ratio?
6. 1 is the first term and .01 is the common ratio?
7. .162 is the first term and .001 is the common ratio?
8. 1 is the first term and .0001 is the common ratio?
9. .4783 is the first term and .0001 is the common ratio?

SECTION XXII.

149. ALLIGATION.

(a.) IT is sometimes desirable to mix articles of the same kind but of different values together, so as to obtain a compound differing in value from any of its ingredients. The problems connected with this subject are called problems in ALLIGATION.

(b.) They are usually considered as coming under one of two classes, viz.—

(c.) ALLIGATION MEDIAL, in which the quantities of the several ingredients and their prices are given, and we are required to find the price of the mixture per pound, per gallon, or per bushel.

(d.) ALLIGATION ALTERNATE, in which the prices of the various ingredients are given, and we are required to find what quantities of each must be taken to make a mixture having a given value per pound, per bushel, or per gallon.

1. A trader mixed together 12 lb. of sugar worth 8 cents per lb., 10 lb. worth 9 cents per lb., and 17 lb. worth 12 cents per lb. What was the mixture worth per lb.?

SOLUTION. — 12 lb. at 8 cents are worth 96 cents.
10 lb. at 9 " 90 "
17 lb. at 12 " 204 "

Hence, the 39 lb. are worth $3.90, and 1 lb. is worth 10 cents.

2. A liquor dealer mixed together 100 gallons of liquor worth 35 cents per gallon, 100 gallons worth 42 cents per gallon, 100 gallons worth 40 cents per gallon, and 100 gallons of water. What was the mixture worth per gallon?

3. A man bought 500 gallons of vinegar at 12 cents per gallon, and 400 gallons at 12½ cents per gallon, and, after mixing the whole with 100 gallons of water, sold it at 13 cents per gallon. What did he gain per gallon on the mixture, and how much did he gain in all?

NOTE. — The above questions are in ALLIGATION MEDIAL. Those which follow are in ALLIGATION ALTERNATE.

(e.) Problems in Alligation Alternate are solved by determining what proportion of the various kinds must be taken in order to make the gains on those kinds which are worth less than the mean price of the mixture, equal the losses on those which are worth more. The things to be done, then, are —

1st. To determine the gain or loss on a unit of each kind.

2d. To select such number of units as may be convenient of kinds which cost either less or more than the mean price, and determine the gain or loss on them.

3d. To determine what quantities of the remaining kinds shall be taken to counterbalance this gain or loss?

4. A merchant has sugars at 6, 8, 11, and 13 cents per pound, and he wishes to make a mixture which shall be worth just 10 cents per pound. How many pounds of each kind may he take?

SOLUTION.—By selling the mixture at 10 cents per pound, he gains 4 cents per pound on the first kind and 2 cents per pound on the second; while he loses 1 cent per pound on the third kind and 3 cents per pound on the fourth. Suppose that he takes 12 lb. of the first and 9 lb. of the second; then he will gain 12 times 4 cents, or 48 cents, on the first, and 9 times 2 cents, or 18 cents, on the second, and his total gains will be 48 + 18, or 66 cents. He must now take enough of the others to lose 66 cents. Suppose that he takes 15 lb. of that at 11 cents; he will lose 15 cents on it, and he must take enough of that at 13 cents to lose 66 — 15, or 51 cents. But as he loses 3 cents on one pound, he must take as many pounds as there are times 3 in 51, which are 17 times. Hence, he must take 17 lb. of the last kind.

The following is a convenient form of writing the work —

Mean Price.	Prices of Ingredients.	Gain or Loss per pound.	Quantities taken.	Sum of Gains or Losses.
10	6	+ 4	12 lb. gain 48 cts.	66 cts. gain.
	8	+ 2	9 lb. gain 18 cts.	
	11	— 1	15 lb. lose 15 cts.	66 cts. loss.
	13	— 3	17 lb. lose 51 cts.	

Proof. — See if 12 lb. at 6 cents per lb., 9 lb. at 8 cents per lb., 15 lb. at 11 cents per lb., and 17 lb. at 13 cents per lb., will make a mixture worth 10 cents per pound.

Note. — There is no limit to the number of answers which may be obtained to such problems as the above, for however many or few pounds of one kind we may take, we can take enough of other kinds to counterbalance the gain or loss.

Below is a part of two other solutions to the last problem. Let the pupil supply the missing number, and prove the correctness of his result.

Quantities taken.	Sum of Gains and Losses.	Quantities taken.	Sum of Gains and Losses.
11 lb. gain $.44	$.68 gain.	47 lb. gain $1.88	$2.30 gain.
lb. gain $.24		21 lb. gain $ .42	
17 lb. lose $.17	$.68 loss.	93 lb. lose $ .93	$ loss.
17 lb. lose $.51		lb. lose $	

5. I have grains worth respectively 42, 46, 52, and 60 cents per bushel. How many bushels of each kind may I take to make a mixture worth 50 cents per bushel?

6. I have coffees worth respectively 10, 13, and 16 cents per pound. How many pounds of each kind may be taken to make a mixture worth 12 cents per pound?

7. A trader wishes to mix 100 gallons of molasses worth 35 cents per gallon, with enough water to make the mixture worth 30 cents per gallon? How many gallons of water must he take?

8. How many gallons of oil worth 55 cents per quart must be mixed with 50 gallons worth 46 cents per quart, to make a mixture worth 50 cents per quart?

9. How many pounds of tea worth 40 cents per pound must be mixed with 12 lb. worth 30 cents per lb., 10 lb. worth 48 cents per lb., and 20 lb. worth 55 cents per lb., to make a mixture worth 45 cents per lb.?

10. I have raisins at 8, 9, 12, and 13 cents per lb. How many pounds of each kind must I take to make a mixture of 100 pounds worth 10 cents per pound?

SUGGESTION.—First get a set of answers, as though the number of pounds in the mixture was not specified. Then find how many times as much must be taken to give the required quantity.

11. I have spices at 20, 25, 33, 35, and 37 cents per lb. How many pounds of each may be taken to produce a mixture of 250 lb. worth 30 cents per lb.?

12. I have oats worth 40 cents per bushel, corn worth 60 cents per bushel, and barley worth 90 cents per bushel, how many bushels of each may I take to make a mixture of 100 bushels worth 65 cents per bushel?

SECTION XXIII.

150. MISCELLANEOUS PROBLEMS.

1. A SUM of money was divided between A and B, in such a way that A had $846, which was $39 more than $\frac{2}{3}$ of what B had. How many dollars had B?

2. How many feet of boards 1 inch thick will it take to make a box, the outside dimensions of which are 8 ft. 7′ long, 6 ft. 5′ wide, and 2 ft. high?

3. Multiply .125 by 22.5, and extract the square root of the result.

4. If the minuend is 4788, and the subtrahend is twice the remainder, what is the remainder?

5. If the minuend is 963, and the remainder is 127 more than the subtrahend, what is the remainder?

6. The sum of the minuend and the subtrahend is 1785, which is 849 more than the remainder. What is the subtrahend and what the remainder?

7. If the subtrahend is 100 more than twice the remainder, and their sum is 400, what is the subtrahend and what the remainder?

8. If the excess of the subtrahend over the remainder is $\frac{1}{3}$ of the subtrahend, and the remainder is 96, what is the subtrahend and what the minuend?

9. I bought apples at the rate of 3 for 2 cents, and sold them at the rate of 2 for 3 cents. What per cent did I gain?

10. What is the value of $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{5}{6} + \frac{7}{8} + \frac{9}{10} + \frac{11}{12} + \frac{14}{15} + \frac{19}{20}$?

11. If my house-lot were 15 feet longer than it now is, it would contain 1200 more square feet than now; while, if it were 15 feet wider, it would contain 1275 more square feet than now. How many square feet does it contain?

12. I bought goods to the amount of $1000, giving in payment my note at 6 months. When 2 months had passed, I sold the goods for $1000 cash, and "shaved" the note I had given for them at 2 per cent per month discount. How many dollars did I gain by the transaction?

13. If the least common multiple of two numbers is their product, what is their greatest common divisor?

14. A farmer bought $6\frac{2}{3}$ yards of cloth, giving in part payment $12\frac{3}{4}$ dozen of eggs at 16 cents per dozen. Now, allowing that the eggs were sufficient to pay for $1\frac{1}{5}$ yards of cloth, what was the value of the cloth which he bought?

15. I bought 2 house-lots, paying for the second $\frac{3}{5}$ as much as for the first. I paid $248 more for the first than for the second. What was the cost of each?

16. What number is that which, increased by $\frac{1}{2}$ of itself, gives for a result 150 less than 3 times itself?

17. If the multiplier is $\frac{8}{11}$ of the multiplicand, and the square of the product is 627264, what is the multiplier and what the multiplicand?

18. If I buy 24 yards of cloth at the rate of 3 yards for a dollar, and 24 yards at $3 per yard, and sell the first lot at the rate of 2 yards for a dollar and the second at $2 per yard, shall I gain or lose, and how many dollars?

19. The town of B is 36 miles due north from the town of A; C is 27 miles due east from B; D is 20 miles due south from C; E is 39 miles due west from D. How far is each of the above-named towns from A?

20. Multiply the sum of $\frac{2}{3}$ and $\frac{3}{4}$ by the difference of $\frac{3}{4}$ and $\frac{2}{3}$, divide the result by the product of $\frac{2}{3}$ and $\frac{3}{4}$, and multiply the result last obtained by the quotient of $\frac{2}{3}$ divided by $\frac{3}{4}$.

21. How many days will it take John and George together, to do a piece of work which John alone can do in 20 days, and which George alone can do in 24 days?

22. A can do a piece of work in 12 days; B can do it in 15 days; A and C together can do it in 8 days; and B and D together can do it in 9 days. In how many days can C and D together do it?

23. By what number must 9 be divided to give 1.6 for a quotient?

24. $\frac{1}{2}$ of a certain number exceeds $\frac{1}{3}$ of that number by 4 more than $\frac{1}{10}$ of the number. What is the number?

25. I bought a house for a certain sum, and, after expending $1000 in repairing it, I sold it at an advance of 25 per cent on its total cost, by which I gained a sum equal to $33\frac{1}{3}$ per cent of what I bought it for. How many dollars did I gain?

26. What per cent shall I gain by buying bank stock at 25 per cent below par, and selling it at 10 per cent below par?

27. If a number be multiplied by itself, and the product be added to 47, the result will equal $2\frac{1}{4}$ times the square of 16. What is the number?

28. $\frac{2}{3}$ of my money equals $\frac{3}{7}$ of Mr. Sumner's, and $\frac{4}{5}$ of Mr. Sumner's equals $\frac{6}{25}$ of Mr. Greene's. Now, allowing that Mr. Greene has $50000, how many dollars have I?

29. A's services are worth $1 per day, B's are worth $1.25 per day, and C's are worth $1.50 per day. They together did a piece of work "by the job" for $75, and they succeeded in performing it in 15 days. How ought the money to be divided?

30. Edward and William start at the same time to walk a distance of 32 miles. Edward walks at the rate of 4 miles per hour and rests every third hour, while William walks at the rate of 3 miles per hour and rests every fourth hour. Which will reach his journey's end first, and how much first?

31. One of two poles is just $\frac{3}{4}$ as long as the other, and they are set up in a vertical position at a distance apart equal to the sum of their lengths. Moreover, a point in the ground between them, just $\frac{3}{7}$ of the distance from the foot of the longer pole to the foot of the shorter, is 30 feet from the top of each. What is the length of each pole? How far is the foot of one from the foot of the other? How far apart are their tops?

32. How many cubic inches of iron are there in a spherical shell, of which the outside diameter is 12 inches, and the inside diameter is 11 inches?

33. A cylindrical pail whose outside diameter and outside altitude are each 1 foot, is made of wood $\frac{3}{4}$ of an inch thick. How many cubic inches of wood does the pail contain?

34. How many gallons of water will the pail described in the last problem contain?

35. I bought a cask of spirits at $2 per gallon, but $\frac{1}{4}$ of it having leaked out, I added 10 per cent of water, and sold the mixture at $2.25 per gallon. Did I gain or lose, and how many per cent?

36. I bought a cask of wine, but $\frac{4}{9}$ of it having leaked out, I poured in $\frac{2}{5}$ as much water as there was wine remaining, and then sold the mixture for as much per gallon as I had given for the wine? What per cent did I gain on the cost of the wine actually sold? What per cent did I lose on the cost of the whole?

37. By selling goods at 10 per cent below their cost, I lost $100 more than I should have gained by selling $\frac{1}{2}$ of them at cost, and the other half at 5 per cent above cost. What was the cost of the goods, and how many dollars did I lose?

38. Divide $\frac{2\frac{1}{4}}{70}$ of $\frac{.08\frac{2}{3}}{1.3\frac{5}{7}}$ by $\frac{.105\frac{7}{8}}{3.7\frac{2}{3}}$ of $\frac{.02}{.003}$ of $\frac{.21\frac{1}{3}}{4.8}$.

39. Divide $\frac{.002\frac{4}{7}}{.4\frac{3}{8}}$ of $\frac{.02}{115\frac{5}{7}}$ of $\frac{.7\frac{1}{5}}{.03\frac{9}{13}}$ by $\frac{.00546}{125.7\frac{2}{3}}$ of $\frac{3\frac{11}{15}}{6\frac{1}{4}}$ of $\frac{1}{.013\frac{1}{11}}$.

40. I bought a lot of goods for a merchant residing in Havana, for which I charged a commission of 2 per cent. I also insured their safe transportation to Havana, for which I charged a premium of 1¼ per cent of their cost. My total charges for the goods, my commission, and the insurance, were $7450. What was the cost of the goods? What was my commission? What was the insurance?

41. Jan. 1, 1858, Gould and Newman purchased a cargo of grain for $2500, giving in payment their note on 4 months. Jan. 17th, they sold the grain at an advance of 25 per cent, receiving in payment a note due in 5 mo.; and, Jan. 19th, they paid $43.25 in cash for storage and other expenses. When the note which they had given for the grain became due, they got the one which they had received discounted at a bank, and made a complete settlement. Money being worth 6 per cent per year, what was their gain by the transaction?

42. Aug. 10, 1858, I purchased for cash 75 bags of coffee, each containing 112 lb., at 9 cents per pound. For how much must I sell it, Sept. 1st, on a credit of 3 months, in order that, if I get the note which I receive for it discounted at a bank, Oct. 1st, the proceeds may be sufficient to give me a clear profit of 10 per cent on the investment, allowing interest at the rate of 6 per cent per year?

43. Robert takes 2½ feet, and Charles takes 3 feet, at each step, but Robert takes 4 steps while Charles takes 3. How long will it take Charles to walk as far as Robert can walk in 12 hours?

44. If $1000 should be so divided between Edward, Arthur, and Daniel, that ¼ of Edward's share should equal ½ of Arthur's, and that $\frac{1}{4}$ of the sum of the shares of Edward and Arthur should equal $\frac{1}{2}$ of Daniel's share, what would be the share of each?

45. A man made a will, in which he bequeathed $\frac{1}{3}$ of his estate to his wife, $\frac{3}{5}$ of the remainder to his son, and the rest in equal shares to his three daughters. He further directed that, in case either heir should die before himself, the property should be divided among the survivors in the proportions indicated by the will. It so happened that his wife and one of his daughters died before he did. If the estate was worth $100000, what was the just share of the son and of each daughter?

46. A lot of land containing 100 acres is divided, by lines running east and west, into three lots of equal value. If the land in the northern lot is 10 per cent better than that in the middle lot, and that in the middle lot is 12$\frac{1}{2}$ per cent better than that in the southern, how many acres are there in each lot?

47. A vessel of war left port with provisions enough to last her crew of 700 men 16 months. 2 months after leaving port, she took a prize, on board which she put 50 men, giving them provisions enough to last them 1$\frac{1}{2}$ months. In three months more she made another prize, from which she took 100 men and provisions enough to last 75 men 6 months. In 4 months more, she captured a large Indiaman, on board which she placed 50 prisoners from the former prize, and 25 of her own men. 2 months afterwards she lost 50 men in an engagement with an enemy's vessel, and sprang a leak, so that $\frac{1}{2}$ of her remaining provisions were destroyed. She reached port the day her provisions were exhausted, but, from the time of her last engagement, each man received only $\frac{3}{4}$ of his usual allowance. How long was her cruise?

48. There are three numbers, the first and second of which are prime to each other, and so also are the second and third. If the least common multiple of the first and second is 1617, of the second and third is 2156, and of the first and third is 132, what are the numbers?

49. If A can do $\frac{2}{3}$ of a certain piece of work in 9 days, and B can do $\frac{3}{4}$ of it in 10 days, how many days will it take A and B together to do a piece of work 3.22 times as large?

50. One pipe will fill a cistern in two hours and another will empty it in three. The cistern being empty, in how long time will it be filled if both pipes are left open?

51. How many balls 3 inches in diameter will be required to weigh as much as a ball 27 inches in diameter, all the balls being of the same material?

52. Two towns are 108 miles apart. A starts from the first and B at the same time starts from the second. They travel towards each other, and meet in 14 hours. If A travels only $\frac{4}{5}$ as fast as B, how many miles does each travel per hour?

53. A owes B $450 payable in 8 months, and B owes A $600 payable in 10 months. If A pays $\frac{1}{3}$ of his debt down, and the rest in 15 months, and B pays $\frac{1}{5}$ of his in 3 months, when ought B to pay the rest of his?

54. Why is not selling 60 apples at the rate of 5 for 2 cents equivalent to selling 30 at the rate of 2 for a cent and 30 more at the rate of 3 for a cent?

55. If a steamboat whose machinery moves uniformly goes at the rate of 15 miles per hour in descending a river, and at the rate of 4 miles per hour in ascending it, how many miles does the current run per hour, and how many minutes will it take the steamboat to go a mile where there is no current?

56. If 60 men can dig a certain trench in 30 days in summer, how many men will it take in winter to dig in 21 days a trench $1\frac{2}{3}$ times as long, $1\frac{1}{6}$ times as wide, and $\frac{4}{5}$ as deep, provided that the men can dig only $\frac{3}{4}$ as many feet per hour, and only $\frac{2}{3}$ as many hours per day in winter as in summer?

57. A farmer owns a triangular field, the sides of which are respectively 30, 40, and 35 rods in length. He wishes to divide it into two parts of equal area by a line running parallel to the longest side. What will be the length of each side of the trapezoid thus cut off? (See **141, II.**)

58. The town A is situated on the banks of a certain river, and at such a distance below the town B, that a boat which sails 7 miles per hour with the current and 3 miles per hour against it, can sail from A to B and back in 12 hours. How far apart are the two towns?

59. I hired a laborer for 60 days, on condition that I should pay him $1.50 for every day he worked, and that he should pay me 50 cents for every day he was idle. At the expiration of the time, I owed him $74. How many days did he work?

60. If Lewis earns $$1\frac{1}{4}$ per week more than Herbert, and both together do a job of work in 8 weeks for $94, how ought the money to be divided between them?

61. By one pipe 4 gallons per hour will run into a cistern capable of holding 100 gallons, and by another pipe 3 gallons per hour will run out of it. If the first pipe is opened for one minute and then closed, and the second one opened for one minute, and so alternately, each pipe being left open for a minute at a time, in how many minutes will the cistern be filled?

62. Twice one number equals 3 times another number, and the sum of the numbers is 1000. What are the numbers?

63. By what number must 7 be multiplied to obtain a result equal to the quotient of $\frac{2\frac{3}{7}}{5\frac{1}{6}}$ divided by $\frac{8\frac{1}{2}}{1\frac{3}{4}}$ of $\frac{4\frac{1}{3}}{4\frac{3}{7}}$.

64. Jan. 1, 1858, I bought on speculation a lot of fancy goods for $500 cash. May 1, 1858, I sold the goods, receiving in payment the following note, which I immediately collected.

$1000. Providence, Sept. 29, 1856.

For value received, I promise to pay Alfred Tyler or order one thousand dollars, on demand, with interest.

James Perrin.

On this note were the following indorsements:

Feb. 3, 1857, received one hundred dollars.
June 10, 1857, received two hundred dollars.
Nov. 5, 1857, received ten dollars.
Jan. 1, 1858, received thirty dollars.
March 5, 1858, received two hundred dollars.

Allowing that money was worth 6 per cent per year, did I gain or lose by the speculation, and how much?

65. I bought a lot of cloth for $1000 cash, and sold it at an advance of 25 per cent, on a credit of 6 months, without grace. It turned out that 10 per cent of my sales were bad debts, and that it cost 4 per cent of the remainder to collect it. What was my net gain?

66. A person bought 25 sheep, agreeing to pay 1 cent for the first, 2 cents for the second, 4 cents for the third, and so on, paying for each sheep twice as much as for the preceding one. How much did the whole cost him?

67. How many feet long is the side of the largest square which can be made in a circle 20 ft. in diameter?

68. When coffee was worth only $\frac{2}{5}$ as much per pound as tea, a person bought 48 pounds of coffee and 36 pounds of tea for $17.25. What was the price of each per pound?

69. A and B trade in company. A put in $\frac{1}{4}$ of the capital and B put in the remainder. A was to manage the business, for which he was to have a salary of $800 per year; and it was agreed that, after paying this salary, the profits or losses should be shared in proportion to the capital stock of the partners. Under this arrangement, B's share of the profits for the first year was $1\frac{1}{5}$ times as much as A's share of the profits added to his salary. Moreover, A's share of the profits added to his salary equalled $\frac{1}{3}$ of his share of the original stock. What was each partner's share of the profits? What was the original stock of each?

70. A merchant invested $1000 more than $\frac{1}{2}$ of his capital in trade, $1000 more than $\frac{1}{2}$ of the remainder in real estate, $1000 more than $\frac{1}{2}$ of the remainder in bank stock, $1000 more than $\frac{1}{2}$ of this remainder in railroad stock, and the rest, $1000, in insurance stock. What was his capital?

71. A retail dealer bought a lot of molasses at the rate of $36.10 per hundred gallons. He calculates that 5 per cent of it will waste in measuring; that 12 per cent of it will be bad debts; and that it will cost 5 per cent of the remainder to collect it; and he wishes to sell it for such a price that, after allowing for all these losses, he may gain 10 per cent of his original investment. For how much per gallon must he sell the molasses?

72. A and B have equal incomes. A saves ⅓ of his each year, and B saves ⅕ of his. At the end of 4 years A has saved $1000 more than ½ as much as B has saved. What are their incomes?

73. A and B hired a pasture together for $64. A pastured 3 horses for 5 weeks and 7 cows for 14 weeks. B pastured 2 horses for 6 weeks, 1 cow for 7 weeks, and 5 cows for 12 weeks. Now, if it is worth 1⅔ times as much per week to pasture a horse as it is to pasture a cow, how many dollars ought each to pay?

74. What number is that the square root of which is 36 less than the square root of 4 times the number?

75. By selling a lot of land for as many dollars per acre as there are acres in the field, I gain 25 per cent of its cost. If I had sold it for $2250 less than I did, I should have sold it for 25 per cent less than cost. How many acres did the lot contain, and how much per acre did I give for it?

76. A certain square garden measures 500 feet on a side, and the owner wishes to lay out a walk completely around it, which shall occupy just $\frac{1}{10}$ of the area of the garden. How wide ought the walk to be?

77. Jan. 1, 1858, I bought a lot of shoes for $5000 cash, and immediately shipped them to New Orleans, paying 2 per cent premium for insurance and $1 for the policy. My agent in New Orleans sold them, Feb. 15, at an advance of 50 per cent on their original cost, charging 3 per cent commission for his services. He paid out $100 for freight and other expenses, and invested the sum due me, after deducting a commission of 1½ per cent on the purchase, in cotton. He shipped the cotton to me in Boston, where I sold it, May 1, at an advance of 25 per cent on the sum which my agent paid for it, receiving in payment a note on 3 months. May 1, I paid $1000 in cash for freight, insurance, etc. May 3, I had the note received for the cotton discounted at a bank, and immediately invested ½ the proceeds in railroad stock at a discount of 15 per cent, and the other half in railroad stock at a discount of 30 per cent. July 1, a dividend of 4 per cent was declared on the stock of the first railroad. This dividend I received July 3. Sept 1, I sold the first lot of stock at 5 per cent above par, and the second at 40 per cent below par. Money being worth 6 per cent what was my gain by these transactions?

78. If I sell a lot of goods at an advance of 25 per cent, and 25 per cent of the sales turn out to be bad debts, shall I gain or lose and how many per cent?

79. If, by selling a lot of goods for $556.10, I shall gain 10 per cent, how many dollars shall I gain or lose by selling ½ of the lot for $200, and the other half at an advance of 25 per cent on their cost?

80. A trader calculates that he can sell $20000 dollars worth of goods per year. He estimates that his expenses will be $800 for clerk-hire, $1000 for rent and other store expenses, $250 for transportation of goods, $600 for interest, and $100 for other expenses. At what per cent advance on the cost of his goods must he sell them to make a net gain of $2000 per year?

81. Albert can run 3 rods in 4 seconds, and Charles can run 4 rods in 5 seconds. How many seconds will it take Charles to overtake Albert if Albert has 10 seconds the start?

82. If the base of a right-angled triangle which contains 18769 sq. ft. is twice its altitude, what is its hypothenuse?

83. The square root of a certain number subtracted from that number, leaves a remainder equal to 16 times the square root of the number. What is the number?

84. The difference between the cube root of a number and the number itself is 120 times the cube root of the number. What is the number?

85. A and B traded in company. A put in 1½ times as much money as B. Moreover, the money which A put in was $600 more than ½ of the whole stock. They gained 25 per cent of their investment. What was each man's share of the gain?

86. Whitney & Dean bought a square lot of land, measuring 350 feet on a side, at 2 cents per square foot. They cut two roads, each 50 ft. wide, through the middle of it, at right angles to each other and to the sides of the field. At how much per square foot must they sell the rest to make a gain of 150 per cent on their original investment, allowing nothing for interest or expenses?

87. I gave my note on 3 months at a bank for such an amount that I realised $4500 on it, and invested the proceeds in flour at $6 per barrel. After incurring expenses on the flour to the amount of 10 cents per barrel, I sold it at an advance of 20 per cent on its first cost; and, after paying the above-named expenses, invested the remainder in cloth at 50 cents per yard. This cloth became so much damaged by a fire, that I was obliged to sell it for ½ what it cost me. I paid $50 for expenses on account of the cloth, and invested the remainder in silks at $1.25 per yard. The very day my note at the bank became due, I sold the silks for cash at an advance of 50 per cent on their cost. Did they bring more or less than the amount of the note due at the bank, and how much?

88. I bought a lot of goods for $250 cash, and at the expiration of 6 months I sold them at an advance of 10 per cent, receiving in payment a note on 60 days. This I immediately got discounted at a bank. What was my real gain, money being worth 6 per cent?

89. If a cubic foot of water weighs 62½ pounds, how many pounds of water will a cylindrical vessel hold, if the inside diameter of its base is 3 ft. and its inside altitude is 6 ft.?

90. A speculator purchased a lot of land. At the end of the first year, its value had increased 25 per cent. At the end of the second year, its value was 30 per cent more than at the end of the first. At the end of the third year, he sold it for 10 per cent less than its value at the end of the second year, receiving in payment a note on 4 months. This note he got discounted at a bank, receiving $9795 on it. Money being worth 6 per cent compound interest, what was his profit on the land?

91. The hour and minute hands of a watch are together at 12 o'clock. When are they next together?

92. At what time between 3 and 4 are the hour and minute hands of a watch together?

93. At what time between 6 o'clock and 7 do the hour and minute hands of a watch make equal angles with the 6 o'clock mark?

94. At what time between 12 o'clock and 1 does the minute hand lack as much of being at the 12 mark as the hour hand is beyond it?

95. May 1, 1856, I borrowed of Charles Rogers $6000 for one year, at 6 per cent interest, and the same day invested it in railroad stock at 20 per cent discount, the par value being $100 per share. July 1, 1856, I received a dividend of $3 per share; and, July 13, I sold the stock for cash at a discount of 10 per cent, sending the proceeds to a commission merchant in New York, with directions to invest it in sugar. This he did, after deducting his commission of 2 per cent on the purchase. Aug. 1, I paid $250 cash for freight and other expenses on the sugar; and, Sept. 1, I sold the sugar to Osgood & Bartlett for $1500 more than the broker paid for it, receiving in payment their note for $3000, due in 4 months, and the balance in cash. This note I got discounted Sept. 17; but when it became due it was protested, and I, as endorser, was compelled to pay it. May 1, 1857, I settled with Osgood & Bartlett for 30 cents on a dollar, and paid the amount due Charles Rogers. Money being worth 6 per cent per year, did I gain or lose by the transaction, and how many dollars?

96. Jan. 1, 1857, I had my note on 4 months for $6000 discounted at a bank, at 6 per cent interest and $\frac{1}{4}$ per cent exchange. With the proceeds I purchased a lot of cloth. March 1, I gave my note on 2 months for $750 in payment of expenses incurred on account of the cloth. March 15, I sold the cloth at an advance of 50 per cent on its first cost, receiving in payment a note on 6 months. When my other notes became due, I had the note received for the cloth discounted at a bank, at 6 per cent interest and $\frac{3}{8}$ per cent exchange, and with the proceeds met my liabilities. What was my net profit?

SECTION XXIV.

151. ACCOUNTS.

NOTE. — The form of Book-Keeping in **67** is designed for those who have but few business transactions to record. Traders and others having accounts with a large number of persons, will find the following form much better adapted to their wants. It requires two account-books, the DAY-BOOK and LEGER.

(a.) The Day-Book, or book of original entries, is ruled as illustrated on pages 96 and 97. At the top of each page is placed the date of the transactions to be recorded. Immediately below the date is written the name of the person with whom, on that date, we first have a transaction on account, followed by the abbreviation "Dr." or "Cr." as the case requires, and also by a statement of the items for which he is debited or credited. The amounts are carried into the money columns at the right. The next transaction in order is then written in the same way, then the next, and so on till the page is filled, or all the transactions of the day are recorded.

(b.) Should there be room left on the page for a part or all the transactions of another day, the new date should be written at the middle of a line, and the entries made beneath it, as just described. The space at the left-hand should be reserved for reference to the leger-page.

(c.) The Leger is used for collecting and arranging the entries of the Day-Book in such a way that each person's account may appear in a condensed form on a page by itself.

(d.) Each page of the Leger is divided into two equal parts, the left-hand part for the debit and the right-hand part for the credit entries. Each part should be divided into five spaces; viz. the first for the dates, the second for the transactions, the third for reference to the pages of the Day-Book on which the original entries are recorded, and the fourth and fifth for the money columns. The left-hand space is sometimes divided into two parts, one for the month and the other for the day of the month.

(e.) The items of the several accounts are not usually given in the Leger, it being sufficient to give the gross amounts, and refer to the Day-Book for particulars. If more than one article is bought on any date, the entry is usually "Mdse." or "Sundries." The specimen pages which follow, will sufficiently illustrate both the Day-Book and the Leger entries.

(f.) The transfer of an account from the Day-Book to the Leger, is called Posting the account.

Note.—The careful accountant will keep his account posted up as fully as possible. A habit of negligence in this respect is almost fatal to accuracy and dispatch in the management of our pecuniary affairs.

(g.) The "Cash Account" (**67**, f and g) may be kept in a separate book, precisely as described in **67**, or the cash entries may first be made in the Day-Book, and then be posted to the Leger, like those of any other account. In the following set of accounts, the students may adopt the latter method.

Note.—Those who do much business usually keep a separate Cash-Book. However kept, the Cash Account should be balanced frequently. (See note, top of page 95.)

(h.) The student may now suppose that, on the first day of July, 1858, he bought the stock in trade of a country merchant for $2000, and commenced business. The first step towards opening his accounts will be to make a careful inventory of his property, which should be recorded either on the first page of his Day-Book or Leger, or in a separate Inventory-Book. In this set, he may write on the first page of his Day-Book as follows:

(i.) Inventory of the assets and liabilities of ———, taken July 1, 1858—

I own cash $200; stock in trade $2000; 1 horse $150, I carriage $100. I owe nothing.

Note.—The above inventory requires an entry in Cash Account, which, with the Day-Book entries required by the following transactions, is illustrated on page 275.

July 2, 1858.

Sell John Ellis 6 lb. B. H. sugar at 8½ cts., 1½ gal. molasses at 48 cts., 4 lb. coffee at 12½ cts. Sell Mrs. Sarah Brown 9¾ yd. lawn at 31 cts., 2½ yd. cambric at 25 cts, ¾ yd. muslin at 42 cts., and 6 skeins of silk at 4 cts. Sell for cash 1 bbl. flour $6.75, 3 bu. meal at 67 cts., and sundries for $8.27. Buy for cash 123 lb. butter at 20 cts.

July 3.

Buy of George Russell 17 bu. of corn at 62 cts., and 20 bu of oats at 37½ cts. Sell Arthur Stanhope 1 pr. boots $2.67, 1 pr. mittens 92 cts., and 2 locks at 46 cts. Credit Edward Jones with 1½ days labor at $1.50. Let Edwin Smith your horse and carriage to go 9 miles at 16¾ cts. per mile. Receive cash for sundries $13.21.

July 6.

Buy for cash 20 bu. potatoes at 50 cts., and 8½ doz. eggs at 14 cts. Buy of Day & Austin 1 cask raisins, 98 lb., at 8¼ cts., 3 boxes raisins each 25 lb. at 12½ cts., and 26 lb. figs at 9 cts. Sell John Ellis 31¼ yd. sheeting at 11 cts. Sell Arthur Stanhope ½ lb. Y. H. tea at 68 cts., and Wm. Jackson 2 gal. molasses at 48 cts. Cash receipts for the day, $11.35.

July 7.

Sell Seth Turner 3 doz. eggs at 16⅔ cts., 2 bu. potatoes at 55 cts., 5 bu. corn at 68 cts. Sell John Harris 15½ lb. cheese at 9 cts., and 7½ lb. butter at 25 cents. Sell for cash 3 yd. broadcloth at $3.25, 1 bbl. flour $6.75, and sundries $9.13.

July 8.

Sell Mrs. Sarah Brown 8 lb. B. H. sugar at 9 cts., 2 lb. cocoa at 18 cts., 2 lb. white sugar at 12 cts. Cash receipts for the day, $15.

July 9.

Sell Edward Jones ¼ lb. tea at 56 cts., 2 hands tobacco at 5 cts., 1 bunch cigars 15 cts. Sell Alice Gray 8 yd. alpine at 96 cts., 4½ yd. cambric at 14 cts., 3 skeins silk at 4 cts., 1 yd. drilling 11 cts. Buy of Charles Hewins 1 bbl. crackers $3.75, 12 loaves bread at 4½ cts., and 5 doz. cakes at 20 cts. Sell Seth Turner 3 loaves bread at 5 cts., 2 doz. cakes at 25 cts. Sell sundries for cash, $10.13.

July 10.

Credit Edward Jones for labor, $1.12. Cash sales for the day, $23.71. Pay cash for sundries, $1.37.

(j.) Posting. This being the close of a week, it will be well to post up our accounts. To do this, look on the Day-Book for the first entry. As no separate Cash-Book is kept, it will be: "July 1.

Cash Dr. To cash on hand, $200." We shall, therefore, proceed to open Cash Account in the Leger. To do it, we write at the head of the first page: "Dr. Cash. Cr." Then on the left side of the page we write, first, July 1; second, To Cash; third, the figure 2, to show on what page of the Day-Book the charge is made; and, fourth, the amount, $200. We also write in the Day-Book, in the space at the left of the entry, the figure 1, to show that the account is posted to the first page of the Leger.

(**k.**) An Index to the Leger is also necessary. If our business is large, one or more pages must be appropriated to each letter of the alphabet; but, for the present set of accounts, three or four lines to each letter will be sufficient. To make it, write at the head of each page the letter A; then, four or five lines below, the letter B; and so on through the alphabet. Under C should be written: "Cash, page 1."

(**l.**) Now post up the other entries in Cash Account, and balance the account as explained in **67**. The specimen account, page 275, will illustrate this.

(**m.**) The careful book-keeper will now compare his Day-Book and Leger, to see if all the entries are posted correctly. He finds the first "Cash" entry in the Day-Book, and, if it is posted correctly, he places opposite to it a mark, as × or ✓. There will thus be found opposite each account in the Day-Book, which has been properly posted, not only the number referring to the Leger page, but the mark to indicate that it has been carefully examined.

(**n.**) The next Day-Book title being "John Ellis," his account should be opened on page 2 of the Leger, exactly as Cash Account was opened on page 1, and under E in the Index should be written: "Ellis, John, page 2." It will not be necessary to balance his account at present.

(**o.**) Proceed in the same way with each of the other accounts till all are posted, and then make the requisite Day-Book entries for the following transactions:

July 12.

Credit Francis Soule for shoeing horse $1.25. Sell Edward Jones 3 lb. sugar at 8 cts., and 1 lb. coffee at 13 cts. Sell Edwin

Smith 2 yd. broadcloth at $3.50, 2¾ yd. doeskin at $1.50, ¾ yd. satin vesting at $3.50, 1 doz. buttons for 42 cts., silk and trimmings for $1.13. Sell sundries for cash, $10.28.

July 13.

Buy of Charles Hewins 12 loaves bread at 4½ cts., 3 doz. cakes at 20 cts., and 4 loaves cake at 10 cts. Sell Freeman Hunt 2 scythes at $1.00, 2 rakes at 37 cts., 1 hay-fork 13 cts. Sell Francis Soule 2½ gal. molasses at 42 cts., 12 lb. B. H. sugar at 9 cts, 3 pk. rye meal at $1.20 per bu., 1 bu. Indian meal at 92 cts., and 1 lb. tobacco at 25 cts. Sell Mrs. Lucy Pond 8 yd. calico at 12½ cts., and ½ lb. tea at 50 cts. Cash receipts for day, $22.71.

July 14.

Buy for cash 12 bbl. flour at $6.25. Sell for cash 24 yd. oil-cloth carpeting at 58 cts. Sell John Ellis 15 lb. potash at 10 cts. Sell Seth Turner 2 loaves bread at 5 cts., 1 gal. oil at $1.17. Credit Edward Jones with labor 87 cts. Buy of Day & Austin 228 lb. sugar at 7 cts., 319 lb. sugar at 7½ cts., and 119 gal. molasses at 33 cts. Sell sundries for cash $11.54.

July 15.

Sell for cash 15 bu. corn at 70 cts., and 13 bu. oats at 41 cts. Sell George Russell 1 pr. calf-boots $3.75. Credit Francis Soule for repairing carriage $1.36. Charge Edwin Smith for use of horse $2. Sell John Harris 2 bu. 3 pk. salt at 63 cts. per bu. Credit Edward Jones with 12 qt. berries at 9 cts. Sell Mrs. Sarah Brown 1 mirror $1.25. Sell sundries for cash $14.25.

July 16.

Sell Alice Gray 1 pr. gaiter-boots $1.25, 2 yd. ribbon at 19 cts. Buy of Charles Hewins 18 loaves bread at 4½ cts., 2 doz. cakes at 20 cts. per doz., and 5 cakes at 10 cts. Pay cash for 1½ tons of hay at $15.50. Sell Edwin Smith 1 box cigars $5.75. Sell Wm. Jackson 1 piece sheeting, 29 yd., at 11½ cts., and 1 piece shirting, 30½ yd., at 13 cts. Sell John Ellis 1 box, 25 lb., raisins, at 15 cts. Credit George Russell for 1 pr. calf-boots returned $3.75, and charge him with 1 pr. patent leather boots $6.50. Cash sales for the day $19.36.

July 17.

Sell Mrs. Lucy Pond 3 lb. soap at 8 cts. Sell John Ellis 8 lb. salt fish at 5 cts., 6 lb. dried beef at 11 cts., 1 gal. molasses at 40 cts. Sell Alice Gray 1½ doz. cakes at 25 cts., 2¼ lb. cheese at 10 cts., and 3 lb. butter at 24 cts. Sell Seth Turner 8 lb. coffee at 11 cts., 2 lb. tea at 46 cts., and 12 lb. sugar at 9 cts. Buy of Mrs. Lucy Pond 7 doz. eggs at 13 cts., and 23 qt. berries at 7 cts.; and sell her 1 oz. nutmegs at 10 cts., and ¼ lb. cheese at 20 cts. Sell John Harris 9 qt. berries at 8 cts. Sell Arthur Stanhope 2 qt. oil at $1.55 per gal., 1 lamp 25 cts., 1 ball wicking 6 cts. Sell sundries for cash $22.31. Pay cash for family expenses $13.58.

(p.) Now post and balance Cash Account, and post your other accounts to date.

July 19.

Buy of Charles Hewins 18 loaves bread at 4½ cts., 2½ doz. cakes at 20 cts., 2 bbl. crackers at $3.50; and sell him 5 bu. potatoes at 60 cts. Credit Francis Soule for work $2.75. Sell John Ellis 1 bbl. flour $6.75, 1 coat $8.37, 1 pr. overalls 75 cts., 1 hat $2.31. Charge George Russell for goods delivered to his wife, as follows: 10 yd. muslin at 37 cts., 4 yd. edging at 20 cts., 1 silk belt 33 cts., 1¼ yd. cotton at 10 cts., 1½ doz. buttons at 16 cts., 1 card hooks and eyes 4 cts. Cash sales for the day $13.31.

July 20.

Sell John Ellis 2 pr. children's shoes at 83 cts., 2 pr. cotton hose at 18 cts., 3½ yd. print at 13 cts., delivered to his wife. Sell Edward Jones 11 lb. salt fish at 5 cts., 6 lb. rice at 5 cts., and 1 pk. meal 20 cts., ½ lb. tea at 46 cts. Pay cash for groceries for store $63.28. Sell John Harris 2 gal. vinegar at 20 cts., 2 lb. saleratus at 12 cts., 1 doz. cakes 25 cts., 2 loaves bread at 5 cts., crackers 25 cts. Cash sales for the day $22.13.

July 21.

Buy of Day & Austin 28 lb. Y. H. tea at 50 cts., 60 lb. Souchong at 35 cts., 98 lb. coffee at 10 cts.; and sell them 1 ream letter paper $3.50, ½ ream bill paper at $3.75, 4 packages envelopes at 10 cts., 1 bottle ink 37 cts., and 1 box pens $1.00. Charge John

Ellis with 1 Colburn's Common-School Arithmetic, delivered to his son Henry, 50 cts. Sell Edwin Smith 22 yd. carpeting at 62 cts., 1 paper tacks 10 cts., 1 small hammer 18 cts. Sundries for cash $18.27.

July 22.

Sell Mrs. Sarah Brown 1 lb. raisins 11 cts., 2 lb. sugar at 10 cts., 1 gal. molasses 38 cts. Sell Edward Jones 1 hand tobacco 5 cts. Sell Wm. Jackson ¼ lb. pepper at 20 cts., 1 pk. salt 24 cts. Buy of Charles Hewins 16 loaves bread at 4½ cts., 8 cakes at 10 cts. Credit Edward Jones with 3 days labor at $1.50; charge him with cash $1. Receive of John Harris cash to balance his account. Sell sundries for cash $16.25.

July 23.

Sell Walter Gay 1 bbl. flour $6.75, 1 bu. meal 84 cts., 8 lb. refined sugar at 12 cts., 12 lb. B. H. sugar at 9 cts., 4 lb. coffee at 13 cts., ¾ lb. tea at 56 cts., 2 lb. chocolate at 18 cts. Sell George Russell 1 axe $1, and 1 hoe 75 cents. Buy for cash 6 bu. beans at $2. Cash sales for the day $10.75.

July 24.

Charge Day & Austin with goods delivered Willard Gay per their order $12.50. Receive of Sarah Brown cash on account $5. Sell Edwin Smith 2 brooms at 25 cts., 1 brush 50 cts., ½ doz. knives and forks at $2.50 per doz. Sell Edward Jones 1½ lb. cheese at 10 cts. Sell Seth Turner 1 pr. thick boots $2.87. Sell Wm. Jackson 1 cap $1.25, and 1 vest $1.75; and credit him with labor $1.20. Sell Arthur Stanhope 1 pk. beans at $3 per bu. Sell sundries for cash $25.38, and pay cash for family expenses $5.23.

(q.) Now post up all your accounts and balance Cash Account.

July 26.

Sell Arthur Stanhope 7 yd. flannel at 58 cts. Sell Walter Gay 1 box, 50 lb., soap at 8 cts., and 1 tub $1.23. Buy of Mrs. Lucy Pond 6 doz. eggs at 14 cts., 13 qt. of berries at 6 cts. Sell Wm. Jackson 1 bu. Indian meal 70 cts. Sell George Russell 1 lb. saleratus 10 cts., and buy of him 25 bu. corn at 65 cts. Sell sundries for cash $9.83.

July 27.

Buy of Charles Hewins 12 loaves bread at 4½ cts., 7½ doz. cakes at 20 cts., and 3 loaves cake at 10 cts. Pay cash for 4 bu. apples at 87 cts, and for 1 ton 3 cwt. hay at $13.50 per ton. Sell Edwin Smith 1 shawl for wife $5.28, 1 set china for $8.27. Sell John Ellis 9 lb. sugar at 8½ cts., 3 lb. crushed sugar at 13 cts., 1 gal. molasses 38 cts., 2 doz. eggs at 16 cts., ¼ lb. cinnamon at 18 cts., ¼ lb. cloves at 16 cts., 2 oz. nutmegs at 12 cts., ¼ lb. ginger at 20 cts., and 1 lb. saleratus 10 cts. Sell sundries for cash $17.28.

July 28.

Exchange horses with Joseph Fisher, paying him cash $50 for difference in value. Buy of Day & Austin 1 hhd. molasses, 137 gal., at 29 cts., 31 gal. oil at 75 cts., 33¾ gal. oil at 92 cts., 100 lb. crushed sugar at 10½ cts. Sell Charles Hewins 1½ lb. tea at 48 cts., and 5 lb. coffee at 14 cts. Sell Alice Gray ¼ lb. tea at 48 cts. Sell Edward Jones 3 lb. salt fish at 5 cts., 1 salt mackerel 10 cts. Cash sales for the day $11.36.

July 29.

Buy for cash 3 doz. brooms at $2.25 per doz., 1 doz. clothes brooms at $1 per doz., ½ doz. hair brushes at 20 cts. each, ¼ doz. hair brushes at 33 cts. each, 1½ doz. tooth brushes at $1.20 per doz., 2 doz. tooth brushes at $1.75 per doz., and ¾ doz. tooth brushes at $3 per doz. Sell Walter Gay 1 pr. calf boots $3.75, 1 pr. slippers 75 cts., 2 pr. children's shoes at $1.12. Sell Mrs. Lucy Pond 1 piece tape 6 cts., 1 paper pins 10 cts., 2 skeins silk at 5 cts. Sell Francis Soule 1 bbl. flour $6.62. Cash sales for the day $23.37.

July 30.

You find a five-dollar gold piece on your way to your store. Sell Sarah Brown 28 lb. flour at 4 cts. Sell Charles Hewins 2 yd. superfine broadcloth at $3.75, ¼ yd. silk velvet at $4.25, 1 satin vest pattern $3, 3 yd. doeskin at $1.50, 1 doz. silk buttons 50 cts., sewing silk and trimmings $1. Sell Freeman Hunt 1 razor 75 cts., 1 strop 37 cts., 1 lather brush 25 cts., 1 cake soap 10 cts Charge John Ellis with goods sold his wife, as follows: 12 yd black silk at $1.05, 3 yd. cambric at 20 cts., sewing silk and trimmings 37 cts. Pay Day & Austin cash on account $50. Cash sales for the day $21.13.

July 31.

Sell Edwin Smith 1 umbrella $1.75, 2 lb. chocolate at 20 cts., 1 lb. tea 56 cts., 8 lb. cheese at 9½ cts. Sell Arthur Stanhope 1 bbl. flour at $6.62, 2 qt. molasses at 38 cts. per gal., 8 lb. sugar at 9 cts. Credit Edward Jones with 2½ days labor at $1.50. Charge Day & Austin for goods delivered Samuel Carey per their order $18.97. Sell Seth Turner 8 lb. salt fish at 5 cts., 2 qt. oil at $1 per gal., 1 lb. tobacco 25 cts., 1 bunch cigars 30 cts. Sell Wm. Jackson 1 coat $7.50, 1 pr. shoes $1.25. You find the owner of the gold piece you found yesterday, and return it. Pay cash $17.31 for family expenses, and pay your clerk for 1 month's services $33.33. Cash sales for the day $18.91.

(r.) Now post up all your accounts, and balance Cash Account.

Aug. 2.

Settle the accounts of George Russell, Arthur Stanhope, Edward Jones, and Seth Turner, with cash. Settle Alice Gray's account by an order on Day & Austin.* Settle Day & Austin's account by note on 60 days. Settle Edwin Smith's account by cash $15, and due-bill for balance. Settle John Ellis's account with note on demand. Close Cash Account and all the other accounts by passing balance to new account. (See second note on page 94.)

(s.) The balance due on each account may be determined by examining the Leger. When the account is *settled*, the closing entry should first be made in the Day-Book. Thus: "To or By Cash, Note, or Order (or whatever it may be), to Balance Account." It should then be posted to the Leger, and the account closed. When an account is closed by "Passing the balance to New Account," no entries are necessary in the Day-Book.

(t.) Fill the blanks in the following inventory with appropriate figures.

Inventory of the assets and liabilities of ———, Aug. 2, 1858.

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* She should be credited, and Day & Austin should be debited, with this order.

ASSETS.

Stock in Trade	$2025
Cash on hand (by Cash Account)	$
1 Horse	$ 200
1 Carriage	$ 100
1 Harness	$ 25
John Ellis owes me on Note	$
Edwin Smith owes me on Due-Bill	$
Mrs. Sarah Brown owes me on Account	$
Wm. Jackson owes me on Account	$
Charles Hewins owes me on Account	$
Francis Soule owes me on Account	$
Freeman Hunt owes me on Account	$
Walter Gay owes me on Account	$

LIABILITIES.

I owe Day & Austin on Note	$
I owe Mrs. Lucy Pond on Account	$
Excess of assets over liabilities	$

(u.) Compare this inventory with the inventory taken on commencing business, and see whether you have gained or lost money during the past month.

(v.) In practical life, we should not be likely to close all our accounts oftener than once a quarter, and perhaps not oftener than once in six months or a year. A great many merchants close all their accounts to the first of January or July of each year, though some only close them to the 1st of January, at which times they generally send bills (see form on page 88) to all who have not called to settle their accounts.

(w.) Should any student desire more practice in Book-Keeping than the above transactions require, he can readily extend his accounts by imaginary transactions.

Providence, July 1, 1858.

	Cash. Dr.		
× 1	To Cash on Hand	$200	00
	July 2.		
× 2	John Ellis. Dr.		
	To 6 lb. B. H. Sugar, @ .08½		51
	" 1½ gal. Molasses @ .48		72
	" 4 lb. Coffee @ .12½		50
		1	73
× 3	Mrs. Sarah Brown. Dr.		
	To 9¾ yd. Lawn @ .31	3	02
	" 2½ yd. Cambric @ .25		63
	" ¾ yd. Muslin @ .42		31
	" 6 skeins Silk @ .04		24
		3	95
× 1	Cash. Dr.		
	To 1 bbl. Flour	6	75
	" 3 bu. Meal @ .67	2	01
	" Sundries	8	27
		17	03
	Cr.		
× 1	By 123 lb. Butter @ .20	24	60

Dr. **Cash.** **Cr.**

1858.		L. P.			1858.		L. P.		
July 1	To Cash	1	$200	00	July 2	By Butter	1	$ 24	60
" 2	" Sundries	1	17	03	" 6	" Sundries	2	11	19
" 3	" "	2	13	21	" 10	" "	4	1	37
" 6	" "	2	11	35	" 10	" Balance		278	90
" 7	" "	3	25	63					
" 8	" "	3	15	00					
" 9	" "	3	10	13					
" 10	" "	4	23	71					
			316	06				316	06
July 10	To Balance		278	90					

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